Models and examples in mathematical optimization
Transportation model

Example 4. Places in 5 different companies have been provided in order to carry out the industrial training of a group of 15 students from Plovdiv University, with each company taking in no more than 4 trainees. Each student has different preferences for each company based on their location. In order to fully satisfy the wishes of the students the dean has asked them to arrange the companies, giving them scores of 1, 2, 3, 4 and 5 (the most desired location being 5 and the least desired – 1). As a result the following table was formed:

C_{ij}	company1	company2	company3	company4	company5
Student 1	2	1	4	3	5
Student 2	5	4	1	2	3
Student 3	2	1	4	3	5
Student 4	4	5	1	2	3
Student 5	3	2	1	4	5
Student 6	5	4	3	2	1
Student 7	3	2	4	1	5
Student 8	2	1	5	3	4
Student 9	1	3	5	4	2
Student 10	4	2	5	1	3
Student 11	5	3	2	4	1
Student 12	3	2	5	4	1
Student 13	4	1	3	2	5
Student 14	2	5	3	1	4
Student 15	5	4	3	1	2

How can the dean distribute the students so that as much of their wishes can be fulfilled?

Solution: We input the variables X_{ij} for i=1,2,...,15; j=1,2,…,5 which can assume values 0 or 1. X_{ij}=1 if the i^{th} student is going to the j^{th} company and 0 if he is not. As a summary evaluation of the satisfaction of the group we input the function

$$Z(X)=\sum_{i=1}^{15}\sum_{j=1}^{5}C_{ij}X_{ij}\longrightarrow \max$$

The constraints of the problem are:

1. Each student must go to only one company:

$$\sum_{j=1}^{5}X_{ij}=1,\forall i=1\div 15$$

2. No more than 4 students can go in any one company:

$$\sum_{i=1}^{15}X_{ij}\le 4,\forall j=1\div 5$$

3. $X_{ij}\in\{0,1\},\forall i=1\div 15;\ j=1\div 5$

Answer

The answer to the problem is the following: max Z = 74, which would mean that only 1 student (student 2) will not be going to their desired company, but their 2^{nd} most desired one. The distribution is given below:

	Example 1	**Example 2**	**Example 3**	**Example 4**	**Example 5**
Student 1	0	0	0	0	1
Student 2	1	0	0	0	0
Student 3	0	0	0	0	1
Student 4	0	1	0	0	0
Student 5	0	0	0	1	0
Student 6	1	0	0	0	0
Student 7	0	0	0	0	1
Student 8	0	0	1	0	0
Student 9	0	0	1	0	0
Student 10	0	0	1	0	0
Student 11	1	0	0	0	0
Student 12	0	0	1	0	0

Student 13	0	0	0	0	1
Student 14	0	1	0	0	0
Student 15	1	0	0	0	0

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