Problems on numerical integration

1) Using the quadratic formulas of rectangles calculate the approximate value of the integral $\int_4^5 \sqrt{2+x^3}\,dx$ by dividing the interval into $n = 10$ subintervals. Evaluate the error. How many characters of intermediate precision do you have to work with?

Answer: $I_1 = 9{,}51$; $I_2 = 9{,}82$.

2) By using the trapezium method calculate the approximate value of the following integrals for assigned n or h:

a) $\int_0^1 \sqrt[3]{x+3}\,dx$, $n = 10$

b) $\int_2^4 \frac{1+x}{1+\sqrt{x}}dx$, $n = 20$

c) $\int_{-1}^1 \ln(5+x)\,dx$, $n = 20$

d) $\int_{-2}^0 \frac{1}{p+x^2}dx$, $h = 0{,}2$, $p = 5$

e) $\int_0^{\pi/2} \cos(x^2)\,dx$, $n = 10$

f) $\int_1^{1{,}5} e^{\sqrt{x}}\,dx$, $h = 0{,}05$.

Answers: a) 1,5171; b) 2,92187; c) 3,20531; d) 0,326; e) 0,845; f) 1,52975; $n = 10$.

3) By using Simpson's rule calculate approximately the integrals from problem 2. Determine the number of characters you have to work with and the precision of the obtained results. Round the final answers correctly.

4) Calculate the integral $\int_{0{,}5}^1 \sin(\frac{x}{1+x})dx$ by the trapezium method with precision $\varepsilon = 0{,}0001$ for initial n=5.

Answer: 0,2056.

5) Calculate the integral $\int_1^2 \sqrt{\frac{1}{x}}\,dx$ by Simpson's rule with precision $\varepsilon = 0{,}000001$.

Answer: 0,828428.

6) Compile a computer program for calculating the approximate value of the following integrals by the trapezium method and Simpson's rule. Compare the results:

a) $\int_1^2 \sqrt[3]{x+\ln(x)}\,dx$, $\varepsilon = 0{,}0001$

b) $\int_0^1 \frac{x^2-\sqrt{x}+2}{1+x}dx$, $\varepsilon = 0{,}00001$

c) $\int_0^1 \frac{\sin(x)}{x}\,dx$, $\varepsilon = 0{,}0000001$

d) $\int_0^\infty \frac{\sin(x)}{x}\,dx$, $\varepsilon = 0{,}00001$

e) $\int_1^2 \frac{\arctan(x+1)}{xe^x}\,dx$, $\varepsilon = 0{,}000001$

f) $\int_0^1 \frac{\arcsin(x)}{1+x}dx$, $\varepsilon = 10^{-6}$

g) $\int_0^{\pi/6} \ln(\cos(x))\,dx$, $\varepsilon = 10^{-7}$

h) $\int_1^4 \frac{x^3}{e^{x^2}+1}dx$, $\varepsilon = 10^{-3}$

i) $\int_0^\infty \frac{1}{x^2+1}dx$, $\varepsilon = 10^{-4}$

j) $\int_0^\infty \frac{\ln(x)}{x^2+1}dx$, $\varepsilon = 10^{-9}$

Answers: a) 1,2259; б) 1,15024 ; в) 0,946083; г) 1,57257; Instruction: Represent the integral as the sum $\int_a^\infty f(x)dx = \int_a^b f(x)dx + \int_b^\infty f(x)dx$. By using an appropriate upper evaluation of the sub integral function, choose such b that the second integral is smaller than $\varepsilon/2$, and calculate the first one again with precision $\varepsilon/2$; e) 0,198912; f) 0,345655; g) –0,0246171; h) 0,326; i) 1,5708; j) 0.

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