LECTURE 9

ELEMENTS OF DYNAMIC OPTIMIZATION

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9.1. Basic notions

1) System

The system is a totality of elements with definite properties (attributes) which are manifested during the concurrent functioning of elements between them. Part of the elements are basic, then in the absence of one or some of them the system doesn't function correctly or doesn't function at all. Other elements are auxiliary and serve to improve or change the activities of the system.

The complex system consists of simpler subsystems.

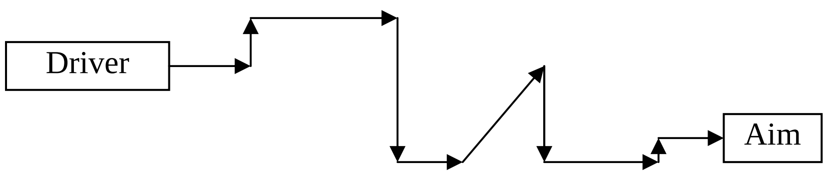
Example 1. A system, for example is a mechanical clock, with elements such as gear-wheels, hands etc. Its basic property is to measure time. In the absence or failure of the hands for example the clock system becomes non-effective.

Example 2. A car - the system has the property of moving through space. It consists of simpler subsystems - engine, electrical etc.

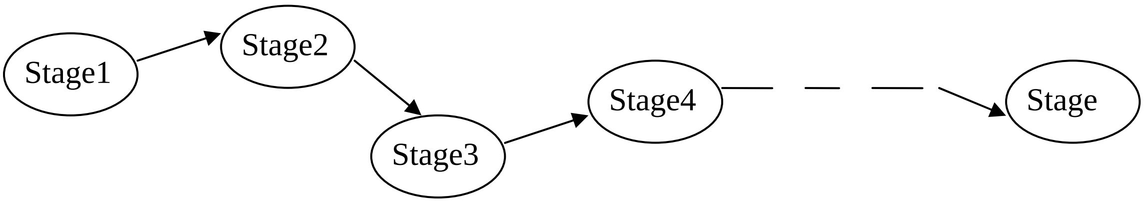
2) Dynamic system.

A system which changes with time we will call dynamic. We will regard the changes as occurring one after the other with the system having an aim.

Example 3. Let's consider a driver moving through a big city via some route and his aim being to arrive at a given address. At every crossing the driver has to take a decision which way to take. This way the movement is divided in time into stages, from crossing to crossing.



Later on we will scrutinize dynamic systems which change from one state into another in the course of time. Every change of the system from one state into the other we will call a stage. Between the stages we will regard that the system has developed according to its own inner rules until the next stage when a new decision has to be taken for a possible new stage. Such a process of development we will call discreet.



Definition. **Dynamic optimization** is a branch of mathematics which solves problems connected with the development of systems by a process discreet in time and aims at finding an optimal, in a definite sense, result (determining the minimum distance, maximum profit etc.).

If the process of development of the system involves n Stages and a decision is taken on every stage, there will be a total of n decisions taken.

Definition. **Strategy or policy** of the system is a ranged set of decisions taken, arranged in the order of the corresponding stages.

9.2. Presenting solutions using graphs

The **graph** is a totality of points called nodes of the graph (set V) and edges or arcs connecting the nodes (set E). If the connective direction is indicated the graph is oriented, otherwise it is unoriented. In many cases every edge is juxtaposed to a number called “weight” or price. It can have different meanings - length, sum of money, time interval etc.

In a sequence of practical problems, the graph is their most natural description.

Example 4. Let the nodes of a graph represent cities, and arcs the roads between them. We will regard the weights as the distances from one node to the other. If the problem is finding the way from city 1 to city 2, it is obvious that there are several solutions, for example 1-2-5, 1-2-4-5 and so on. Another problem is finding the shortest path itinerary between 1 and 5.

Likewise many other problems, connected for example with planning production, investment, project management and so on, can also be described using graphs. The different possible states of the corresponding dynamic system in time can be shown as nodes of the graph and the transition from one state to the other as arcs.

