Difference equations – examples

Example 7. With the help of difference equations calculate the values of the determinants of order *n* where we assume that $ac<0$:

a) $A_n = \begin{vmatrix} -2 & 1 & 0 & 0 & \ldots\ldots & 0 \\ 1 & -2 & 1 & 0 & \ldots\ldots & 0 \\ 0 & 1 & -2 & 1 & \ldots\ldots & 0 \\ & \ldots\ldots & \ldots & \ldots & \ldots & \\ 0 & \ldots\ldots & 0 & 1 & -2 & 1 \\ 0 & \ldots\ldots & & 0 & 1 & -2 \end{vmatrix}$, b) $B_n = \begin{vmatrix} -b & c & 0 & 0 & \ldots\ldots & 0 \\ a & -b & c & 0 & \ldots\ldots & 0 \\ 0 & a & -b & c & \ldots\ldots & 0 \\ & \ldots\ldots & \ldots & \ldots & \ldots & \\ 0 & \ldots\ldots & 0 & a & -b & c \\ 0 & \ldots\ldots & & 0 & a & -b \end{vmatrix}$

SOLUTIONS, ANSWERS

Example 7. a) By developing the determinant in the last line we find the recurrent dependency: $A_n = -2\,A_{n-1} - A_{n-2}$. Then we calculate the initial conditions: $A_0 = 1$, $A_1 = -2$. Since the corresponding characteristic equation $\rho(z) = z^2 + 2z + 1 = 0$ has a double root $z_{1,2} = -1$, then the general solution of the difference equation is: $A_n = C_1(-1)^n + C_2 n(-1)^n$. By substitution with the initial conditions we determine the final formula, namely: $A_n = (-1)^n(n+1)$.

b) Answer: $B_n = -\dfrac{1}{\sqrt{D}}\left(z_1^{\,n+1} - z_2^{\,n+1}\right)$, where $D = b^2 - 4ac > 0$, $z_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2}$.

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