

## $S_N$ Notation

**Aim**

To introduce the  $S_N$  notation.

**Learning Outcomes**

At the end of this section you will:

- Understand what the  $S_N$  notation is,
- Know how to calculate the sum of  $N$  terms of a series.

$S_N$  is generally used to denote the sum to  $N$  terms of a series. In the series  $1 + 3 + 5 + 7 + 9 + \dots$ ,  $S_4 = 16$ . That is, the sum of the first 4 terms is 16.

In general  $S_N = a_1 + a_2 + a_3 + a_4 + \dots + a_N$

$$\Rightarrow S_N = \sum_{r=1}^N a_r$$

Example

Consider the sequence  $a_r = \frac{x^{r-1}}{(r-1)!}$  for any given  $x$ .

$$\{a_r\} = 1, x, \frac{x^2}{2!}, \frac{x^3}{3!}, \frac{x^4}{4!}, \dots$$

$$\begin{aligned} S_N(x) &= \sum_{r=1}^N a_r \\ &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \end{aligned}$$

Remember that  $x$  is a fixed real number.  $S_1, S_2, S_3, \dots$  define a sequence of numbers that are defined in terms of  $x$ . i.e.  $1, 1 + x, 1 + x + \frac{x^2}{2!}, 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}, \dots$

This is in fact a converging sequence whose limit is  $e^x$ . Therefore it is possible to define a limit in terms of a series,

$$S_\infty(x) = \sum_{r=1}^N a_r = e^x \quad (\text{in this example})$$

**Definition:** If a sequence  $\{a_r\}$  converges to a limit  $\ell$  we can write

$$\ell = \sum_{r=1}^{\infty} a_r$$

To test and see if a series converges we use the ratio test - see next section.

### Related Reading

Gersting, J.L. 2007. *Mathematical Structures for Computer Science*. 6<sup>th</sup> Edition. Freeman & Company.

Morris, O.D. 1992. *Text & Tests 4*. The Celtic Press.