

# Reusable Learning Objects in Mathematical e-Learning<sup>1</sup>

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## 1. Introduction to Current e-Learning Strategies and Solutions

Since the Internet has become a widely used educational environment, education communities have at first adopted a concept of creating educational materials in the form of the complex full text electronic courses, comprising all course material in large educational modules. Traditionally, content came in a several hours chunk called a course. Currently a new concept for e-learning has been adopted, and it is the object-based learning. This approach is a new way of thinking about instructional content. The idea is to decompose existing course material into smaller units, o-called "learning objects" (1). Learning objects are self-contained, modular pieces of course material appropriately annotated with metadata. They may be combined to form larger educational interactions, traditional modules, but in a modular way. The goal is to develop an open architecture for online learning that will allow teaching to be centred around the needs and interests of the learner, enabling learning to occur anytime, anyplace and to allow for greater customization and flexibility of the learning environment.

Learning objects have emerged as instructional technology's new paradigm. This idea has gained such broad acceptance that the IEEE has formed the Learning Technology Standards Committee to pursue the creation of common standards for the description, interchange, and management of learning objects (1). As time passes, eventually object-based technologies are replacing classical instructional design approaches. This leads to reuse, and reuse leads to faster development and higher quality instructional contents. In addition, as object-based systems are easier to adopt and easier to scale, different re-assembling of reusable learning objects can create large and colourful instructional materials.

This paper aims to describe how reusable learning objects might be used in teaching Mathematics with e-technology, in order to enhance better conceptual understanding, and to introduce some from the numerous advantages of the ICT utilisation in teaching maths. Few comments on the design of data-driven reusable learning object-based lesson that meet the learners and instructional design requirements is given. First some background and key aspects of learning object design principles and instructional properties are given. This is followed by an explanation of how reusable object-based lesson model with sound instructional design is defined. Several examples are then provided to illustrate how to implement the reusable object-based lesson, module and a complete course.

## 2. What is a Reusable Learning Object

In the last few years, learning objects have been widely discussed in the open literature in many different perspectives. There can be find many terms that are used in the literature and industry, besides „learning objects,, by IEEE Learning Technology Standard Committee (1). Other terms that imply the general intention to take an object-oriented approach to online learning are "knowledge objects", "pedagogical documents", "online learning materials", and "educational software components" (2, 3). The Learning Technology Standards Committee had chosen the term "Learning objects" to describe these small instructional components and provided a working definition: "any entity, digital or non digital, which can be used, reused or referenced during technology supported learning. Examples of technology-supported learning include computer-based training systems, interactive learning environments, intelligent com-

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puter-aided instruction systems, distance learning systems, and collaborative learning environments. Examples of learning objects include multimedia content, instructional content, learning objectives, instructional software and software tools, and persons, organizations, or events referenced during technology supported learning" (4).

Currently, we can see more and more instructional content developed specifically to be deployed as learning objects in multiple learning contexts due to its potential for reusability, interoperability, discoverability, and manageability. There can be described two approaches to design the reusable object-based lesson: first - by searching and using the existing instructional materials which can be considered as learning objects from the online learning sources, such as the Internet, second - design or convert the instructional materials into object forms. The key underlying principle of creating object-based lessons from learning objects is that sound instructional design practices must be followed. Standardized principles for design of learning objects and instructional properties are necessary in the design process so that instructional contents can eventually be used for facilitating intended learning outcomes and reused within and between different learning contexts at an appropriate level of granularity.

An instructional content is only considered as learning object if with the following instructional properties:

a) Learning objects must be instructional objects

A learning object is not just a piece of information. It must provide deliberate instruction at appropriated level with meaningful interaction, in order to retain skills or key concepts by the learner. Often if the object is an informational object, the intention is just to inform, but if the object is a learning object, then it will provide an environment that is much more conducive to facilitate learning and reinforce the recognition of skills or key concepts.

b) Learning objects must be relatively small

The learning objects for instructional contents should focus on a single learning objective, so that it will be relatively small, discrete or unit of knowledge to support flexible, individualized learning. It is important to note that simply being physically small does not qualify as a learning object. To equate a learning object, information or content must be small and focused. If each learning object is based upon a single objective, and the granularity is small enough, then each learning object will be "appropriately" small.

c) Learning objects must be extractable or stand-alone

A learning object must be self-sufficient to provide instructional information in the form of modular units and it must not rely on previously learned information, references or examples in order to clearly provide instruction on a concept (5). The lack of self-sufficiency is one of the reasons why most existing educational materials do not qualify as learning objects.

d) Learning objects must be usable on a standard platform.

Learning objects should work in a variety of standard Web platforms. It should require only web browsers and some common plug-ins which are available free for download from Internet for viewing objects created with Flash, Director, Shockwave, Java Applet, and Java Scripts. No prior software component installations should require other than those provided by the web browsers and operating systems.

e) Learning objects must be tagged and searchable.

Learning objects must be shared, accessible and discoverable by others across learning environment, in order to be used, and reused widely to meet real-world performance criteria. They must be labelled as to what they contain, what they teach, and what requirements exist for

using them so that users understand what a learning object is about without ever seeing them (9, 10). Currently there are initiatives for building the metadata schema that will allow for the universal sharing of learning objects. Examples of existing metadata standard which can be used are Dublin Core Meta-data (6) and IMS Metadata (7).

