# The Development of the Cluj (Kolozsvár – Klausenburg) School of Mathematics between 1872–1919

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## 1. Prolog

At the centurial commemoration of János Bolyai's birth in Kolozsvár (Cluj) the board of Hungarian Academy of Sciences announced the foundation of the Bolyai prize, intending to award it in five-year periods to a mathematician with outstanding achievements during the previous years. The person who first was honored with this prize was the French mathematician, Henri Poincaré. With regard to this Jenő Gergely, who once had been the disciple of Frigyes Riesz, then professor at the Bolyai University, related the following story.

Many prominent representatives of the Hungarian scientific life waited in excitement the arrival to the Eastern Railway Station in Budapest of the Paris express. One of the greatest scientists of those times was coming to Budapest to receive the Bolyai prize. After the welcoming speech Poincaré started to speak: "Where is Feujé", he asked. The Hungarians looked at each other in confusion. Who is Feujé? Suddenly they realized that Poincaré was talking about Lipót Fejér, a professor at the Kolozsvár (Cluj) university, who, despite of his young age (he was only 25 years old at that time), was one of the most famous Hungarian mathematicians of those times. His results related to the trigonometric series had been published in *Comptes Rendus* (Paris) when he had been only twenty years old. Since then the main periodicals published regularly important results achieved by Fejér. Thus we can easily understand Poincaré's wish to meet the young scientist during his short visit to Hungary. What was to be done? Thanks to an efficient mediation a train consisting only of an engine and one carriage started to rush within a few hours from Cluj to Budapest...

Whether it is true or not, the story is interesting, also because it reflects the condition of mathematics in Cluj at that time. Lipót Fejér was not the only mathematician, whose activity had a great influence on the scientific development in the 20$^{th}$ century, and who was working for a while at the university of Cluj. It is remarkable that it was the Franz–Josef University of Cluj, not that one in Budapest among Hungarian universities that first became an important centre of mathematics. During the decades preceding to the world war the school of mathematics in Cluj became one of the best centres of mathematics in the world. Its representatives gained imperishable merits for themselves and for the Hungarian scientific life. It is not probable that a week day passes unless mentioning somewhere in the world the name of Gyula Farkas, Lipót Fejér, Alfréd Haar or Frigyes Riesz.

How did mathematics in Cluj reach so high at the beginning of the 20$^{th}$ century? What personal conditions, social and economic facts, strategies aiming the scientific life and the development of education contributed to this development?

## 2. The first decades

In 1872, when the university of Cluj was established, the situation concerning mathematics wasn't promising at all. Apart from the activity of the Bolyais the mathematical literature of that time was quite poor here. The main event worthy to mention is related to Sámuel Brassai, who translated into Hungarian the book of Euclid called *Elements*.

The first professors in Mathematics and Mathematical Physics were **Sámuel Brassai** (1800–1897) – a former high-school teacher at Cluj, **Lajos Martin** (1827–1897) – former

officier of the engineer corps, **Mór Réthy** (1848–1925). The first two of them have not any worth mentioning original mathematical result.

Mór Réthy was one of the extremely talented young mathematicians, who acquired thorough knowledge and doctor's degree at famous German universities and who started to work at the university of Cluj. He studied at the universities of Vienna, Göttingen and Heidelberg. He took his doctorate in Heidelberg in 1874. He became in that year extraordinary professor at the university of Cluj, after two years the full professor at the Department of Mathematical Physics. He was the head of the Department of Elementary Mathematics between 1884-1886. After that he moved to the Technical University in Budapest. He was one of the first Hungarian professors in theoretical physics.

In 1874 he presented a lecture in Cluj on Bolyai's geometry; this was the first lecture on this issue in Hungary. We note here that the excellent Austrian mathematician, Johannes Frischauf already had course-like lectures in the academic year of 1871-72 at the university of Graz on the non-Euclidean geometry. These lectures comprised the first detailed overview of János Bolyai's brilliant work, the *Appendix*. The lectures were published in Leipzig in 1872 under the title *Absolute Geometrie nach Johann Bolyai*. This booklet was the only treaty for a long time, which constructed the absolute geometry using elementary methods, relying on the *Appendix.*

We continue the enumeration of important personalities of the university with **Gyula Vályi (1855–1913)**, the most famous disciple of Réthy. He was the first important mathematician teaching at the university of Cluj, who came from Transylvania. He graduated his studies in 1877 in Cluj, then with the support of the university attended the lectures of Weierstrass, Kirchhoff, Kronecker, Borchardt and Kummer in Berlin for four semesters. He sustained his thesis entitled *Contributions to the theory of second-order partial differential equations* in 1880 in Cluj. He became an associated professor in 1881 at the university of Cluj. He was promoted full professor in 1884 of mathematical physics, then in 1886 of elementary mathematics..

