Calculus WIZ and *The Mathematical Explorer*

Acquiring *Calculus WIZ* and *The Mathematical Explorer*

Both *Calculus WIZ* and *The Mathematical Explorer* are stand-alone software products for exploring mathematical ideas from various fields of mathematics (see below in more detail). They are commercial products developed by Wolfram Research, Inc. (http://wolfram.com/) and are sold by local resellers worldwide. More information about your local reseller can be retrieved from http://store.wolfram.com/. There are currently versions available for Windows and Mac OS.

Installation

The products are distributed on a CD in a box. Booklets containing installation instructions are enclosed. Both products require a license number and valid password to run with full functionality. These numbers are printed on the cover of the CD. In general, the installation process is very easy and the installation Wizard will guide you throughout the installation process.

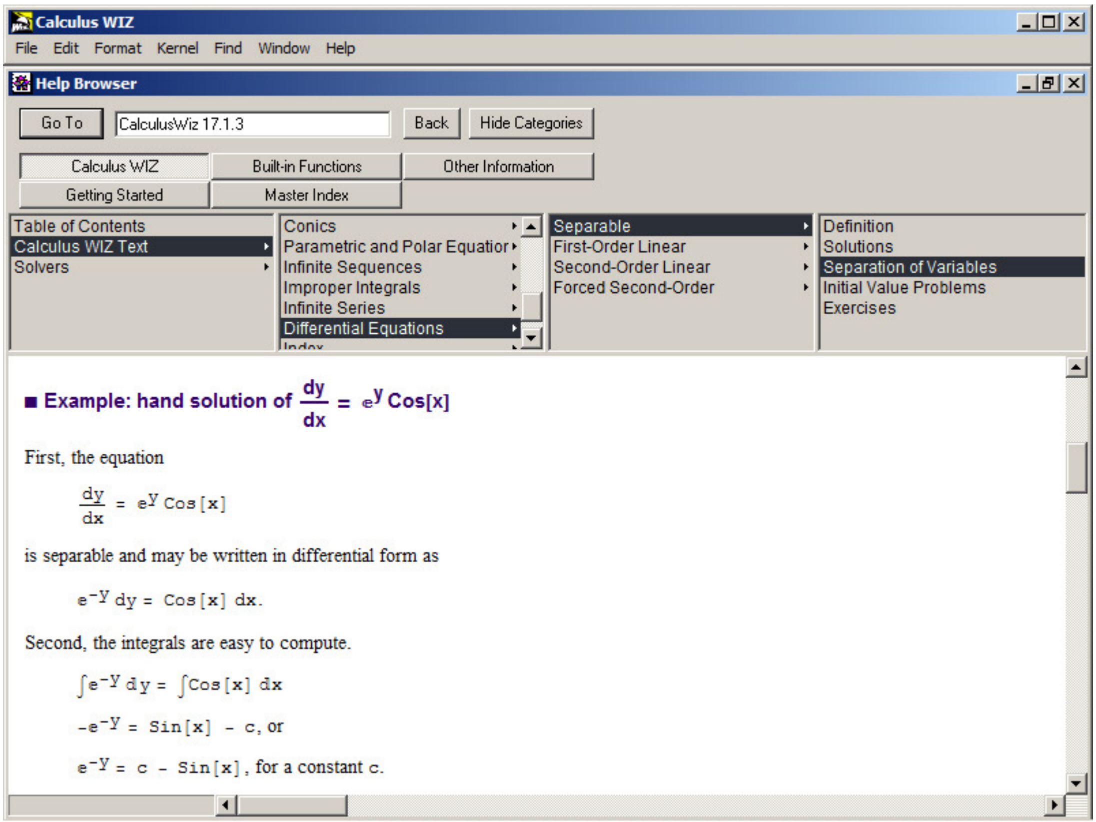
Calculus WIZ

Calculus WIZ is a tool that can provide help in solving most of the mathematical problems in the traditional first-year calculus course. Calculus problems can be solved simply by clicking a computer button and filling in information