Power method for solving the partial eigenvalue problem

Sometimes only the modularly largest eigenvalue λ is needed. It is well known, that

$$\frac{A^{k+1} y_j^0}{A^k y_j^0} \approx \lambda, \ j = \overline{1, n},$$

where $\vec{y}^{\,0} = \left(y_1^0, ..., y_n^0\right)$ is arbitrary n – dimensional vector and $\vec{y}^{\,0} \neq \vec{0}$.

It is recommended that the derived vector is normalized on every step, so that its largest component is 1. On the last step the normalizing multiplier will give the value of the modularly biggest eigenvalue. Also, the eigenvector is derived.

Example. Find the modularly largest eigenvalue of the matrix

$$A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

and its corresponding eigenvector.

Solution:

Let $y^0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is the non-zero initial vector. Then we calculate

$$Ay^0 = \begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix} \approx \begin{pmatrix} 1 \\ 0{,}8 \\ 0{,}4 \end{pmatrix}, \qquad A^2 y^0 = \begin{pmatrix} 4{,}8 \\ 3 \\ 1{,}2 \end{pmatrix} \approx \begin{pmatrix} 1 \\ 0{,}625 \\ 0{,}25 \end{pmatrix},$$

$$A^3 y^0 = \begin{pmatrix} 4{,}625 \\ 2{,}5 \\ 0{,}875 \end{pmatrix} \approx \begin{pmatrix} 1 \\ 0{,}5405 \\ 0{,}1892 \end{pmatrix}, \qquad A^4 y^0 = \begin{pmatrix} 4{,}5405 \\ 2{,}2702 \\ 0{,}7297 \end{pmatrix} \approx \begin{pmatrix} 1 \\ 0{,}499 \\ 0{,}1607 \end{pmatrix},$$

$$A^5 y^0 = \begin{pmatrix} 4{,}4999 \\ 2{,}1605 \\ 0{,}6606 \end{pmatrix} \approx \begin{pmatrix} 1 \\ 0{,}4801 \\ 0{,}1468 \end{pmatrix}, \qquad A^6 y^0 = \begin{pmatrix} 4{,}4801 \\ 2{,}107 \\ 0{,}6269 \end{pmatrix} \approx \begin{pmatrix} 1 \\ 0{,}4703 \\ 0{,}1399 \end{pmatrix},$$

$$\frac{A^6 y_j^0}{A^5 y_j^0} = (1;\ 0{,}98;\ 0{,}95)^T \to A^6 y^0 \approx A^5 y^0$$

with an accuracy of $\varepsilon = 0{,}05 \to \lambda = 4{,}48; \quad x = (1;\ 0{,}47;\ 0{,}14)^T$.

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