Example

Evaluate,

$$\int x^2 \sin x \, dx$$

Solution

This problem requires us to use integration by parts to solve it. Recall that the formula for integration by parts is

$$\int u dv = uv - \int v du$$

Therefore we choose

$$\begin{array}{cc} u = x^2 & dv = \sin x \, dx \\ du = 2x \, dx & v = -\cos x \end{array}$$

Substituting into the formula yields

$$\begin{aligned} \int x^2 \sin x \, dx &= x^2(-\cos x) - \int (-\cos x) 2x \, dx \\ \Rightarrow \int x^2 \sin x \, dx &= -x^2 \cos x + 2 \int x \cos x \, dx \end{aligned}$$

The last integral in the above equation cannot be integrated directly so we must use integration by parts again to calculate $\int x \cos x \, dx$. We choose

$$\begin{array}{cc} u = x & dv = \cos x \, dx \\ du = \, dx & v = \sin x \end{array}$$

Applying integration by parts we now have

$$\begin{aligned} \int x^2 \sin x \, dx &= -x^2 \cos x + 2 \int x \cos x \, dx \\ &= -x^2 \cos x + 2 \left(x \sin x - \int \sin x \, dx \right) \\ &= -x^2 \cos x + 2x \sin x + 2 \cos x + c \end{aligned}$$

Problems

Evaluate the following:

1. $\int x \cos x \, dx$ [Solution: $x \sin x + \cos x + c$]
2. $\int x^2 \ln x \, dx$ $\left[\text{Solution: } \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c\right]$
3. $\int e^x \sin 4x \, dx$ $\left[\text{Solution: } \frac{1}{17}e^x \sin 4x - \frac{4}{17}e^x \cos 4x + c\right]$