What is a Sequence?

Aim

To define what a sequence is.

Learning Outcomes

At the end of this section you will:

- Understand what a sequence is,
- Know how to represent sequences,
- Know that a sequence is a special type of function.

Definition: A sequence is a finite/infinite list of terms (or numbers) arranged in a definite order, that is, there is a rule by which each term after the first may be found.

Consider the following examples,

1. 1,2,3,4,5,...
2. 1,4,9,16,25,...

Notice the "..." at the end of the sequences. This infers that the sequence continues ad infinitum (up to infinity).

More precisely, a sequence is actually a function $f : \mathbb{N} \to \mathbb{R}$, where $\mathbb{N}$ is the set of natural numbers — 1,2,3,...

In example (1) above $f(n) = n$ for every $n \in \mathbb{N}$ and in example (2) $f(n) = n^2$ for every $n \in \mathbb{N}$.

The n^{th} term in the sequence is $f(n)$ where f is the function defining the sequence, but we more commonly refer to this as a_n instead of $f(n)$.

In example (1), $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, ... In example (2) $a_1 = 1, a_2 = 4, a_3 = 9, ...$

A sequence is sometimes written

$$a_1, a_2, a_3, a_4, ...\text{etc.}$$

Shorthand for such a sequence is

$$\{a_n\}_{n=1}^{\infty}$$

Using this notation the rule for obtaining each term is usually defined by the n^{th} term.

Example

List the first 4 terms of the following sequence,

$$a_n = n^2 + 1$$

The first four terms are a_1, a_2, a_3 and a_4. Therefore using the rule we get

$$a_1 = (1)^2 + 1 = 2; a_2 = (2)^2 + 1 = 5; a_3 = 10; a_4 = 17$$

Related Reading

Gersting, J.L. 2007. *Mathematical Structures for Computer Science.* 6^{th} Edition. Freeman & Company.

Morris, O.D. 1992. *Text & Tests 4.* The Celtic Press.