## **Difference equations – examples**

**Example 5**. Choose suitable initial conditions in such a way that the following difference equations have bounded solutions.

a)  $2u_{n+2} - 5u_{n+1} + 2u_n = 0$ b)  $u_{n+2} + u_{n+1} - 2u_n = 0$ c)  $3u_{n+3} + 2u_{n+2} - 7u_{n+1} + 2u_n = 0$ d)  $2u_{n+2} - 2u_{n+1} + u_n = 0$ e)  $2u_{n+2} - 3u_{n+1} + 10u_n = 0$ 

## SOLUTIONS, ANSWERS

**Example 5.** a) The solutions will be bounded if there is no member bigger than one, i.e. if we take only those roots of the characteristic equation, the absolute value of which is smaller than 1. It is sufficient for the other constants in the general solution to be equal to 0.

For case a) we have a characteristic equation

$$2z^2 - 5z + 2 = 0.$$

It has roots  $z_1 = \frac{1}{2}$ ,  $z_2 = 2$ . From the general solution we need to leave out the member with  $2^n$ . Answer:  $u_n = C\left(\frac{1}{2}\right)^n$ .

- b) Answer:  $u_n = C$ .
- c) Answer:  $u_n = C_1 + C_2 \left(\frac{1}{3}\right)^n$ .
- d) Answer:  $u_n = C_1 \left(\frac{1}{\sqrt{2}}\right)^n \cos n \frac{\pi}{4} + C_2 \left(\frac{1}{\sqrt{2}}\right)^n \sin n \frac{\pi}{4}$ .

e) As the absolute value of all the roots of the characteristic equation is bigger than one, the equation cannot have bounded solutions in this case.