

1. Find the following limits, if they exist.

- a.  $\lim_{(x,y) \rightarrow (1,3)} e^{2x+y}$
- b.  $\lim_{(x,y) \rightarrow (\infty,1)} \frac{\arctan 2x}{y^2+1}$
- c.  $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin xy}{xy}$
- d.  $\lim_{(x,y) \rightarrow (0,0)} \frac{1}{x^2+y^2} e^{-\frac{1}{x^2+y^2}}$
- e.  $\lim_{(x,y) \rightarrow (3,1)} \frac{xy}{x^2+y^2}$
- f.  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$

2. Find the limits, if they exist, of the indicated branched functions.

- a.  $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$ , where  $f(x,y) = \begin{cases} y \sin \frac{1}{x}, & \text{if } x \neq 0, \quad y \in \mathbb{R} \\ 0, & \text{if } x = 0, \quad y \in \mathbb{R}. \end{cases}$
- b.  $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$ , where  $f(x,y) = \begin{cases} \frac{\sin xy}{xy}, & \text{if } x \neq 0, \quad y \neq 0 \\ 0, & \text{otherwise.} \end{cases}$
- c.  $\lim_{(x,y) \rightarrow (\frac{3}{\pi}, 0)} f(x,y)$ , where  $f(x,y) = \begin{cases} y \sin \frac{1}{x}, & \text{if } x \neq 0, \quad y \in \mathbb{R} \\ 0, & \text{if } x = 0, \quad y \in \mathbb{R}. \end{cases}$
- d.  $\lim_{(x,y) \rightarrow (2,2)} f(x,y)$ , where  $f(x,y) = \begin{cases} \frac{x^2-y^2}{x-y}, & \text{if } x \neq y \\ 2x, & \text{otherwise.} \end{cases}$

3. Determine the points for which the functions are continuous. At the points, where the function is discontinuous, tell why.

- a.  $f(x,y) = e^{x-3y}$
- b.  $f(x,y) = \frac{xy}{x^2+y^2}$
- c.  $f(x,y) = \frac{x^2y}{3+y^2}$
- d.  $f(x,y) = \ln(x^2 + y^2)$
- e.  $f(x,y) = \begin{cases} y \sin \frac{1}{x}, & \text{if } x \neq 0, \quad y \in \mathbb{R} \\ 0, & \text{if } x = 0, \quad y \in \mathbb{R}. \end{cases}$
- f.  $f(x,y) = \begin{cases} \frac{x^2-y^2}{x-y}, & \text{if } x \neq y \\ 2x, & \text{otherwise.} \end{cases}$

$$\mathbf{g.} \quad f(x, y) = \begin{cases} \frac{\sin xy}{xy}, & \text{if } x \neq 0, \quad y \neq 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathbf{h.} \quad f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{otherwise.} \end{cases}$$