Rules of Integration

Aim

To introduce the rules of integration.

Learning Outcomes

At the end of this section you will be able to:

- Identify the different rules of integration,
- Apply the rules of integration to find the integral of a given function.

The basic rules of integration are presented here along with several examples. The basic rules of integration, as well as several common results, are presented in the back of the log tables on pages 41 and 42.

Rule 1

Since the derivative of x^{n+1} is $(n+1)x^n$ it follows that

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

provided $n \neq 1$. Thus to integrate a power of x, **we increase the power by 1 and divide by the new power**.

This rule can be used to integrate any power of x except x^{-1}, since the integration of x^{-1} using this rule would result in division by 0, which is undefined.

$$\text{In this case } \int x^{-1} = \ln x.$$

Rule 2

$$\int a\ x^n dx = a \int x^n dx,$$

for any constant a. Any constant factor may be taken outside of the integration sign.

e.g. $\int 3x^2 dx = 3\int x^2 dx = \frac{3x^3}{3} + c = x^3 + c.$

Rule 3

$$\int a\ dx = ax + c,$$

where a and c are constants, since $\dfrac{d}{dx}(ax + c) = a$.

Thus, $\displaystyle\int 3\ dx = 3x + c.$

Rule 4

$$\int [f(x) \pm g(x)] = \int f(x) \pm \int g(x),$$

that is, if a function consists of two or more terms, each of the terms is integrated separately.

Example

Find

$$\int (x^3 - \frac{1}{x^2} + \sqrt{x})dx$$

$$\begin{aligned}\int (x^3 - \frac{1}{x^2} + \sqrt{x})dx &= \int (x^3 - x^{-2} + x^{\frac{1}{2}})dx \\ &= \frac{x^4}{4} - \frac{x^{-1}}{-1} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c \\ &= \frac{x^4}{4} + \frac{1}{x} + \frac{2}{3}x^{\frac{3}{2}} + c\end{aligned}$$

Related Reading

Jacques, I. 1999. *Mathematics for Economics and Business.* 3^{rd} Edition. Prentice Hall.

Morris, O.D., P. Cooke. 1992. *Text & Tests 5.* The Celtic Press.