## Using the Del operator

Let us introduce the real valued vector function (or 3 variable scalar function) type $F : V_3 \longrightarrow \mathbb{R}$, for which $F(\langle x, y, z\rangle) = F(x, y, z) \in \mathbb{R}$, and the vector valued vector function (shortly vector function) type $\overrightarrow{w} : V_3 \longrightarrow V_3$, for which $\overrightarrow{w}(\langle x, y, z\rangle) = \overrightarrow{w}(x, y, z) = \langle F_1(x, y, z), F_2(x, y, z), F_3(x, y, z)\rangle$, where $F_i(x, y, z)$ $(1 \leq i \leq 3)$ are of the first type.

We define three operations using a formal vectors, called the vector Nabla, Del operator or Nabla operator, denoted $\overrightarrow{\bigtriangledown} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$.

The gradient of a 3 variable scalar function $F : V_3 \longrightarrow \mathbb{R}$, for which $F(\langle x, y, z\rangle) = F(x, y, z) \in \mathbb{R}$, is the vector we obtain by mulpilying the vector $\overrightarrow{\bigtriangledown}$ by the scalar $F$. In other words $\overrightarrow{grad\ F} = F \cdot \overrightarrow{\bigtriangledown} = \left\langle \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right\rangle$.

The divergence of a vector function $\overrightarrow{w} : V_3 \longrightarrow V_3$, for which $\overrightarrow{w}(\langle x, y, z\rangle) = \overrightarrow{w}(x, y, z) = \langle F_1(x, y, z), F_2(x, y, z), F_3(x, y, z)\rangle$ by the following "scalar product" $div\ \overrightarrow{w} = \overrightarrow{\bigtriangledown} \cdot \overrightarrow{w} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$.

The curl (rotation) of a vector is $curl\ \overrightarrow{w} = \overrightarrow{\bigtriangledown} \times \overrightarrow{w} = \begin{matrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{matrix}$