Problems on interpolation

1) Using an interpolation polynomial, built by the table:

x	0	1	3	5
y	-5	0	2	8

find the approximate value of the function for $x = 1{,}5$.

Answer: $f(1{,}5) \approx 1{,}0875$.

2) Obtain an interpolation polynomial by the table

x	0	1/27	1/8	1
$\sqrt[3]{x}$	0	1/3	1/2	1

With its help calculate $\sqrt[3]{0{,}5}$ and $\sqrt[3]{1{,}5}$. Can the error be estimated in this case?

3) The function $y = \sqrt[3]{x}$ is tabulated in the points 1; 1,1; 1,3; 1,5 and 1,6. By using $L_2(x)$, which is built by the obtained table, calculate $\sqrt[3]{1{,}15}$ and estimate the error. Which points have to be used?

4) There is given the table

x	-1	0	1	2	4
$f(x)$	-7	-4	-2	4	8

By means of a reverse interpolation find an approximate solution to the equation:

а) $f(x) = 6$; Answer: $x \approx 2{,}388$

б) $f(x) = 0$; Answer: $x \approx 1{,}690$.

в) $y(x) = 0$ by the table:

x	1	2	2,5	3
y	-6	-1	5,625	16

Answer: $x \approx 2{,}122$.

5) The function $y = \sqrt{x}$ is tabulated in $[1,2]$ by step h. How big must the step be (in how many steps must the function be calculated, $h = (b-a)/N$), so that the interpolation error with a polynomial of the second degree does not exceed 10^{-6}.

Answer: $h \le \sqrt{3}/50$; $N \ge 29$.

6) Find the minimal degree of the interpolation polynomial and the location of the interpolation points in such a way that the interpolation error of $y = \sqrt{x}$ in [1,2] does not exceed 10^{-5}.

Answer: $n = 6$, instruction: $M_{n+1} = (2n\text{-}1)!!/2^{n+1}$.

7) Find all the divided differences for the function $f(x) = 3x^3 - 2x^2 + 1$ on the points $x_k = x_0 + kh,\ k = \overline{-2,2}$.

8) Show that

$$f[x_0, x_1, \ldots, x_n] = \frac{\begin{vmatrix} 1 & x_0 & x_0^2 & \ldots & x_0^{n-1} & f_0 \\ 1 & x_1 & x_1^2 & \ldots & x_1^{n-1} & f_1 \\ . & . & . & \ldots & . & . \\ 1 & x_n & x_n^2 & \ldots & x_n^{n-1} & f_n \end{vmatrix}}{\begin{vmatrix} 1 & x_0 & x_0^2 & \ldots & x_0^{n-1} & x_0^n \\ 1 & x_1 & x_1^2 & \ldots & x_1^{n-1} & x_1^n \\ . & . & . & \ldots & . & . \\ 1 & x_n & x_n^2 & \ldots & x_n^{n-1} & x_n^n \end{vmatrix}}.$$

9) Build an interpolation polynomial with finite/divided differences for the table and calculate $y(0,5)$ with its help:

a)

x	-1	0	1	2
y	1	1	1	7

Answer: $y(0,5) \approx 0{,}625$

б)

x	-1	0	1	2
y	-3	-1	1	8

Answer: $y(0,5) \approx -0{,}3125$

10) With the help of an interpolation polynomial with finite differences find the sums:

a) $S_n = 1^2 + 2^2 + \ldots + n^2 + (n+1)^2$

b) $S_n = 1^3 + 2^3 + ... + n^3$

c) $S_n = 4 + 7 + ...(3n + 1)$

d) $S_n = 5 + 9 + ... + (4n + 1)$

e) $S_n = 1^2 + 3^2 + ... + (2n - 1)^2$

f) $S_n = 2^2 + 4^2 + ... + (2n + 2)^2$

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