in a form. *Calculus WIZ* combines problem solving forms (capable of solving most textbook problems) with an introductory survey of underlying theory (e.g. mean value theorem, convergence of power series etc.) and some information beyond that provided by the usual textbooks. *Calculus WIZ* is a stand-alone product based on established *Mathematica* technology (also from Wolfram Research, Inc.). The Mathematica 4 Kernel is used as computing engine. As with most Wolfram products it uses the Mathematica Notebook hypertext document that combines text, graphics, and formulas in an easy-to-use interface that is completely interactive and allows creation of animations. Mathematical typesetting can be entered either using special keyboard shortcuts or by using palettes.

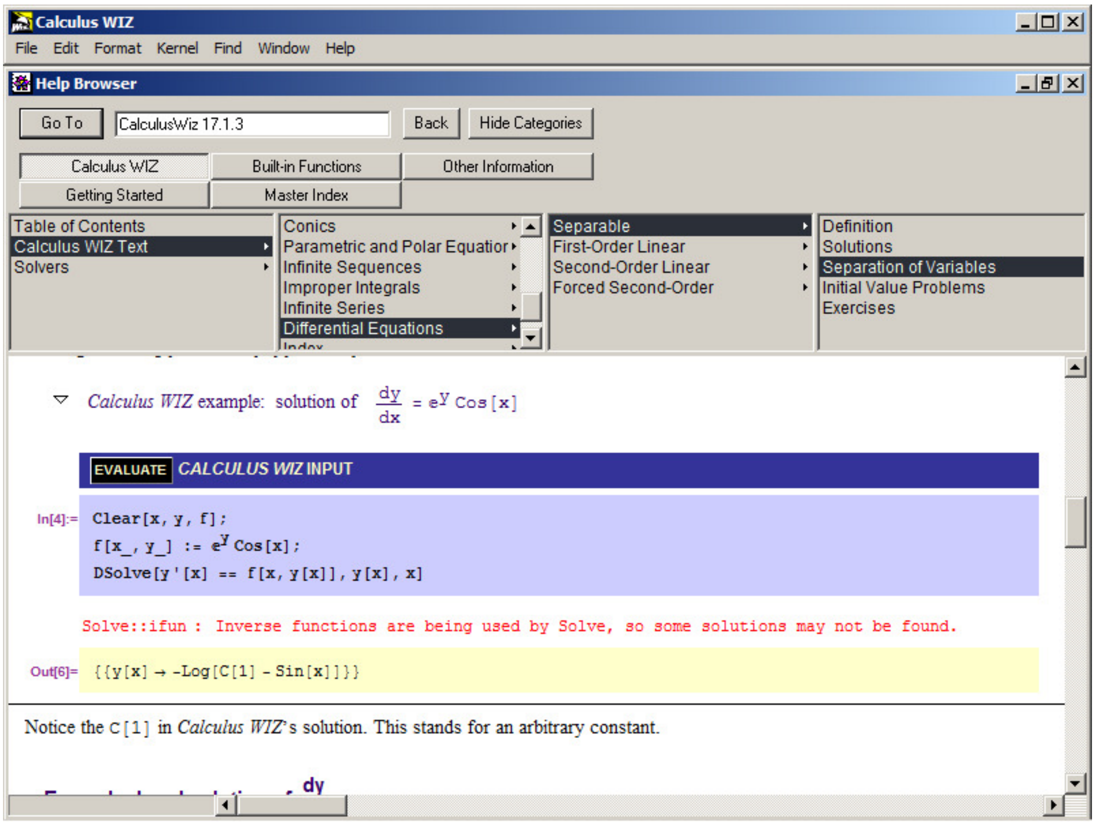
Calculus WIZ is organized in chapters and sections that follow the outline of regular calculus textbooks: Functions and Graphs, Limits, Differentiation, The Mean Value Theorem, Curve Sketching, Applications of Derivatives, Integration, Applications of Integration, Transcendental Functions, Techniques of Integration, Parametric and Polar Equations, Infinite Sequences, Improper Integrals, Infinite Series, Differential Equations.