According to the curriculum of the university it was Gyula Vályi who first presented course-like lectures on the *Appendix* in the second semester of the 1891-1892 academic year. He repeated this popular course almost invariably every four years. The lithographed lecture notes of 102 pages were published in 1904 in Cluj, with the title *Bolyai János Appendixéről (On János Bolyai's Appendix)*. He completed the original demonstrations by explanations. When comparing the absolute and the hyperbolic geometry he made use of Lobatschewsky's results in some places.

It is also due to Gyula Vályi's enthusiastic work that Cluj became the centre of the Bolyai cult, and many of his disciples had achievements in the further development of the Bolyai geometry.

### 3. New impulses

At the beginning of the last decade of the 19$^{th}$ century important changes occurred in the university life. By that time the number of departments increased to 51, the members of the teaching staff to 68, while the number of the scientific employees comprising the teaching assistants and trainees to 119. During the first 25 years the number of students increased from 233 to 702. Similarly the number of mathematicians also increased.

We must mention the good result of the Hungarian high-school reforms. The law on public education linked with the name of József Eötvös meant major changes in the field of mathematics and physics too. A modern network of schools was built up by the end of the nineties, where the education became demanding and effective. Excellent high-school teachers showed up, whose tireless work won many young people over to sciences.

Loránd Eötvös (1848–1919) and Gyula Kőnig (1849–1913) founded in 1891 the Society of Mathematics and Physics. Loránd Eötvös formulated the purpose of the Society as follows: "Let's learn from each other, thus we teach better." The teacher training colleges and training schools established on the basis of the concepts of Mór Kármán (the father of the world-famous physicist, Tódor Kármán) next to the universities in Budapest and Cluj (like the Demonstration School in Budapest) resulted in the increase of the teachers' training level. The specific Hungarian teacher-training model was completed with the establishment of the Eötvös College. In this college talented students could learn under the guidance of famous professors.

The "Hungarian wonder", the *Középiskola Matematikai Lapok* (High School Mathematics Journal) launched by the young professor from Győr, Dániel Arany in 1893 played an important role in mathematics and physics training. Among such periodicals this is the journal lasting for the longest time in the world. (After three years the legendary teacher of the Lutheran College in Budapest, László Rátz took over the editing of the journal.) It turned out very soon how useful was when gifted young people were racking their brains on dozens of exercises for years and they wrote down their thoughts. Part of the students educated by the journal called today officially KöMaL became scientists, the others "only" very good professionals or professors. The best Hungarian mathematicians of the 20$^{th}$ century come among the students solving the exercises, like those who developed mathematics in Cluj before the First World War to the state-of-the-art.

**Gyula Farkas (1847–1930)** studied at the university of Pest, where Ányos Jedlik had a great influence upon him. Among his youthful achievements in mathematics we mention here only his examinations referring to Farkas Bolyai's trinom equation, which had referred to the algorithm for the approximation of the roots briefly presented in the *Tentamen*. Thus the Bolyai algorithm became well known, and many Hungarian and foreign mathematicians have been interested in its generalization, applicability, and the related convergence issues, even in our days. Gyula Farkas was an associated professor until 1887 in Pest, when he was nominated extraordinary professor at the university of Cluj. In 1888 he became a full professor at the university, he held this position until 1915.

As a professor his interest focused mainly on problems related to theoretical physics, but he elaborated the background of the examined physical problems in such deepness that there are classical mathematical results as well among these elaborations. Dating from the nineties he was particularly preoccupied by the Fourier principle of mechanics. His studies specify the necessary condition of balance in case of conditions given by inequalities. For this he demonstrates his theorem on linear inequality, which is one of the most famous Hungarian mathematical achievements, known as the *Farkas-lemma*. On the basis of these works it is obvious today that Gyula Farkas was one of the modern creators of the optimisation theory.

**Lajos Schlesinger (1864–1933**) was the best known professor abroad at the turn of the century. He studied in Heidelberg and Berlin. He was the disciple and the son-in-law of the famous professor from Berlin, Lazarus Fuchs, that is why people said about him that *"the talent is inherited mainly by the son-in-law"*.

