OPTIMIZING THE PARAMETERS OF POLARIZED ELECTROMAGNETIC CONSTRUCTION USING MATHEMATICA

Physical and mathematical formulation of the problem

We are looking at a polarized mechanism (figure 1), which consists of an excitation winding *1* and a permanent magnet *2* moving along the axis. By means of key *3* the winding is connected for a period of time t_1 to a source of voltage U - *4* and after a short pause t_p, it is is once again connected to the source for time t_2 but the polarity of the voltage is changed. The periods of connected state t_1 and t_2, and the period of the pause t_p are chosen so that during the time t_1 the permanent magnet is drawn to the center of the winding. The pause t_p follows and after the magnet bypasses the center the key *3* is switched over for the time t_2. The voltage and the current reverse their direction and this causes the magnet to be pushed up in the same direction.

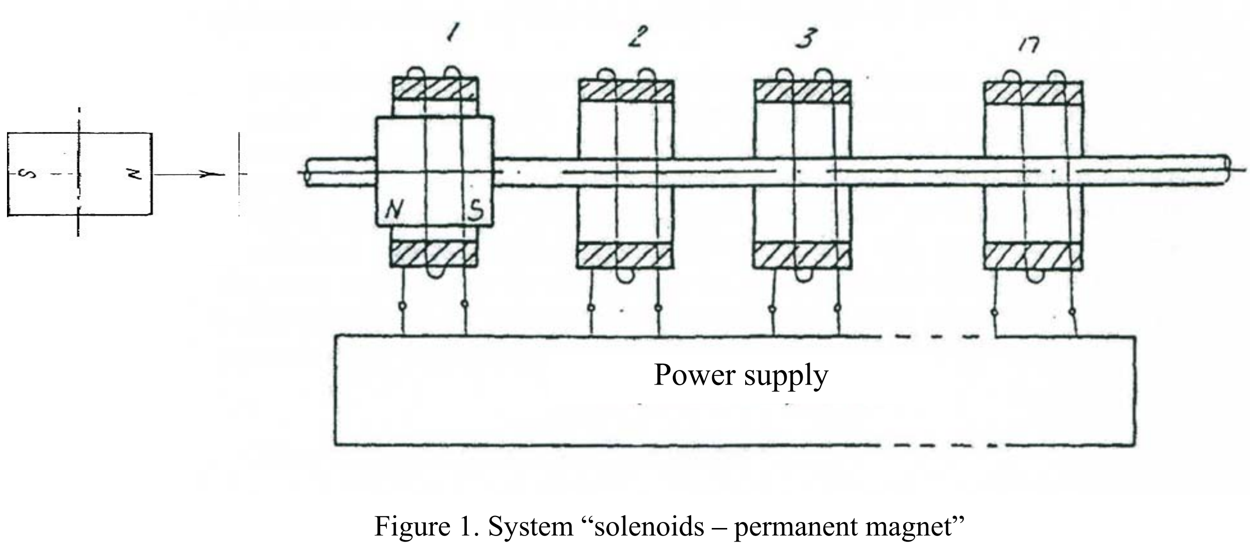


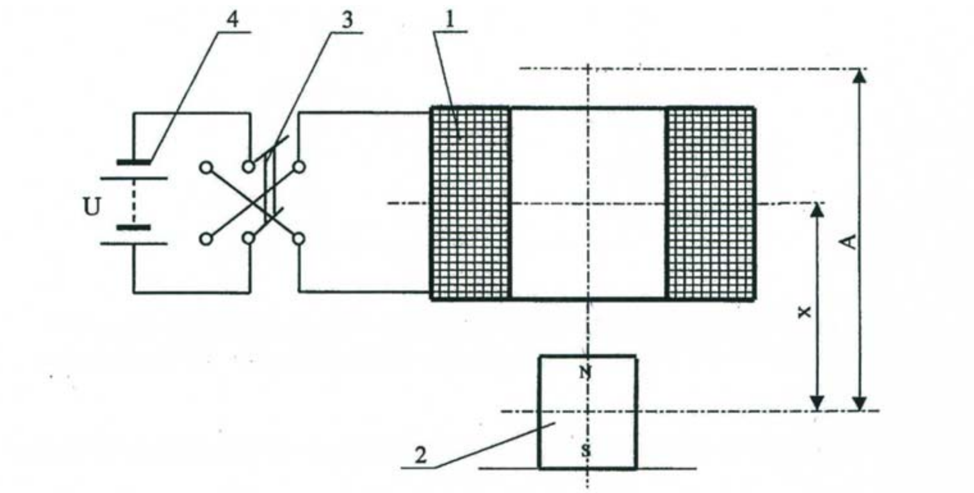
Figure 1. System “solenoids – permanent magnet”

This is opposed to a permanent magnet moving at a definite speed can be stopped without a hit by means of another solenoid (a system of solenoids) the excitation of which causes a force with direction opposite to the movement (negative acceleration).

