

1. Find the divergence and the circulation of the following vector fields:

(a) $\mathbf{v} = (yz, zx, xy)$

(b) $\mathbf{v} = (y \sin z, z \cos x, \cos xz)$

(c) $\mathbf{v} = \mathbf{r} \ln |\mathbf{r}|$, where $\mathbf{r} = (x, y, z)$

(d) $\mathbf{v} = \ln(x + y + z) \mathbf{i} + \ln(x + y + z) \mathbf{j} + \ln(xyz) \mathbf{k}$

(e) $\mathbf{v} = (6x^2y - 4yz^3, 2x^3 - 4xz^3, -12xyz^2)$

2. Prove the identities:

(a) $\text{curl}(\mathbf{v} + \mathbf{w}) = \text{curl}(\mathbf{v}) + \text{curl}(\mathbf{w})$

(b) $\text{curl}(f \cdot \mathbf{v}) = f \cdot \text{curl}(\mathbf{v}) + (\text{grad}f \times \mathbf{v})$

(c) $\text{div}(f \cdot \mathbf{v}) = f \cdot \text{div}(\mathbf{v}) + (\text{grad}f \cdot \mathbf{v})$

(d) $\text{curl}(\text{grad}f) = \mathbf{0}$

(e) $\text{div}(\text{curl}\mathbf{v}) = 0$