Example

Determine $\dfrac{dy}{dx}$ for the following function,

$$x^3 + 3x^2y + y^2 = 0.$$

Solution

Differentiate each term separately with respect to x and we get

$$\begin{aligned}
\frac{d}{dx}(x^3 + 3x^2y + y^2 = 0) &= 3x^2 + \left(6xy + 3x^2\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0 \\
\Rightarrow \left(3x^2 + 2y\right)\frac{dy}{dx} &= -3x^2 - 6xy \\
\Rightarrow \frac{dy}{dx} &= \frac{-3x^2 - 6xy}{3x^2 + 2y} \\
\Rightarrow \frac{dy}{dx} &= \frac{-3x(x + 2y)}{3x^2 + 2y}
\end{aligned}$$

Problems

Determine $\dfrac{dy}{dx}$ for the following functions:

1. $3x^3 + 3xy + 4y^2 = x$ $\quad\left[\text{Solution: } -\left(\dfrac{9x^2 + 3y - 1}{3x + 8y}\right)\right]$
2. $3x + 3x^2y + 4x^3y^2 = 4$ $\quad\left[\text{Solution: } -\left(\dfrac{3(1 + 2xy + 4x^2y^2)}{x^2(3 + 8xy)}\right)\right]$
3. $2y^2 + 4xy^2 = 4y$ $\quad\left[\text{Solution: } -\left(\dfrac{y^2}{y + 2xy - 1}\right)\right]$