Tasks for individual work

on the subject of Variational methods for solving boundary problems for ordinary differential equations (ODE) of the second order

Use the Ritz and Galerkin's methods and a different **number** of basis functions to solve the following boundary problems:

Problem 1. $(e^{-x}y')' = -1, \quad y(0) = 1, \quad y(1) = 0$

with a basis: $\varphi_0 = 1 - x, \ \varphi_1 = x(1-x), \ \varphi_2 = x^2(1-x)$

Problem 2. $y'' - (x+1)y' - y = -1, \qquad y(-1) = y(1) = 0$.

Problem 3. $y'' - y = 0, \qquad y(0) = 0, \ \ y'(1) = 1$, by using the following functions as a basis: $\varphi_0 = x$, $\varphi_1 = x(x-2), \varphi_2 = x(x^2-3)$ and compare the approximate solution with the exact one: $u = 0.324(e^x - e^{-x})$.

Solution. $y = x - 1.3392x(x-2) + 1.1137x(x^2-3)$

Problem 4. $y'' + (x+1)y' - \alpha y = (\alpha - 1)(x+1)^{\alpha-1}, \qquad y(0) = \frac{1}{\alpha}, \ \ y(1) = \frac{2^\alpha}{\alpha}$

for α = 1, 2, 3. Compare the approximate solution with the exact one: $u = \frac{(x+1)^\alpha}{\alpha}$.

Problem 5. $y'' + xy' - 2y = 2(\lambda^2 - 1), \qquad y(0) = 1, \ \ y(1) = \lambda^2 + 1, \ \lambda = \frac{1}{2}(a+1)$, where

a is a parameter, taking values 0, 1, ..., 9. Compare the approximate solution with the exact one: $u = \alpha^2 x^2 + 1$.

Author: Luba Popova
Plovdiv University