Vector product of two vectors (cross product)

We define the internal operation type $V_3 \times V_3 \longrightarrow V_3$ in the following way:

Given any two vectors $\overrightarrow{v_1} = \langle a_1, b_1, c_1 \rangle$, $\overrightarrow{v_2} = \langle a_2, b_2, c_2 \rangle \in V_3$,their vector product, (named sometimes cross product) is:

$$\overrightarrow{v_1} \times \overrightarrow{v_2} = \langle b_1 c_2 - b_2 c_1, a_2 c_1 - a_1 c_2, a_1 b_2 - a_2 b_1 \rangle \in V_3.$$

This definition can be easier memorised if taking the following formal definition (using a formal determinant):

$\overrightarrow{v_1} \times \overrightarrow{v_2} = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$, where $\overrightarrow{i} = \langle 1, 0, 0 \rangle$, $\overrightarrow{j} = \langle 0, 1, 0 \rangle$, $\overrightarrow{k} = \langle 0, 0, 1 \rangle$ are the coordinate vectors already mentioned.

Properties

$\overrightarrow{v_1} \times \overrightarrow{v_2} \perp \overrightarrow{v_1}, \overrightarrow{v_1} \times \overrightarrow{v_2} \perp \overrightarrow{v_2}$.

$\overrightarrow{v_1} \times \overrightarrow{v_2} = -\overrightarrow{v_2} \times \overrightarrow{v_1}$ anti-commutativity

$\overrightarrow{v_1} \times (\overrightarrow{v_2} + \overrightarrow{v_3}) = \overrightarrow{v_1} \times \overrightarrow{v_2} + \overrightarrow{v_1} \times \overrightarrow{v_3}$ and

$(\overrightarrow{v_1} + \overrightarrow{v_2}) \times \overrightarrow{v_3} = \overrightarrow{v_1} \times \overrightarrow{v_3} + \overrightarrow{v_2} \times \overrightarrow{v_3}$ linearity

We can express the vector product of two vectors $\overrightarrow{v_1} = \langle a_1, b_1, c_1 \rangle$, $\overrightarrow{v_2} = \langle a_2, b_2, c_2 \rangle \in V_3$ as a vector perpendicular the same time on both vectors, and with the lenght: $|\overrightarrow{v_1} \times \overrightarrow{v_2}| = |\overrightarrow{v_1}| \, |\overrightarrow{v_2}| \sin\varphi$, oriented acording to the "right hand" rule, i.e. the vectors $\overrightarrow{v_1}$, $\overrightarrow{v_2}$ and $\overrightarrow{v_1} \times \overrightarrow{v_2}$ are in the similar position to the first 3 fingers on anybody's rigth hand.

Geometric application.

$|\overrightarrow{v_1} \times \overrightarrow{v_2}|$ = the area of the parallelogramm spanned by the vectors $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$.