The results from an analysis on the dynamics of such a mechanism in acceleration mode are presented here. The same method without any changes in principle can also be applied when working in a halt mode.

A basic algorithm is developed to determine the dynamic mode of such a mechanism containing only one solenoid which is controlled by means of impulses generated by a source of direct current. We view a simplified model composed by a cylindrical solenoid with vertically orientated axis line and a cylindrical permanent magnet which is coaxially orientated according to the solenoid and is driven in the direction of the same axis line (figure 2).

Figure 2. Polarized mechanism supplied by a source of voltage:
1-excitation winding;
2- permanent magnet;
3- key;
4- source of voltage.



The dynamic problem for the drive is reduced to defining the covered distance $x(t)$ under the influence of a "driving out" electric impulse. The differential equation calculating the movement of the body for the time of the impulse is

(1) $$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} = F_{ел} - F_c = F$$,

where r is the coefficient of friction and F_c - the force of the counteracting mechanism, which in most cases is a function of x. In order to simplify the physical experiment it is accepted that this force is a constant. It is replaced by the weight of the permanent magnet F_g. In addition, if the speed is relatively low, the force of friction could be ignored and if the speed is high but constant the force of friction is a constant and could be transferred to the right side, therefore the equation (1) is simplified to the following:

(2) $$m\frac{d^2x}{dt^2} = F_{ел} - F_c = F \cdot$$

As simple as it looks no analytic solution can be found for this equation because of the difficulties of defining the function $F_{ел}(x)$. The solution of this equation is reduced to a system of two first order equations after the respective replacing.

(3) $$\left| \begin{array}{l} x' = v \\ mv' = -rv + F \end{array} \right.$$

$$x(0) = x_0; v(0) = v_0$$

In this way, following the described algorithm the calculations continue in order to match the given duration t_1 of the first voltage impulse. After the voltage is switched off (it is assumed that the permanent magnet is close but it has not reached its central position yet and $x = A$ (figure 2), it continues to move by its own momentum in the same direction as a free body with initial speed that is already calculated and known. After the beginning of the second impulse the defining of the parameters of the movement continues but the initial conditions of position and speed are different from zero. The algorithm of such a solution can be applied in case of brake mode, where the permanent magnet moves (for example falls) with a definite kinetic energy and in a given moment t=0 the winding is connected to the source in such way that it creates a counteracting force opposing the movement during both impulses.

Numerical experiments with the basic algorithm

A. Defining the electromotive force

The defining of the function of the electromagnetic force $F_{ел}(x)$ of interaction between the magnet and the field of the solenoid (with constant current $I = const$ through it), is done using the method of the finite elements. Calculations are carried out using David Meeker's **FEMM 3.3 package** with triangular finite elements and linear functions. In the figure below are displayed the interpolation polynomials of the tabular values calculated by FEM (Finite Element Method) electromotive forces, which correspond to four windings with different lengths. The coordinate x is defined as distance between the centers of the solenoid and the permanent magnet.

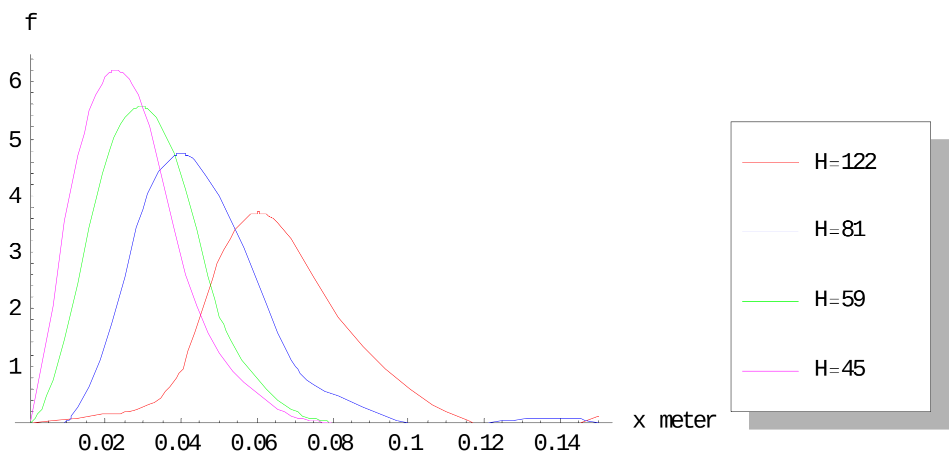


Figure 3. Electromotive force for different windings

B. Defining the Law of Motion

The solution of the above mentioned ODE system (Ordinary Differential Equation) is done by means of **NDSolve** operator of the **Mathematica**. The initial position of the body is chosen so that the electromotive force is higher than its weight in order to begin a movement in the desired direction.

