Interpolation and extrapolation

Formulation of the problem. Let the function $y = f(x)$ be defined in an interval with a known table of its values

x_i	x_0	x_1	...	x_n
y_i	y_0	y_1	...	y_n

We assume that the points x_i are different and sorted increasingly in value: $x_0 < x_1 < ... < x_n$.

We need to find an approximate function $\varphi(x)$, which satisfies

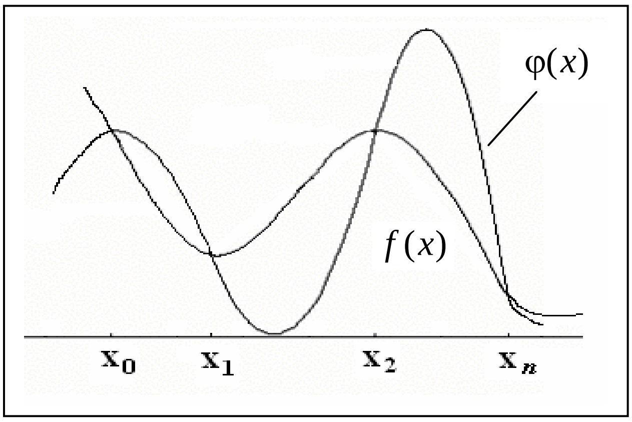
$$\varphi(x_i) = y_i,\ i = 0,1,\ldots,n. \qquad (1)$$

This problem is solved by choosing some class of functions $\varphi(x)$.

By (1) and a given class of functions we can obtain a formulae $\varphi(x) \approx f(x)$ which is usually used for approximate calculation of the function $f(x)$ at arbitrary point x^*. If $x^* \in [x_0, x_n]$, then $\varphi(x)$ is called to "interpolate" $f(x)$ in x^*. If $x^* \notin [x_0, x_n]$, i.e. the point is not in the interval $[x_0, x_n]$, then $\varphi(x)$ "extrapolate" $f(x)$ in x^*.

Definition. The points $x_0, x_1, \ldots, x_n$ are called points or knots of interpolation.

Graphically condition (1) means that the approximate function $\varphi(x)$ passes through the points (x_i , y_i) seeing as there are the same values in x_i as in $f(x)$ - see the next fig.



Examples

1) If we select the system of functions $1, x, x^2, \ldots, x^n$ and explore all linear combinations in this system we will obtain the class of the polynomials of the n-th power $P_n(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$. The interpolation polynomial is defined by the condition

$$P_n(x_i) = y_i, \quad i = 0, 1, \ldots, n. \qquad (2)$$

2) If we select the system of functions

$$\frac{1}{2}, \cos(x), \sin(x), \cos(2x), \sin(2x), \ldots, \cos(nx), \sin(nx), \ldots$$

we will obtain the class of the trigonometrical polynomials of the n-th power

$$T_n(x) = a_0 + a_1 \cos(x) + b_1 \sin(x) + \ldots + a_n \cos(nx) + b_n \sin(nx)$$

etc.

According to the type of the function $f(x)$ the first thing to do is define the appropriate class of approximate functions and after that the corresponding interpolation function (polynomial) $\varphi(x)$ satisfying the conditions (1).