In 1897, when he became a full professor at the university of Cluj, he already was a well known scientist. The Teubner Publishing House in Leipzig had published the first two volumes of his famous book (*Handbuch der Theorie der Linearen Differentialgleichungen)* that he had written as the associate professor of the Berlin university. He published the third volume in 1898 as a professor in Cluj. The society of mathematicians knew him by that time as one of the prominent authorities in the theory of differential equations built on the complex analysis. Many parts of this significant theory were enriched by Schlesinger.

After arriving to Cluj he got involved in education with a great enthusiasm besides the continuation of his research. Fifteen of his lecture notes still can be found in the mathematics library in Cluj.

These lectures are fascinating due to their clear, distinct style and the accurate treatment of the referred latest results. There is no doubt that the above-mentioned lectures belong to the best results of mathematical education in Cluj.

The book entitled *Vorlesungen über lineare Differentialgleichungen* and published in 1908 by the Teubner Publishing House can be considered an important result of Lajos Schlesinger's scientific activity carried out in Cluj. This is not the revision of the theory elaborated in the above-mentioned *Handbuch*, but a treatment of linear differential equation systems using totally new methods. This is the first monograph treating the linear differential equation systems with variable coefficient using the product integral as interpreted by the Italian mathematician, V. Volterra.

It can be considered a recognition of his authority that the Teubner Publishing House published in 1909 his book entitled *Bericht über die Entwicklung der Theorie der linearen Differentialgleichungen seit 1865,* edited by the German Mathematics Society. He wrote his famous book, *Automorphe Funktionen* that still can be found in the mathematics library in Cluj as a professor at the university of Giessen.

During his stay in Cluj Schlesinger contributed significantly to the advancement of the local mathematics. Together with Gyula Farkas and Gyula Vályi he had a basic role in the establishment of an excellent mathematics library within the university.

**Lipót Klug (1854–1944**) took a diploma in mathematics at Budapest. He was a professor in Pozsony (Bratislava), then in Pest. Between 1897–1917 he was a professor at the department of descriptive geometry at the university of Cluj.

**4. The peak**

Besides his up-to-day valid scientific activity Gyula Farkas had a predominant role in the development of the Mathematics Seminary in Cluj. Started out almost from nothing the institute became after a quarter of century one of the best scientific workshops of the Monarchy. Due to his commitment to sciences and outstanding virtues Farkas had a great authority among his colleagues and students, which he made use of in the university administration. He was dean for seven times and once even the rector of the university. His word was always decisive. He was exigent, he expected much from himself and others. He appreciated the human values, and helped a lot those deserving it. He made use of his influence to improve the material and personal conditions of the university activity. It was due exclusively to his intercession that such brilliant young mathematicians came to the university like Lipót Fejér (1905), Frigyes Riesz (1911) and Alfréd Haar (1912), who together with the elder professors raised the quality of the mathematics training to such a high level in the first decades of the 20$^{th}$ century, that this level, considering the international reputation of the professors, would be hard to surpass.

Who were the above-mentioned representatives of the new wave?

**Lipót Fejér (1880–1959)** studied at the Technical University of Budapest, but after a semester he moved to the so-called general section, as a student in mathematics and physics. There he attended mainly the lectures of Gyula Kőnig, József Kürschák and Gusztáv Rados, then he moved to the University of Budapest. During the academic year 1899-1900 he studied at the University of Berlin, where he attended the courses of L. Fuchs, G. Frobenius and H. A. Schwarz. Once returned home he spent the 1900-1901 academic year at the University of Budapest. He published in *Comptes rendus* his theorem on Fourier's series that made his name all at once worldwide known. He wrote his dissertation in mathematics on Fourier's series, in physics on the phenomena of diffraction. From September 1$^{st}$, 1901 he