For the windings in view we have accepted $x_0 = -0.065\,m$, which is experimentally established.

The covered distance and the speed as a function of time for the four different windings are displayed in different colors in the two figures below.

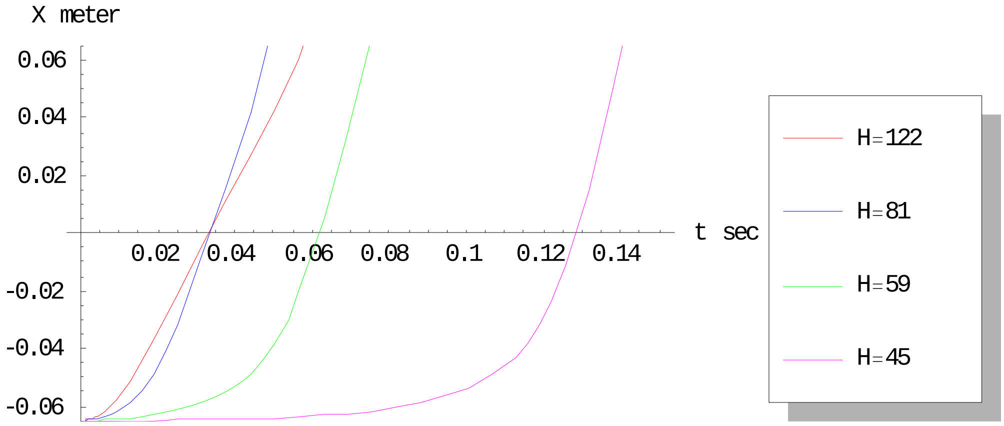


Figure 4. Representation of the covered distance X(t) for a definite time

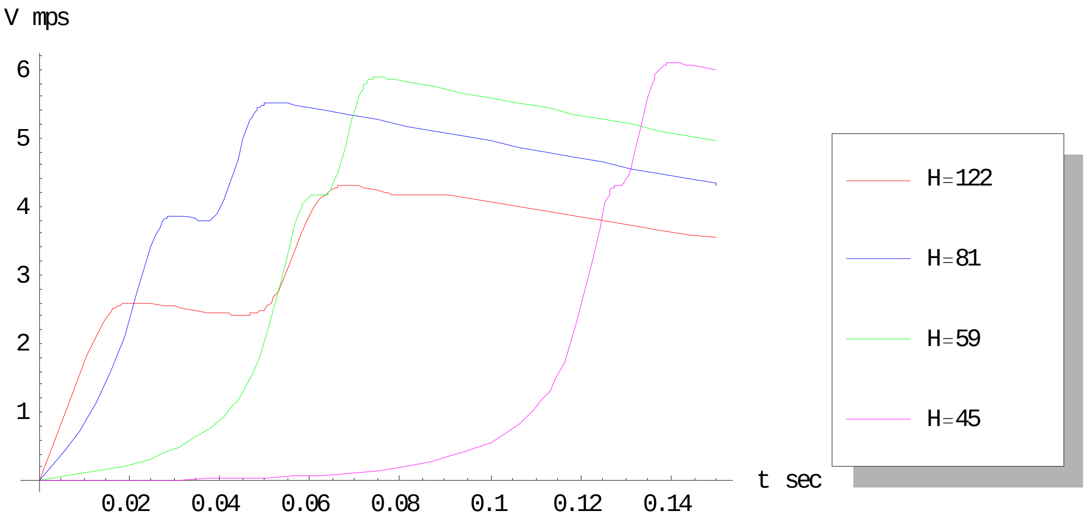


Figure 5. Representation of the speed as a function of time

In order to obtain the curve of speed as a function of the covered distance we make a list of values for the two above mentioned functions (different for the different lengths of the windings) for the same values of time and we draw the functions received after interpolation.