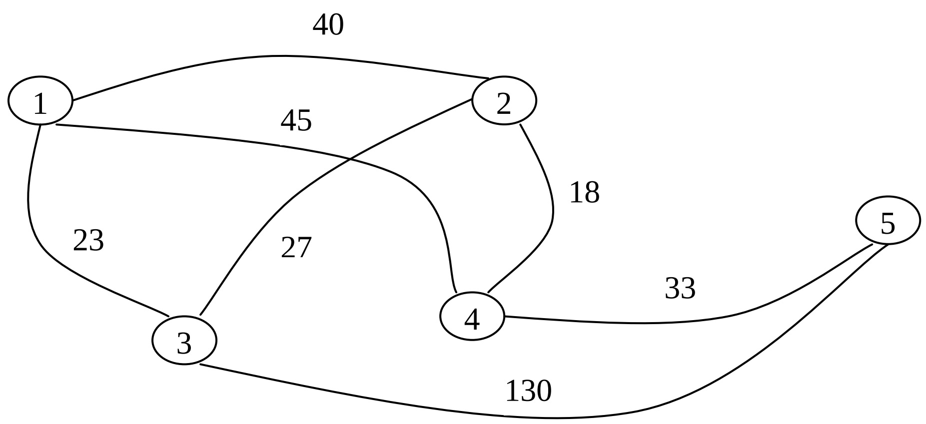


Fig.1. Example of an unoriented weight graph

Example 5. Company *X* has two types of machines - type *A* and type *B*, respectively 4 and 5 in number. Due to excellent profits made in the next few months the company plans to expand its production, so it needs to buy additional equipment - 8 more machines of type *A* and 5 machines of type *B*. Each month the company can set aside enough money to buy 2 type *A* machines or 1 type *B* machine.

a) Determine the possible states of the system and the number of stages for making a decision.

b) Write down two strategies.

Solution: a) Obviously the states of a finite number and the stages correspond to every monthly purchase. To arrange the states we represent them using the oriented graph from fig. 2.

The nodes of the graph have been marked with circles and a number, while the arcs using arrows, showing the possible transitions. The beginning of the graph is node (1) with 4 machines type *A* and 5 machines type *B*, the aim at the end being node (25) with 14 machines type *A* and 9 machines type *B*. Two main directions of movement have been shown - right with an increase in number of machines type *A* by two a month and up with an increase of machines type *B* by 1 a month. Starting from (1) two decisions can be made - left or right and to move the arrow to the next state (2) or (3). This is the first stage of making a decision. Likewise after that there is a choice of direction and there comes the second stage of possible decisions (4), (5) or (6) and so on. The total number of stages is seven because at the last transition to state (25) no decision is made. Therefore company *X* can carry out its planned production expansion in eight months.

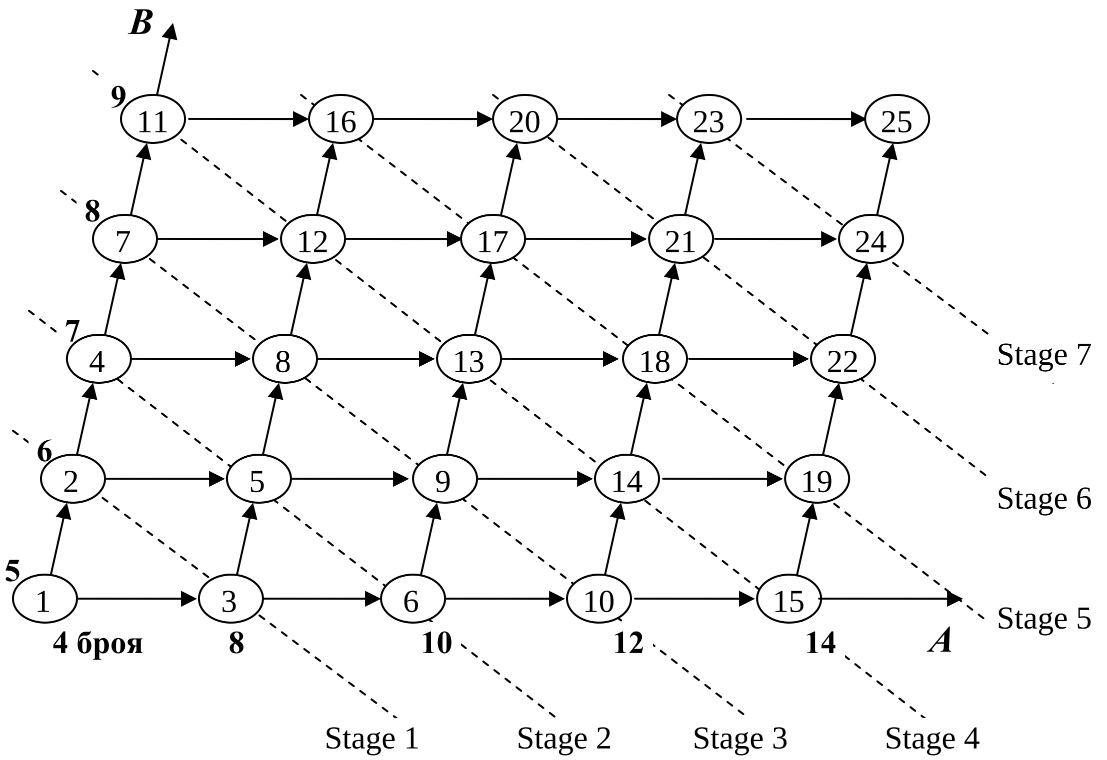


Fig.2. Oriented graph for example 5.

b) Possible strategies are:
<1, 3, 5, 8, 13, 18, 22, 24, 25> and <1, 2, 4, 7, 12, 17, 21, 24, 25>.

If every edge of the graph is juxtaposed to a number there can be different problems whose solution requires finding an optimal strategy. It can be found using different methods at the base of which lies Bellman’s optimality principle.