was an associated professor at the University of Budapest, in spring 1902 he acquired the doctor's degree there. He spent the first semester of 1902–1903 in Göttingen, where he attended mainly the lectures of D. Hilbert and H. Minkowski. The spent the second half of the same year in Paris, attending the lectures of E. Picard and J. Hadamard. He had nine publications until 1905, three among these in the *Comptes rendus*, one in the *Matematische Annalen*. After such preliminaries, as Gyula Farkas expressed it, no one would have liked to "loose him in favour of some foreign universities". The university of Cluj ensured him the possibility of advancement. He was employed as an associate professor at the Department of Mathematical Physics (its head being Gyula Farkas). Besides the proofs of his brilliant talent in mathematics a reason for this could have been that he had learnt physics earlier, and he was interested in theoretical mechanics. Following his habilitation on June 23rd, 1905 at the Faculty of Mathematics and Natural Sciences of the Franz-Josef University with his thesis called *Stability and instability examinations in the mechanics of mass point system* his star was raising rapidly. In the first half of the 1905–1906 academic year he was the assistant lecturer of Lajos Schlesinger, in the second semester the associate professor of analysis and analytical mechanics, in September 1906 senior lecturer, and in 1911 extraordinary professor. He finished his activity at the university of Cluj in summer that year, as from September he became full professor at the University of Budapest. In 1908 they elected him a correspondent member of the Hungarian Academy of Sciences. During his stay in Cluj he published around 30 scientific treaties, mainly in famous journals of mathematics. His treaties relate to Fourier's series, theoretical mechanics and analytical functions.

**Frigyes Riesz (1880–1956),** who directed the Department of Higher Mathematics between 1912 and 1919 started his university studies at the Technical University of Zurich. However his vocation for science prevailed over the attraction to make a career as an engineer, thus he continued his studies from 1899 at the University of Budapest, and then spent a year in Göttingen. In Budapest the lectures of Gyula Kőnig and József Kürschák, in Göttingen the lectures of Hilbert and Minkowski had the greatest influence on him. In 1902 he was conferred a doctoral degree in Budapest. He published his first scientific discovery, the so-called Riesz-Fischer theorem when he was 27 years old. Later the identity of the two composition of the quantum mechanics, the wave mechanics of Schrödinger and the matrix mechanics of Heisenberg has been proved using this theorem. It is also linked to Riesz the widely applied discovery of the space of the integrable function on the $p^{th}$ exponent, the representation of the linear functionals defined on the set of the continuous functions in the form of the Stieltjes integral, respectively the definition of the dual of the Hilbert spaces. In 1908 he defined in his presentation held at the international mathematics conference in Rome the term of the topological space. Due to all these youthful discoveries he became famous before coming to Cluj.

Frigyes Riesz is not only one of the excellent Hungarian mathematicians, but he is considered worldwide as one who had the greatest effect on the grounding and development of modern branches of mathematical analysis. The terms and methods he had introduced, the achievements related to him belong today to the classical material of the real function theory, the functional analysis and the general topology. These results were partly included in his famous book written jointly with Béla Szőkefalvy-Nagy entitled *Leçons d'analyse fonctionnelle*, which was translated into several languages, and is known worldwide.

The last young mathematician who came to Cluj from the present territory of Hungary was **Alfréd Haar (1885–1933)**. He studied in Budapest and Göttingen. He attended the lectures and seminars of Beke, Eötvös, Frölich, Kürschák, Rados, Scholtz in Budapest, respectively of Carathéodory, Hilbert, Klein, Minkowski, Prandtl, Runge, Schwarzschild, Voigt and Zermelo in Göttingen. He took his doctor's degree under the guidance of Hilbert. After that he was an associate professor at the Technical University in Zurich. In 1912 he

was nominated to one of the physics department at the university of Cluj, first as extraordinary professor, then in 1917 as full professor. His work had an acknowledged effect on the modern development of mathematics. His talent was linked with the conscientiousness of the real scientist. His research covered a wide area. His results related to the systems of orthogonal functions, to the variation calculus, to the singular integrals, to the theory of sets, the function approximation, the linear equations and the topological groups are famous even today. The terms Haar's basis, the function systems having Haar features, but especially the term Haar's integral are definitely part of the every-day tools of mathematicians. The present effect of Haar's results are well proved by the lectures kept at the centenary scientific meeting in 1985, which are collected in the volume entitled *Alfred Haar Memorial Conference*.

**5. Epilogue**

In 1919 the university of Cluj entered under Romanian authority, therefore most of the professors – like Alfréd Haar and Frigyes Riesz – moved to Szeged, where they laid the foundations of a new university. In the meantime the Hungarian "mathematicians producing machine" continued its operation. A short time after the war great scientists showed up, like Tódor Kármán, György Pólya, Gábor Szegő, John von Neumann and others. However they didn't start towards the eastern regions of the Carpathian Basin anymore, alike their colleagues a few years before, but towards... But this one is another story to tell.