Chapters and sections are easily accessible from the Help Browser and built-in hyperlinks throughout *Calculus WIZ*. Although *Calculus WIZ* is a complete reference to calculus, as with any calculus textbook, it is mainly intended to assist with homework. It contains a template for teachers to generate homework to each student individually.

Calculus WIZ is organized in the Help Browser by title, chapter, section, and subsection. The Homework with the *Calculus WIZ* notebook has examples of medium-difficulty textbook exercises. They are solved in three different ways: with automatic *Calculus WIZ* solvers, by textbook methods, and with short *Calculus WIZ* template programs. The three solutions let you compare the approaches. Here is an example from the Help Browser. The hand calculation is explained as follows.



This is the way the same problem can be solved by short *Calculus WIZ* template program:



The Mathematical Explorer

The Mathematical Explorer is an electronic book divided into 15 chapters: Prime Numbers, Calculus, Formulas for Computing π, Square Wheels, The Power of Check Digits, Secret Codes, Recreational Mathematics, Exploring Escher Patterns, Varieties of Roses, Turtle Fractalization, Patterns in Chaos, Fermat's Last Theorem, The Riemann Hypothesis, Unusual Number Systems, and The Four Color Theorem. Each chapter has several subchapters. Much more mathematical topics are covered than one would expect from the chapter titles. The reader will become familiar with continued fractions, Diophantine equations, modular arithmetic, the Buffon Needle Problem, Fibonacci numbers, the Brachistochrone Problem, space filling curves etc. Each chapter is endowed with historical remarks and short biographies of the greatest mathematicians that contributed to a given subject (including Euclid, Fermat, Gauss, Euler, Newton, Riemann, Wiles and many more). In the Help Browser, Section Demos, one will find many interesting examples of symbolic-numeric computations such as (Thirty-Three Representations of Catalan's Constant, Series Solution of Newton's Equation, Computation of the Multipole Field etc.). These demos come as additional information and are not related to the main topics of the book.

The Mathematical Explorer is a stand-alone product based on the *Mathematica* technology (the *Mathematica* 4 Kernel is used as computing engine). The intended primary user interface to *Mathematical Explorer*'s electronic text is through *Mathematica*'s Help Browser. This allows one to select chapters, sections and subsections through a mouse-click interface. It also allows one to search for concepts, mathematician biographies, references, etc., and it also provides reference material on much of *Mathematica* itself. In order to explore or experiment, the user is directed to click on a *Mathematica* expression (or program) which is activated by holding down the Shift key and pressing Enter. The results can be impressive — graphs and tables are created, often a complicated algebraic expression is manipulated and simplified and consequently tedious hand calculations are eliminated. The text is written in an approachable and friendly tone. The reader is challenged with a number of exercises covering each topic.

