

## Vector algebra- vector multiplied by scalar

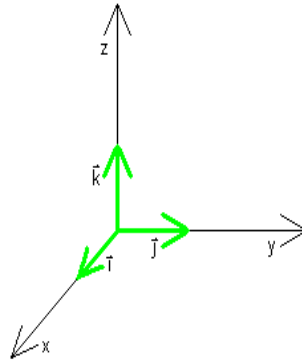
In the sequel we consider only the three dimensional Euclidian vector spaces, denoted by  $V_3$ .

Based on the usual notations in  $\mathbb{R}^3$ , a point  $P_0$  can be written in Cartesian coordinate form as  $P_0 = (x_0, y_0, z_0)$ .

We will denote by  $\overrightarrow{OP_0}$  the oriented line sequence, the position vector of the point  $P_0$ , and for which we introduce the similar coordinates  $\overrightarrow{OP_0} = \langle x_0, y_0, z_0 \rangle$ .

We will have for two different points  $P_1 = (x_1, y_1, z_1)$  and  $P_2 = (x_2, y_2, z_2)$  in  $\mathbb{R}^3$ , the oriented line sequence will be denoted by  $\overrightarrow{P_1P_2}$ , and we will use the coordinate form

$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$ , as obviously  $\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$ . The class of congruence  $\{ \overrightarrow{P_1P_2} = \langle a, b, c \rangle \mid P_1, P_2 \in \mathbb{R}^3 \}$  is by definition the vector  $\vec{v} = \langle a, b, c \rangle \in V_3$ . We mention 3 special vectors denoted  $\vec{i} = \langle 1, 0, 0 \rangle$ ,  $\vec{j} = \langle 0, 1, 0 \rangle$ ,  $\vec{k} = \langle 0, 0, 1 \rangle$ , the unit vectors of the 3 coordinate axis, named coordinate vectors.



### Basic vector operations

We have  $\vec{v} = \langle a, b, c \rangle = a\vec{i} + b\vec{j} + c\vec{k}$ .

#### Vector multiplied by a scalar

Let us take the field  $(\mathbb{R}, +, \cdot)$  and the Abelian group  $(V_3, +)$ . We define an external operation type  $\mathbb{R} \times V_3 \rightarrow V_3$ , i.e. the operation of multiplying the vector  $\vec{v} = \langle a, b, c \rangle \in V_3$  by the scalar  $\lambda$ , denoted by  $\lambda\vec{v} = \langle \lambda a, \lambda b, \lambda c \rangle \in V_3$ .

#### Properties

The vector  $\lambda\vec{v}$  will be parallel with  $\vec{v}$ , except for  $\lambda = 0$ .

Example. For  $\vec{v} = \langle 2, -1, 3 \rangle$  and  $\lambda = 5$  will furnish the vector  $5\vec{v} = \langle 10, -5, 15 \rangle$

We will be able to check this using e.g their vector product, see below.

Further properties:

$$\lambda(\vec{v}_1 + \vec{v}_2) = \lambda\vec{v}_1 + \lambda\vec{v}_2$$

$$(\lambda_1 + \lambda_2)\vec{v} = \lambda_1\vec{v} + \lambda_2\vec{v}$$

$$\lambda_1(\lambda_2\vec{v}) = (\lambda_1\lambda_2)\vec{v} \text{ and}$$

$$1\cdot\vec{v} = \vec{v}, \text{ where } 1 \text{ is the unit in } \mathbb{R}.$$