## Problems on the square root method for systems of linear algebraic equations

1) Is it possible to apply the square root method for the system?

$$\begin{cases} 1{,}2x_1 - 1{,}5x_2 + 7{,}2x_3 = 16{,}8 \\ 2{,}2x_1 + 5{,}5x_2 - 1{,}5x_3 = 10{,}55 \\ 6{,}1x_1 + 2{,}2x_2 + 1{,}2x_3 = 16{,}55 \end{cases}, \qquad A = \begin{pmatrix} 1{,}2 & -1{,}5 & 7{,}2 \\ 2{,}2 & 5{,}5 & -1{,}5 \\ 6{,}1 & 2{,}2 & 1{,}2 \end{pmatrix}$$

**Instruction:** Rearrange some of the rows and/or columns.

2) By means of the square root method decompose the matrix $S$ as a product of two triangular matrices and calculate its determinant.

$$S = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 8 & 11 \\ 3 & 8 & 14 & 20 \\ 4 & 11 & 20 & 30 \end{pmatrix}.$$

Answer: $T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{pmatrix}$.

3) By means of the square root method solve the system and find the determinant of the respective matrix

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 4 \\ 2x_1 + 5x_2 + 8x_3 + 11x_4 + 14x_5 = 10 \\ 3x_1 + 8x_2 + 14x_3 + 20x_4 + 26x_5 = 18 \\ 4x_1 + 11x_2 + 20x_3 + 30x_4 + 40x_5 = 28 \\ 5x_1 + 14x_2 + 26x_3 + 40x_4 + 55x_5 = 39 \end{cases}.$$

**Recommendation:** Use a computer to solve this problem.

Answer: $y = \begin{pmatrix} 4 \\ 2 \\ 2 \\ 2 \\ 1 \end{pmatrix}$, $x = \begin{pmatrix} 2 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$, $T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 \\ 4 & 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix}$.

4) By means of the square root method solve the system

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1 + 3x_2 + 3x_3 = 2 \\ x_1 + 3x_2 + 6x_3 = 5 \end{cases}.$$

Answer: (-1, 0, 1).

5) There is given the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}.$$

By means of the square root method calculate the determinant of the matrix $B = A.A^T$.

Answer: det $B$=16.

Author: Iliya Makrelov, ilmak@uni-plovdiv.bg

Plovdiv University