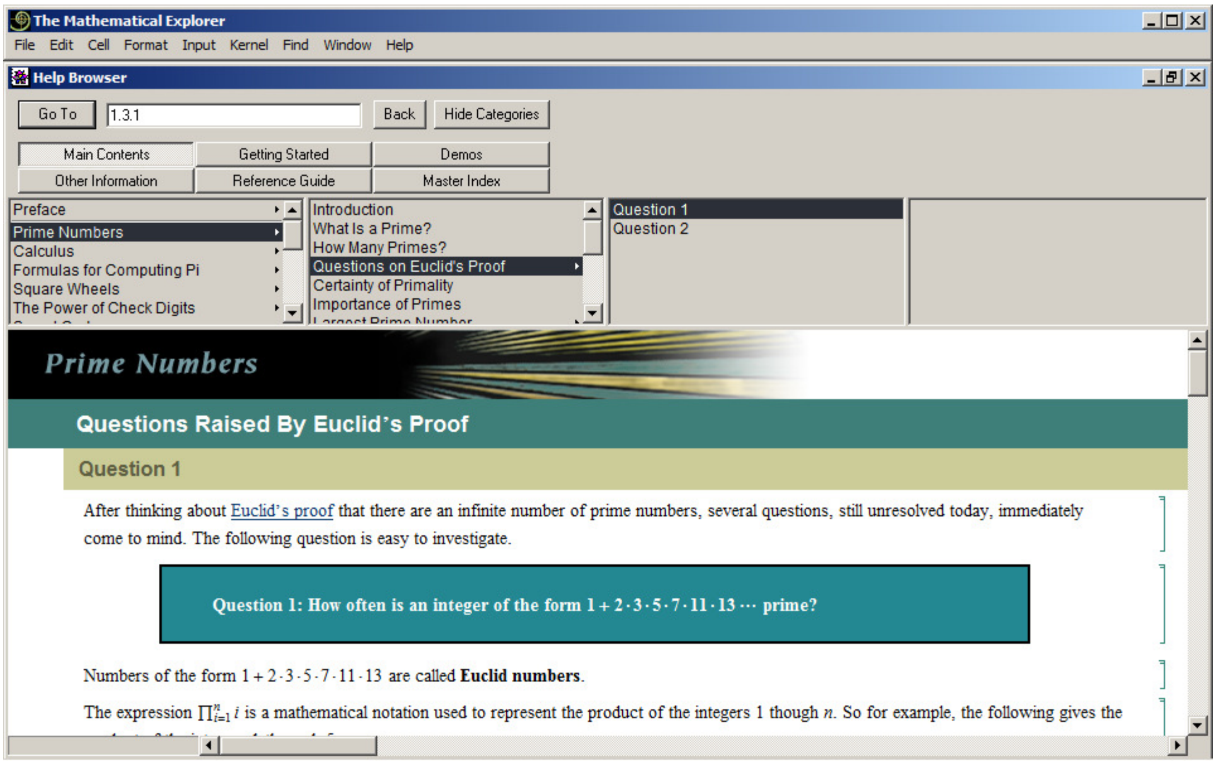
The mission statement of *The Mathematical Explorer* is best explained in its Introduction:

"The Mathematical Explorer is an interactive journey through some of the most fascinating problems in the history of mathematics — problems that have challenged mathematicians from the ancient Greeks up to the modern day. It includes topics on questions that were only very recently solved, such as Fermat's Last Theorem and the computer proof of the Four-Color Theorem, and also explores as yet unsolved problems such as the Riemann Hypothesis.

