

The Limit of a Sequence

Aim

To show how to calculate the limit of a sequence.

Learning Outcomes

At the end of this section you will:

- Know how to calculate the limit of a sequence.

We have already seen that all convergent sequences have a limit, that is, the sequence converges to a specific value.

In order for a given sequence $\{a_n\}_{n=1}^{\infty}$ to converge to a limit,

$$a_n \rightarrow L \text{ as } n \rightarrow \infty \quad (L = \text{some number}),$$

what we really mean is that the distance between a_n and L becomes almost nothing as n is increased to infinity,

$$|a_n - L| \rightarrow 0 \quad \text{as } n \rightarrow \infty.$$

Not all sequences have a limit. To find the limit of a sequence, if it exists, we must calculate

$$\lim_{n \rightarrow \infty} a_n$$

We have already seen the procedure for calculating the limit of a sequence - divide each term in the sequence by the highest power of “x” in the sequence and then take the limit of the resulting sequence.

Example 1

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^3}{x^2 - 4} &= \lim_{x \rightarrow \infty} \frac{\frac{x^3}{x^3}}{\frac{x^2}{x^3} - \frac{4}{x^3}} \\ &= \lim_{x \rightarrow \infty} \frac{1}{\frac{1}{x} - \frac{4}{x^3}} = \frac{1}{\frac{1}{\infty} - \frac{4}{\infty}} \\ &= \frac{1}{0 + 0} = \infty \end{aligned}$$

Example 2

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{x^2 + 3}{x^3 - 4} &= \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^3} + \frac{3}{x^3}}{\frac{x^3}{x^3} - \frac{4}{x^3}} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{1}{x} + \frac{3}{x^3}}{1 - \frac{4}{x^3}} = \frac{\frac{1}{\infty} + \frac{3}{\infty}}{1 - \frac{4}{\infty}} \\ &= \frac{0 + 0}{1 - 0} = \frac{0}{1} = 0\end{aligned}$$

Related Reading

Gersting, J.L. 2007. *Mathematical Structures for Computer Science*. 6th Edition. Freeman & Company.