1. Sketch the region of the following integrations and change of variables:

   (a) $\int\limits_{y=0}^{1} \int\limits_{x=y}^{\sqrt{y}} f(x, y)dxdy$

   (b) $\int\limits_{x=1}^{2} \int\limits_{y=x}^{2x} f(x, y)dydx$

   (c) $\int\limits_{x=0}^{4} \int\limits_{y=0}^{4x-x^2} f(x, y)dydx$

   (d) $\int\limits_{x=-1}^{1} \int\limits_{y=0}^{\sqrt{1-x^2}} f(x, y)dydx$

   (e) $\int\limits_{y=0}^{2} \int\limits_{x=y-1}^{y+2} f(x, y)dxdy$

   (f) $\int\limits_{x=0}^{1} \int\limits_{y=-x}^{x^2} f(x, y)dydx$

2. Compute the following integrals:

   (a) $\int\limits_{y=0}^{\frac{\pi}{2}} \int\limits_{x=1}^{2} x \sin^2 y dxdy$

   (b) $\int\limits_{0}^{3} \int\limits_{0}^{3-y} e^{2x+3y} dxdy$

   (c) $\iint\limits_{T} \frac{dxdy}{(x+y+1)^2}$, where $T = \left\{(x, y) \in \mathbb{R}^2, 0 \le x \le 1, 0 \le y \le 1\right\}$

   (d) $\int\limits_{0}^{1} \int\limits_{y}^{\sqrt{y}} \left(x^2 + \sin xy\right) dxdy$

   (e) $\int\limits_{0}^{1} \int\limits_{0}^{\pi} (\cos(y + x) + arctg 2x) \, dydx$

   (f) $\iint\limits_{x^2+y^2 \le 9} (5 - 3x + 4y) \, dxdy$

   (g) $\iint\limits_{x^2+y^2 \le 4x} \sqrt{16 - x^2 - y^2} dxdy$

   (h) $\iint\limits_{T} arctg \frac{y}{x}$, where $T = \left\{(x, y) \in \mathbb{R}^2, x^2 + y^2 \le 9, 0 \le y\right\}$.

3. Evaluate the following triple integrals and sketch the regions of the integrations:

   (a) $\int\limits_{x=0}^{1} \int\limits_{y=0}^{2} \int\limits_{z=0}^{3} dzdydx$

(b) $\int\limits_{z=-1}^{1}\int\limits_{y=0}^{1}\int\limits_{x=0}^{\sqrt{1-y^2}}(y+xz)\,dxdydz$

(c) $\int\limits_{x=0}^{1}\int\limits_{y=0}^{\sqrt{1-x^2}}\int\limits_{z=0}^{x^2+y^2}xydzdydx$

(d) $\iiint\limits_V \frac{dxdydz}{(1+x+y+z)^3}$, where the region V is bounded by the planes: $x+y+z=1$, $x=0$, $y=0$ and $z=0$

(e) $\iiint\limits_V \sqrt{x^2+y^2}dxdydz$, where the region V is bounded by the cone $x^2+y^2=z^2$ and by the plane $z=1$

4. Evaluate the following integrals. If it is necessary convert to polar-cylindrical or spherical coordinates.

(a) $\int\limits_{x=0}^{3}\int\limits_{y=-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int\limits_{z=-1}^{4}dzdydx$

(b) $\int\limits_{0}^{2}\int\limits_{0}^{\sqrt{2x-x^2}}\int\limits_{0}^{a}z\sqrt{x^2+y^2}dzdydx$

(c) $\int\limits_{-R}^{R}\int\limits_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\int\limits_{-\sqrt{R^2-x^2-y^2}}^{0}(x^2+y^2)dzdydx$

(d) $\iiint\limits_V \left(x^2+y^2\right)dxdydz$, where the region V is characterized by the inequalities $r^2\leq x^2+y^2+z^2\leq R^2$ and $0\leq z$

(e) $\iiint\limits_V \sqrt{x^2+y^2+z^2}dxdydz$, where the region V is bounded by the surface $x^2+y^2+z^2=z$

(f) $\int\limits_{0}^{1}\int\limits_{0}^{\sqrt{1-x^2}}\int\limits_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}}dzdydx$

5. Compute the volume of the solids which are bounded by the following surfaces:

(a) $z=x^2+y^2, \qquad z=2x^2+2y^2, \qquad y=x, \qquad y=x^2$

(b) $x^2+y^2+z^2=4, \qquad x^2+y^2=3z$

(c) $z=x+y, \qquad z=xy, \qquad x+y=1, \qquad x=0, \qquad y=0$

(d) $z=6-x^2-y^2, \qquad z=\sqrt{x^2+y^2}$

(e) $x^2+y^2+z^2=2az, \qquad x^2+y^2\leq z^2$

(f) $\left(x^2+y^2+z^2\right)^2=a^3x$

(g) $\left(x^2+y^2+z^2\right)^3=3xyz$