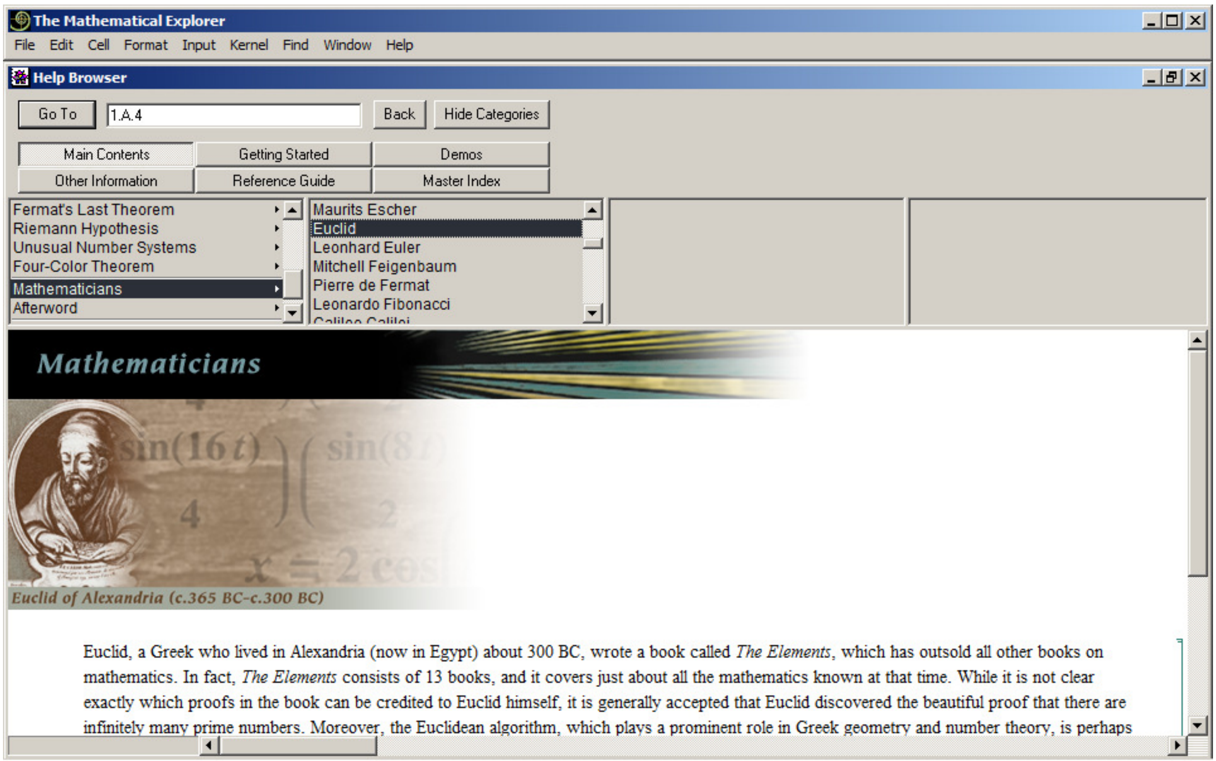
"The treatment of each topic is designed to be educational as well as entertaining; it includes a clear explanation of the important concepts along with fascinating cultural and historical details. Many topics have a strong computational thread, while still others are best understood through graphical visualization. Integrated with *The Mathematical Explorer* are a powerful computational engine and interface that rely on technology from the creators of *Mathematica*, the award-winning technical computing system from Wolfram Research. With *The Mathematical Explorer*, you can perform a wide range of numerical and symbolic calculations as well as create an unlimited array of graphics to help you better understand the concepts you are exploring.

"The Mathematical Explorer is intended as an open-ended, interactive resource to the world of modern mathematics, one that allows you to walk in the computational footsteps of the great mathematicians and experience the wonder of discovery that has fascinated amateurs and professionals alike throughout the ages."

Example from Chapter Prime Numbers:



Example of a biography of a mathematician:



Stan Wagon, author of the software, says “With each passing year, more and more mathematics becomes experimental in nature, with many hours of computations serving to uncover new relationships and formulas. One consequence of this computational shift in mathematics is that more of the field, both classic and modern, has become accessible to those without specialized training. This has occurred because sophisticated algorithms and methods of visualization that were once the domain of only a few specialists are now opened up to the entire world. The Mathematical Explorer is an attempt to show how elementary computations can shed light on many fascinating constructions, from the easy-to-understand 4-Color Theorem and Fermat's Last Theorem, to the more abstract and more important, Riemann Hypothesis."

Other features of Calculus WIZ and The Mathematical Explorer

As *Mathematica* based products, *Calculus WIZ* and *The Mathematical Explorer* both support procedural, functional (LISP programming style using pure functions) and rule based programming paradigms. For that reason, their capabilities are not limited to predefined templates and they can be used in a creative way by writing the user's own programs.

We can easily generate tables of function values:

```
Table[Sin[x], {x, 0, Pi, Pi/6}]
```

$$\left\{0, \frac{1}{2}, \frac{\sqrt{3}}{2}, 1, \frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right\}$$

