Increasing & Decreasing Sequences

Aim

To define what is meant by saying that a sequence is increasing or decreasing.

Learning Outcomes

At the end of this section you will:

- Understand what an increasing sequence is,
- Understand what a decreasing sequence is,
- Know how to check if a sequence is increasing or decreasing.

Definition: Given that a_n is the n^{th} term of a sequence, then if $a_{n+1} > a_n$, for all n, the sequence is said to be increasing. This implies that

$$\frac{a_{n+1}}{a_n} > 1 \qquad \text{for all } n$$

Definition: Given that a_n is the n^{th} term of a sequence, then if $a_{n+1} < a_n$, for all n, the sequence is said to be decreasing. This implies that

$$\frac{a_{n+1}}{a_n} < 1 \qquad \text{for all } n$$

Look at the following sequence.

$$a_n = \{n\}_{n=1}^{\infty}$$

It is obvious to see that $a_{n+1} > a_n$ for any given value of n. Therefore this is an example of a increasing sequence.

On the other hand, if we look at the sequence

$$a_n = \{1 - n\}_{n=1}^{\infty}$$

it is obvious that this sequence is decreasing.

Is it possible for a sequence to neither increase nor decrease?

$$a_n = \left\{1 + \frac{(-1)^n}{n}\right\}_{n=1}^{\infty}$$

This is an example of a sequence that neither increases nor decreases. Look at the first few terms

$$a_1 = 0,\ a_2 = \frac{3}{2},\ a_3 = \frac{2}{3},\ a_4 = \frac{5}{4}$$

This sequence keeps bouncing backwards and forwards around 1 on the numberline and so neither increases nor decreases.

Related Reading

Gersting, J.L. 2007. *Mathematical Structures for Computer Science.* 6^{th} Edition. Freeman & Company.

Morris, O.D. 1992. *Text & Tests 4.* The Celtic Press.