Problems on solving equations with one unknown variable

1) Calculate the real root of the equation with accuracy ε by the indicated method after localizing it in advance:

a) $x^3 + 2x + 7{,}8 = 0$, $\varepsilon = 10^{-2}$, bisection method

b) $x + e^x = 0$, $\varepsilon = 10^{-3}$, method of chords

c) $e^x - x^2 - 2x - 2 = 0$, $\varepsilon = 10^{-3}$, Neuton's method

d) $x^4 - 4x - 1 = 0$, $\varepsilon = 10^{-2}$, bisection method

e) $x^4 + 2x^3 - x - 1 = 0$, $\varepsilon = 10^{-3}$, combined method

f) $3x - \cos(x) = 1$, $\varepsilon = 10^{-3}$, tangents

g) $x\cos(x) = \ln(x)$, $\varepsilon = 10^{-2}$, bisection method

h) $\lg(x) = 3x$, $\varepsilon = 10^{-4}$, the smallest positive root with Neuton's method

i) $x\lg(x) = 1$, $\varepsilon = 10^{-4}$, combined method

j) $x^4 - 9x^3 - 2x^2 + 120x - 130 = 0$ has exactly four different real roots in $[-6;10]$. Specify all four of them with $\varepsilon = 10^{-3}$ using a different method for each one.

2) Build a convergent MCA (method of consecutive approximations) to specify the smaller root of the equation $0{,}1x^2 - x + 1 = 0$. How many iterations must be made in order to guarantee accuracy 10^{-3} for $x_0 = 2$? Make three iterations.

3) Build a convergent MCA (method of consecutive approximations) to specify the root in the given interval. How many iterations must be made in order to guarantee accuracy 10^{-3} for the given x_0? Make three iterations for:

a) $x^3 + x - 1000 = 0$, [9;10] ; $x_0 = 9{,}5$

b) $x^2 - x - 0{,}5 = 0$, [-1;0] ; $x_0 = 0{,}5$

c) $x\lg(x) = 1$, [2;3] ; $x_0 = 2{,}5$

d) $x - \sin(x) = 0{,}25$, [1;1,5] ; $x_0 = 1{,}25$

4) It is known that the equation $x + \ln(x) = 0$ has a root $\alpha \approx 0{,}5$. For its calculation there are suggested four iteration formulas:

A) $x_{n+1} = -\ln(x_n)$, B) $x_{n+1} = \exp(-x_n)$, C) $x_{n+1} = 0{,}5(x_n + \exp(-x_n))$,

D) $x_{n+1} = \sin(x_n) + 0{,}25$.

a) which of the formulas above can be used?

b) make three iterations using the fastest formula.

5) Prove that the iteration process $x_{k+1} = \frac{1}{2}\left(x_k + \frac{a}{x_k}\right)$ for $a > 0$, for calculating $\sqrt{a}$ has a quadratic speed of convergence. By using this formula, calculate $\sqrt{10}$ with four correct decimal digits (Heron's formula).

6) Work out an iteration formula for calculating $\sqrt[n]{a}$ by building Newton's method for solving the equation $x^n - a = 0$.

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