Finite difference

Let the interpolation points $x_0, x_1, \ldots, x_n$ be equidistant with a step of $h > 0$, in other words $\boxed{x_k = x_0 + kh,\ k = 0,1,\ldots,n}$. And let the table of values for the function $y = f(x)$ be given**:**

x_i	x_0	$x_1 = x_0 + h$	...	$x_n = x_0 + nh$
y_i	y_0	y_1	...	y_n

Definition 1. Finite differences of the first order are called:

- in x_0: $\Delta_0 = y_1 - y_0$; in x_1: $\Delta_1 = y_2 - y_1$, ..., in x_{n-1}: $\Delta_{n-1} = y_n - y_{n-1}$ or in the general case: x_i: $\Delta_i = y_{i+1} - y_i$, $i = 0,1,...,n-1$.

Definition 2. Finite differences of the second order are called:

- in x_0: $\Delta_0^2 = \Delta_1 - \Delta_0$; in x_1: $\Delta_1^2 = \Delta_2 - \Delta_1$, ..., in x_{n-2}: $\Delta_{n-2}^2 = \Delta_{n-1} - \Delta_{n-2}$ or in the general case: x_i: $\Delta_i^2 = \Delta_{i+1} - \Delta_i$, $i = 0,1,...,n-2$.

Definition 3. Finite differences of *k*-th order are the numbers:

$$\Delta_i^k = \Delta_{i+1}^{k-1} - \Delta_i^{k-1},\ i = 0,1,...,n-k,\ k = 1,...,n.$$

We receive the following triangular table of the finite differences:

i	x_i	y_i	Δ_i	Δ_i^2		Δ_i^{n-1}	Δ_i^n
0	x_0	y_0	Δ_0	Δ_0^2		Δ_0^{n-1}	Δ_0^n
1	x_1	y_1	Δ_1	Δ_1^2		Δ_1^{n-1}	
2	x_2	y_2	Δ_2	Δ_2^2			
…							
n-2	x_{n-2}	y_{n-2}	Δ_{n-2}	Δ_{n-2}^2			
n-1	x_{n-1}	y_{n-1}	Δ_{n-1}				
n	x_n	y_n					

Example. To create the table of the finite differences of the values of the function $y = e^x$:

x_i	3.60	3.65	3.70	3.75	3.80
y_i	36.598	38.475	40.447	42.521	44.701

(*)

We define $h = 0.05$, a constant step, so we can calculate the finite differences. The table is:

i	x_i	y_i	Δ_i	Δ_i^2	Δ_i^3	Δ_i^4
0	3.60	36.598	1.877	0.095	0.007	-0.003
1	3.65	38.475	1.972	0.102	0.004	
2	3.70	40.447	2.074	0.106		
3	3.75	42.521	2.180			
4	3.80	44.701				

Note that for the given function the finite differences decrease in value.