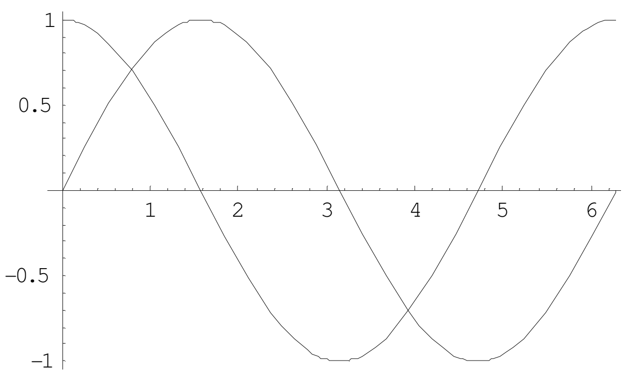
Taking advantage of the functional programming style, we can write short and elegant programs. Here we create a table of derivates and integrals for a given list of functions:

```
({#, D[#, x], Integrate[#, x]} & /@ {Sin[x], Cos[x], Tan[x], Cot[x],
ArcTan[x]}) // TableForm
```

Sin[x]	Cos[x]	-Cos[x]
Cos[x]	-Sin[x]	Sin[x]
Tan[x]	$\text{Sec}[x]^2$	-Log[Cos[x]]
Cot[x]	$-\text{Csc}[x]^2$	Log[Sin[x]]
ArcTan[x]	$\frac{1}{1+x^2}$	$x\,\text{ArcTan}[x] - \frac{1}{2}\text{Log}\left[1+x^2\right]$

It is also easy to create graphs of functions and combine them:

```
Plot[{Sin[x], Cos[x]}, {x, 0, 2 Pi}]
```



Rule based programming can be used to perform e.g. substitutions in equation solving.

```
Sin[x]^2+Cos[x]==1/2  /. Sin[x] -> Sqrt[1-Cos[x]^2]
```

$$1 + \text{Cos}[x] - \text{Cos}[x]^2 == \frac{1}{2}$$

The symbol % refers to result of previous calculation:

```
% /. Cos[x] -> z
```

$$1 + z - z^2 == \frac{1}{2}$$

The command Solve solves the previous equation to which we refer by %:

```
Solve[%, z]
```

$$\left\{\left\{z \to \frac{1}{2}\left(1 - \sqrt{3}\right)\right\}, \left\{z \to \frac{1}{2}\left(1 + \sqrt{3}\right)\right\}\right\}$$

Results can immediately be used for back-substitution:

```
Cos[x] == z /. %
```

$$\left\{\mathrm{Cos}[x] == \frac{1}{2}\left(1 - \sqrt{3}\right), \mathrm{Cos}[x] == \frac{1}{2}\left(1 + \sqrt{3}\right)\right\}$$

and we can solve the first one of these two goniometric equations

```
Solve[%[[1]], x]
```

Solve::ifun Inverse functions are

being used by Solve, so some solutions

may not be found.

$$\left\{\left\{x \to -\mathrm{ArcCos}\left[\frac{1}{2}\left(1 - \sqrt{3}\right)\right]\right\}, \left\{x \to \mathrm{ArcCos}\left[\frac{1}{2}\left(1 - \sqrt{3}\right)\right]\right\}\right\}$$

Note that the warning message was generated as the function is not invertible on the entire domain. Since both *Calculus WIZ* and *The Mathematical Explorer* support computations with exact quantities, the previous output is not written as a number. We can, however, ask them for numeric solutions:

```
N[%]
```

$$\{\{x \to -1.94553\}, \{x \to 1.94553\}\}$$

Conclusion

Both *Calculus WIZ* and *The Mathematical Explorer* are stand-alone software products based on solid *Mathematica* technology. They are suitable not only for exploring mathematical ideas by means of templates, but also for using work created by students through programming. Both products support the *Mathematica* programming core language (with some restrictions) with its rule-based, functional and procedural paradigms. Both products also support many *Mathematica* commands for formula manipulation, symbolic/numeric integration etc. They do not support *Mathematica* commands for processing huge data sets as these commands are not supposed to be used by high school students and undergraduate students. There are also some other restrictions with respect to the full version of *Mathematica*. *Calculus WIZ* and *The Mathematical Explorer* are both recommended for later courses at high school and/or for teaching undergraduate courses at university and college level.