### 3. Designing RLOs in Mathematics

World Wide Web provides great opportunities to Math educators to utilize and create on-line educational materials. Not only text and picture, but also animation and sound can be used in educational materials on Internet. Learning objective, introduction, and summary are additional learning strategies, which must be designed and integrated into every RLO in order to create a complete instructional experience. Many companies made web-ready software, such as Math View by Waterloo Maple Inc, which can be used to manipulate mathematical symbols and draw 2D and 3D graphs in browser windows, or the RLO-CETL, the Centre for Excellence for the design, development and use of learning objects, the partner institutions are London Metropolitan University, the University of Cambridge and the University of Nottingham. At the same time, interactive learning materials from individuals such as Java applets, and Flash are now growing rapidly on Internet. We might be able to find what we need in the near future, and utilize these educational materials for other purposes by making links.

Many existing on-line databases contain reusable learning objects bringing instructional study material from different parts of mathematics. Majority of them work in so called „closed-shop” exclusively, and these provide access to the study material after the required login and password input. Among those freely accessible on Internet, one can find EVLM - European Virtual Laboratory of Mathematics, product of the European project within the Leonardo da Vinci programme scheme for vocational training (8, 9). EVLM provides on-line database storing different types of e-learning materials distributed into several mathematical subjects (Fundamental Maths, Algebra, Calculus, Geometry, Difference and Differential Equations, Multivariable Calculus, Probability, Statistics, Numerical Analysis, Optimisation and History of Maths) and material types (FACTs, RLOs, MODULEs and PROBLEMs). When learners needs a specific piece of information, they can navigate in the EVLM database by material type in a request, and get the relevant, only needed learning objects. Figure 1 shows the interface page to search the appropriate material type and the object details that appears on the learner’s screen after retrieve from the repository. This particular object is a RLO web page reference that describes what different reusable learning object types of materials are available in the EVLM database repository in Geometry. Besides being used to recombine to form larger educational interaction, e. g. modules in some particular geometric theme, each of these RLOs can be used by learners as an independent learning material to enhance certain concept, principle or idea.

Most of the learners can use e-learning resources as a supplement to the traditional lecture and textbook and this would provide substantial benefits. Students at the Faculty of Mechanical Engineering, Slovak University of Technology were asked to indicate their interest in the ability to access the most updated learning contents from a pool of learning objects to tailor learning content for multiple learners, and to personalise learning content to the needs of individuals offered by object-based learning systems. In each case, a majority of students considered these capabilities to be highly beneficial. Because the learning object approach allows teaching and learning to be centred on the needs and interests of the learners, enabling learning to occur anytime, anyplace and to allow for greater customisation and flexibility of learning contents at the level of granularity desired.

