1. Partial derivatives revisited

   Given an n-variable real function, $f : D \to C$, where $D = \{(x_1, x_2, ..., x_n) \mid x_i \in \mathbb{R})\} \subseteq \mathbb{R}^n$, and $C \subseteq \mathbb{R}, n \geq 2$, we will have

$$\frac{\partial f}{\partial x_i}(x_1, x_2, ..., x_n) = \lim_{h \to 0} \frac{f(x_1, x_2, ..., x_i + h, ..., x_n) - f(x_1, x_2, ..., x_i, ..., x_n)}{h}$$

   .

   Ex.: $\frac{\partial f}{\partial x}(e^{\sin x} y) = e^{\sin x} y. \cos x$, while $\frac{\partial f}{\partial y}(e^{\sin x} y) = e^{\sin x}$.

   Problems

2. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ (in the last case $\frac{\partial f}{\partial z}$, too).

   (a) $f(x, y) = x^2 - 5xy + 3y^2$
   (b) $f(x, y) = e^{\sin x} y$
   (c) $f(x, y) = \frac{xy^2}{x+y}$
   (d) $f(x, y) = \ln(x^2 + y^2)$
   (e) $f(x, y) = x^{2y}$
   (f) $f(x, y) = e^{-x} y$
   (g) $f(x, y) = x \ln y + tg\,(xy)$
   (h) $f(x, y, z) = x^2 y - \frac{y}{z} + 2\sqrt[3]{\sin z}$

3. State the second derivatives of the following functions.

   (a) $f(x, y) = ch\,(xy) + x \ln y$
   (b) $f(x, y) = x^3 y^5 - 2x^2 y^3 + 8x$
   (c) $f(x, y) = e^{-\left(x^2+y^2\right)}$, $P_0(2, 0)$

4. Investigate the stationary points of the following functions, deciding in each case whether the point is a maximum or a minimum.

   (a) $f(x, y) = 3x^2 + 2xy + y^2$
   (b) $f(x, y) = xy$
   (c) $f(x, y) = x^2 - y + e^y$
   (d) $f(x, y) = e^{-\left(x^2+y^2-xy\right)}$
   (e) $f(x, y) = \frac{1}{x} + \frac{1}{y} + xy$
   (f) $f(x, y) = 4x^2 + 2xy - 5y^2 + 2$