



Problems on numerical solving of tri-diagonal matrices

1) Solve the systems using the double sweep method:

$$\text{a) } \begin{cases} 5x_1 + 2x_2 & = 9 \\ -x_1 + 3x_2 + 2x_3 & = 11 \\ 2x_2 - 3x_3 + 2x_4 & = 3 \\ -x_3 + x_4 & = 1 \end{cases},$$

$$\text{b) } \begin{cases} 4x_1 + x_2 & = 1 \\ x_1 + 4x_2 + x_3 & = 2 \\ x_2 + 4x_3 + x_4 & = 3 \\ x_3 + 4x_4 & = 4 \end{cases},$$

$$\text{c) } \begin{cases} 2x_1 + x_2 & = 0 \\ x_1 + 2x_2 + x_3 & = 2 \\ x_2 + 4x_3 + 2x_4 & = -3 \\ x_3 + 5x_4 & = 2 \end{cases},$$

$$\text{d) } \begin{cases} 3x_1 - 2x_2 & = 1 \\ -4x_1 + 6x_2 - x_3 & = 3 \\ -2x_2 - 5x_3 + x_4 & = 6 \\ -x_3 + 2x_4 - x_5 & = 5 \\ -2x_4 + 3x_5 & = 0 \end{cases},$$

$$\text{e) } \begin{cases} 1,2x_1 - 2x_2 & = 3,1 \\ -4x_1 + 6,5x_2 - x_3 & = -3,5 \\ -2x_2 - 5,2x_3 + 1,2x_4 & = 7,2 \\ -x_3 + 2x_4 - 0,5x_5 & = 4,8 \\ -2x_4 + 3x_5 & = -0,2 \end{cases}.$$

Answers: a) $x = (1; 2; 3; 4)$; b) $x = (0,162679; 0,349282; 0,440191; 0,889952)$;

c) $x = (-1,25; 2,5; -1,75; 0,75)$; d) $(1; 1; -1; 3; 2)$; e) $(38,2819; 21,4191; -10,4032; -3,38191; -2,32128)$

2) Create a computer program for the calculation of a system using the double sweep method for an arbitrary number of equations and with its help solve the systems above.

- 3) Deduce the formulas for the double sweep method for the solution of five-diagonal linear systems of equations.

Hint: Substitute $x_k = \alpha_k x_{k+1} + \beta_k x_{k+2} + \gamma_k$.

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