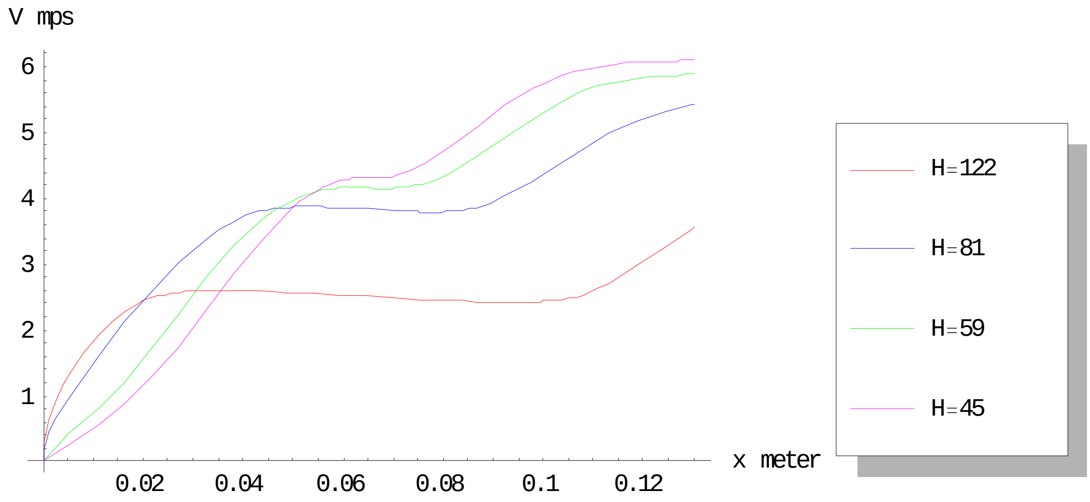


Figure 6. Representation of the speed as a function of the covered distance

We define the time, for which the body in hand will reach a position symmetric to the initial in relation to the center of the solenoid, by defining the root of the equation $x(t)=x_{end}$. It is numerically done according to the type of the function $x(t)$ by means of Solve operator of *Mathematica*..

Winding length (mm)	**H=122**	**H=81**	**H=59**	**H=45**
Time (sec)	**0.0580**	**0.0485**	**0.0748**	**0.1405**

From the above displayed values of the time needed to reach the definite state we see that there are considerable differences depending on the winding length. It is quite natural to set the problem for defining the "optimum" length of the winding, namely the length that guarantees minimum time for covering a definite distance.

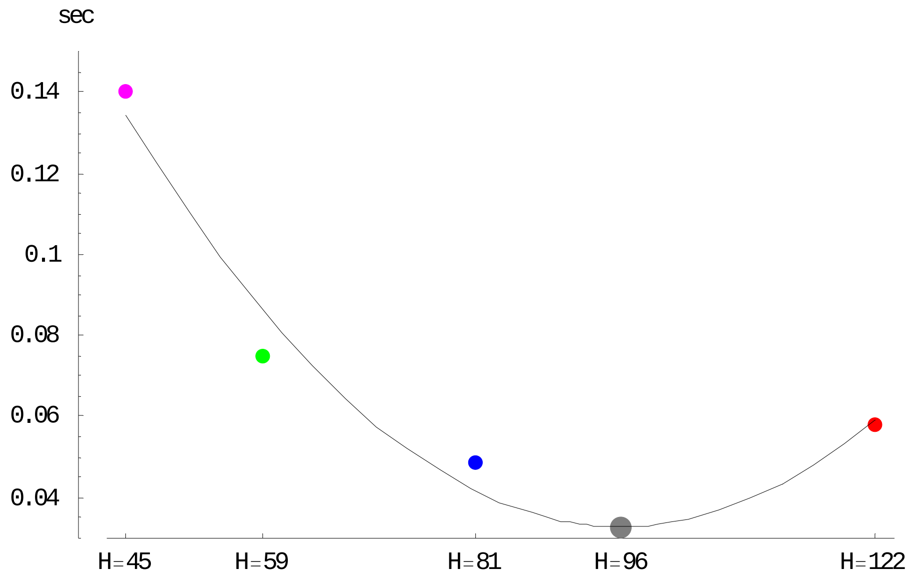


Figure 7. Representation of the time for covering the full distance for the four different winding lengths (A=0.13)

During the work process this is done by approximating the above displayed table by the Least Squares Method with a second order polynomial. The optimum winding length corresponds to the top of the obtained parabola.

Using a program we can do this using the **Fit** and **Minimize** operators of the ***Mathematica***. The results from these calculations are shown in figure 7 above. The bigger point corresponds to the obtained minimum value.