



Models and examples in mathematical optimization

Transportation model

Example 3. An intermediary company is hired to deliver specified materials to 5 sites, with the amount of materials being 300, 310, 550, 620 and 200 items, respectively, which can be bought from 4 warehouses. The available number of items for each warehouse is 750, 500, 550 and 450 items, respectively, and are sold at 20, 22, 23 and 24 levs per item, respectively. The transport costs from each warehouse to each site are listed in the table below:

	site 1	site 2	site 3	site 4	site 5
warehouse 1	2	2	5	1	1
warehouse 2	3	3	4	2	2
warehouse 3	5	4	3	4	3
warehouse 4	1	7	2	3	2

It must be decided how many items to buy from each warehouse for each site so that the site needs are met and the expenses (both item price and transport cost) be minimal.

Solution:

Let the variable X_{ij} show the number of items purchased from warehouse i to site j .

Then the transport expenses are:

$$Z_1 = 2X_{11} + 2X_{12} + 5X_{13} + X_{14} + X_{15} + 3X_{21} + 3X_{22} + 4X_{23} + 2X_{24} + 2X_{25} + \\ + 5X_{31} + 4X_{32} + 3X_{33} + 4X_{34} + 3X_{35} + X_{41} + 7X_{42} + 2X_{43} + 3X_{44} + 2X_{45}$$

Item purchase expenses are:

$$Z_2 = 20(X_{11} + X_{12} + X_{13} + X_{14} + X_{15}) + 22(X_{21} + X_{22} + X_{23} + X_{24} + X_{25}) + 23(X_{31} + X_{32} + X_{33} + X_{34} + X_{35}) \\ + 24(X_{41} + X_{42} + X_{43} + X_{44} + X_{45})$$

The sum of the two types of expenses $Z = Z_1 + Z_2 \rightarrow \min$.

The constraints of the problem are two-fold:

1. Constraints by the available number of items – the amount purchased materials from each warehouse cannot be more than the materials present at said warehouse:

- for the 1st: $X_{11}+X_{12}+X_{13}+X_{14}+X_{15} \leq 750$
- for the 2nd: $X_{21}+X_{22}+X_{23}+X_{24}+X_{25} \leq 500$
- for the 3rd: $X_{31}+X_{32}+X_{33}+X_{34}+X_{35} \leq 550$
- for the 4th: $X_{41}+X_{42}+X_{43}+X_{44}+X_{45} \leq 450$

2. Constraints by the orders – there must be an exact number of items delivered to each site. In other words, for each site we have:

- for the 1st: $X_{11}+X_{21}+X_{31}+X_{41} = 300$
- for the 2nd: $X_{12}+X_{22}+X_{32}+X_{42} = 310$
- for the 3rd: $X_{13}+X_{23}+X_{33}+X_{43} = 550$
- for the 4th: $X_{14}+X_{24}+X_{34}+X_{44} = 620$
- for the 5th: $X_{15}+X_{25}+X_{35}+X_{45} = 200$

Additionally we add the natural constraint $X_{ij} \geq 0$ for $i=1,2,3$ and 4 and $j=1,2,3,4$, and 5 .

Answer

The answer to the problem is shown in the table below:

	for site 1	for site 2	for site 3	for site 4	for site 5
From warehouse 1	0	0	0	550	200
From warehouse 2	0	310	0	70	0
From warehouse 3	0	0	550	0	0
From warehouse 4	300	0	0	0	0

Transport costs are $Z1=3770$, amount paid for the materials – $Z2=43210$. Total expenses: $Z=Z1+Z2=46980$.

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