Problems on numerical differentiation

1) By tabulating the values of the function $y = f(x) = \ln(x^2)$ in the interval $[2,3]$ wit step h=0.1, find: y'_2, y'_3, y'_9 by the formulas of the first and the second type. Answer: $y'_2 = 0{,}90908,\ y'_3 = 0{,}86955,\ y'_9 = 0{,}68965$.

2) From the table of the function from the previous problem 1) find approximately the first derivatives by using three-point patterns. Compare the respective results, obtained by the other methods.

3) By the experimental data from table 1, calculate the derivatives using methods I and II. Compare the results.

Table 1

i	x_i	y_i
0	1,00	-0,2475
1	1,05	-0,2490
2	1,10	-0,2498
3	1,15	-0,2500
4	1,20	-0,2496
5	1,25	-0,2485
6	1,30	-0,2469
7	1,35	-0,2446
8	1,40	-0,2417

4) Make a table of the function $f = \dfrac{x^3 + 2x - 1}{x + 2}$ in the interval [-1; 0] with the step $h = 0{,}1$ and fill in the table of the final results. With its help calculate the approximate values of the first derivatives of the function in the points $x_0 = -1,\ x_1 = -0{,}9$ и $x_{10} = 0$. Be careful with the stability. Compare the results with the exact solution. Answer: $f'_0 = 8{,}987,\ f'_1 = 6{,}934,\ f'_{10} = 1{,}249$.

5) Make a table of the function $f = \dfrac{4x-1}{x^2+1}$ in the interval $[5, 6]$ with the step $h = 0{,}2$ and fill in the table of the final results. With its help calculate the approximate values of the first derivatives of the function in the points $x = 5{,}05;\ x = 5{,}9$. Compare the results with the exact values, which are: $f'(5{,}05) = 0{,}12516$ and $f'(5{,}9) = -0{,}09626$.

6) Calculate the approximate values of the derivative of the function $f = \sqrt{x^2 - x + 3}$ in the point $x = 1$. Choose an appropriate formula and step in such a way as to have a theoretical error $\varepsilon = 0{,}001$. Answer: $f_0' = 0{,}288$, $h = 0{,}2$.

7) Make a table of the function $f = \sin(x + \pi/7)$ by dividing the interval $[0, \pi/6]$ into five equal subintervals. Calculate the approximate values of the first and the second derivatives of the function in the boundary points. Determine the error of approximation.

8) Calculate the first and the second derivatives of the function of the previous problem 7) in all the points by using appropriate three-point patterns. Evaluate the error.

9) By using formulas for numerical differentiation with error $O(h^2)$, fill in the blanks in the table.

x	0,6	0,7	0,8	0,9	1,0
y	31	44	59		95
y'					
y''			200		

Instruction. Use the formula for the central difference for $y''(0{,}8)$ in order to find the value of $y(0{,}9)$. How would you solve the problem if $y''(0{,}8)$ weren't known?

Author: Snezhana Gocheva-Ilieva, snow@uni-plovdiv.bg
Plovdiv University