10.2. Network planning (problem of finding the shortest and longest path in a network)

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A network is given. We'll examine the following

Basic problem 1

Find the shortest (or longest) path from the beginning to the end of the network if distances (weights) of arcs in the network are given.

Many practical activities can be reduced to solving this problem, which belongs to the so called network planning problems. Such are, for example:

- Selecting a route from one city to the other without returning. In this case the shortest path in relation to the distance can be sought, or the cheapest, or the one which can be the fastest to travel (minimum time);
- Compiling an algorithm for finding the shortest possible at a given moment path for transferring an electronic message from a one point of a computer network to another. In this case contours should be avoided, as well as overloading the network.
- Planning and managing a project which has to start and finish within a time limit and has to be carried out in a particular sequence of stages naturally described by means of a network. Individual tasks for every stage, which may or may not be interconnected, are described as nodes in a graph, and the possible connecting arcs can represent execution times or evaluation points for achieved results or another indicator.
- Planning the construction of a new building divided into parallel or sequenced in time stages;
- Organizing new production and so on.

In many cases this type of problems can be directly solved using Bellman's principle.

I.e. we start from the end node of the graph and go backwards selecting in sequence the nodes at which we can take the optimal decision. In this way the stages of the problem are determined until the starting node is reached. After that, by following the selected optimal decisions the corresponding optimal solutions to the problem are found.

Example 2. Find the minimal and maximal path from node 1 to node 7 in the network shown in fig.4.

Solution: In the upper part of the circle of every node numbered i the values of medial maximums d_i are written and in the lower part are written the minimums. We will put forward the explanations for finding the maximal path, the other case we leave to the reader. Calculations go through the following stages:

1. In accordance with Bellman's principle we start from node 7 and value $d_7 = 0$.
2. Node 7 is incoming for arcs from nodes 5,6,4 and 3. Of all of them at this stage only node 5 can be calculated because we have all the necessary information about it. The distance from node 5 to node 7 is $d_5 = 1$, which we write in the upper half of its corresponding circle, and mark the direction of movement with .
3. Directly connected with nodes 5 and 7 are nodes 6,4,3,2,1. Of all of them only nodes 4 and 6 are satisfactory as all outgoing arcs lead to nodes with already known medial maximums. We calculate for node 4 $d_4 = \max\{6, \underline{10+1}\} = 11$, directed to node 5. For node 6 we find $d_6 = \max\{\underline{3}, \underline{2+1}\} = 3$, with two possible directions leading to nodes 5 and 6. Maximum values are underlined.
4. At this stage the maximum for node 2 is determined. We get $d_2 = \max\{9+1, 16+3, \underline{4+11}\} = 15$, directed towards node 4.
5. Next is node 3. We have: $d_3 = \max\{15, 8+3, 7+11, \underline{4+15}\} = 19$, directed towards node 2.
6. The last stage is for node 1: $d_1 = \max\{5+15, 12+1, \underline{7+19}\} = 26$, directed towards node 3.

We found the maximal path with a distance of 26 and route: 1, 3, 2, 4, 5, 7.

In the case of a minimal path the solution has been shown with optimal directions marked using .

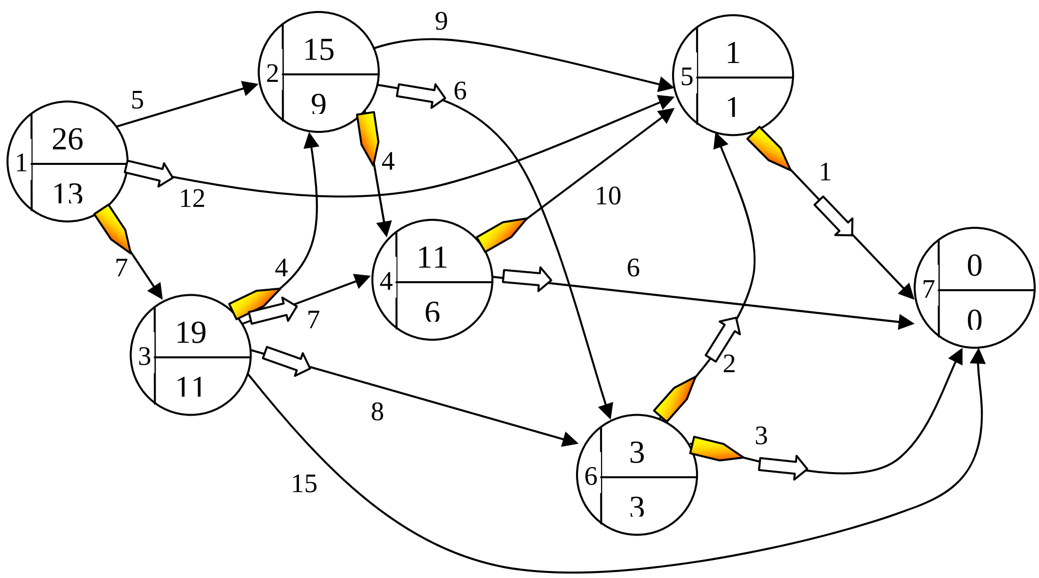

Fig.4. Finding minimal and maximal path from node 1 to node 7 using Bellman's principle.