Rates of Change

Aim

To explain the concept of rates of change.

Learning Outcomes

At the end of this section you will:

- Understand the difference between average speed and instantaneous speed,
- Understand that the derivative is a measure of the instantaneous rate of change of a function.

Differentiation can be defined in terms of rates of change, but what exactly do we mean when we say rates of change ? Consider the following example. Imagine you are driving from Limerick to Cork. You start your journey at midday and obey all the speed limits. Assume that when you reach your destination in Cork you have travelled exactly 100 kilometers and that it took you exactly 2 hours. It is easy to see that we have averaged 50 kilometers per hour during this journey. This means that your **average speed** was 50 km/h. If you had looked at your speedometer at a particular time during your journey you would have seen the **instantaneous speed** you were travelling.

The average speed of the car during the journey is measured by dividing the distance it has travelled by the time it has taken to travel that distance. In this example

$$\frac{\text{Distance travelled}}{\text{Time taken}} = \frac{100}{2} = 50 \text{ kilometers per hour}$$

Average speed is the rate of change of distance with respect to time and is calculated from the ratio of distance travelled to the time taken. Instantaneous speed, as opposed to average speed, is the rate of change of distance with respect to time *at a specific time* and because of this we cannot calculate instantaneous speed from a ratio because the denominator - at a specific time - is zero. To overcome this problem we calculate the derivative. The derivative of a function is a measure of the instantaneous rate of change of the function - that is, it is a way of measuring how the function changes at each separate point.

Formal Definition

The **average rate of change** of a function $f(x)$ with respect to x over an interval from a to $a+h$ is

$$\frac{f(a+h)-f(a)}{h}.$$

The (**instantaneous**) **rate of change** of f with respect to x at $x = a$ is the derivative

$$f^{'}(x) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h},$$

provided the limit exists.

It is conventional to use the word *instantaneous* even when x does not represent time, although the word is frequently omitted. When we talk about rates of change we are talking about instantaneous rates of change.

Related Reading

Booth, D.J. 1998. *Foundation Mathematics.* 3^{rd} Edition. Pearson Education Limited.

Adams, R.A. 2003. *Calculus: A Complete Course.* 5^{th} Edition. Pearson Education Limited.