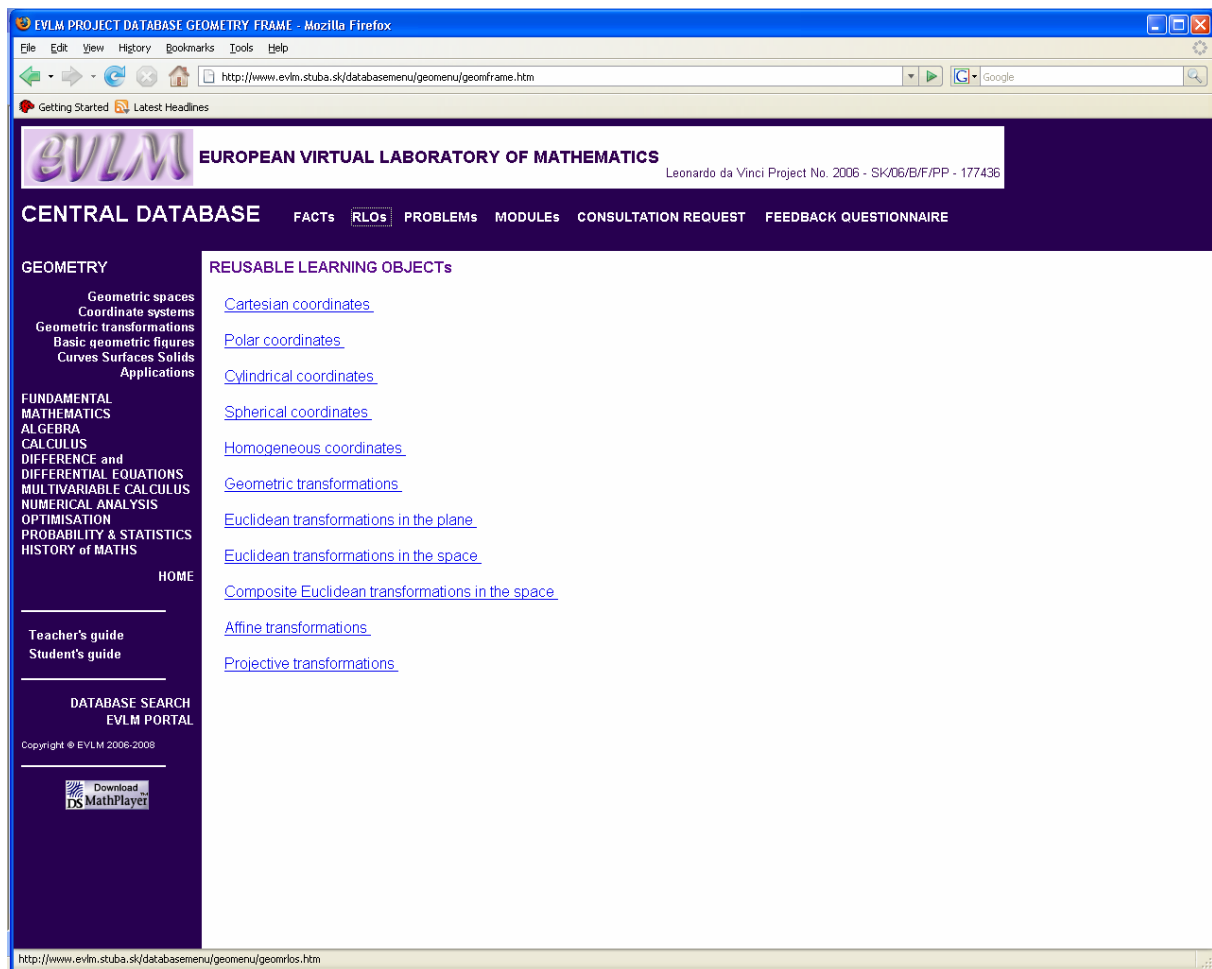


Fig. 1: EVLM database contents of RLO in Geometry

A basic problem faced by the learning community is how to produce and deliver quality content for online learning experiences. Online learning content typically contains:

- a) Text, tables, graphics, on-line calculations, or video shots and movies,
- b) A navigation scheme (easily a table of contents and/or buttons).

It may also contain collaboration tools as well as other interactive elements and graphical elements designed to produce a unified or branded look and feel, but the above list is basic.

To be learning content, the content should also be aware of learners. At a minimum, learning content should recognize who the learner is and record information about the learner's experience. To make this possible, learning content has generally been developed in conjunction with some sort of learning system that keeps track of learners. Learners log on to the system and launch the content. As the learners interact with the content, results are passed back to the system. If the system allows it, the content can also change its behaviour based on learner information stored in the system. For example, learners might be sent to different places in the content based on test scores, language preferences, learning style inventories, competencies, certifications, organizational roles, and other data.

Interoperability (content from multiple sources working equally well with different learning systems) and reusability (content developed in one context being transferable to another context) are imperative to the sustainability of the reusable learning object design. Without them, anyone with a significant investment in either content or a learning system is locked in to that particular content or system. Without them, every time a course, (whatever it is) or an interac-

tive electronic training manual needs to be updated, far more of the material must be rewritten than is necessary or desirable. Without them, the process of developing high-quality content is prone to unnecessary duplication of effort, driving up the cost, possibly past what the market will bear.

Reusable learning objects represent an alternative approach to content development. In this approach, content is broken down into chunks. From a pedagogical perspective, each chunk might play a specific role within an instructional design methodology. The requirements for each chunk are:

- Each chunk must be able to communicate with learning systems using a standardized method that does not depend on the system
- What happens within a chunk is the chunk's business.
- How a learner moves *between* chunks is controlled by the learning system.
- Each chunk must have a description that enables designers to search for and find the right chunk for the right job.

Such chunks are called learning objects. There is no standard for the size (or *granularity*) of a learning object. Larger learning objects are typically harder to reuse, and smaller learner objects save less work for those who are assumed to reuse them.

For learning objects to be used they must be found. It is not easy to find anything in a large distributed online environment like the World Wide Web or a large intranet. The solution is to store not only learning objects but also descriptions of the learning objects. Thinking of the learning objects as data, the descriptions are data about the data, or metadata. Learning object metadata potentially includes information about the title, author, version number, creation date, technical requirements and educational context and intent.

### **Pros and Cons**

There are some of the pros and cons of learning objects that you should be aware of:

#### 1. Production Costs

**Pro:** By properly breaking content into learning objects, different parts can be maintained and updated separately. If a suitable learning object can be found, a new one does not need to be created. These are costs savers.

**Con:** Changing to a learning object approach from a "self-contained system" approach involves retooling and retraining costs.

#### 2. Flexibility

**Pro:** As more and more standards-based learning objects become available, increased choice will translate into more flexibility for designers.

**Con:** Using standards-based learning objects restricts the scope of learner information that is accessible by content if total interoperability is maintained.

#### 3. Pedagogy

**Pro:** Learning objects fit nicely into many ISD theories. Instructional templates can be created with slots for specific types of learning objects. Learning objects may encourage designers to operate in more disciplined ways with a positive effect.

**Con:** Restrictions on learner information available could restrict pedagogical approaches. Approaches using lengthy discursive material may not benefit from the use of learning objects.

#### 4. End User Cost

Pro: The learning object approach prevents consumers from being locked in to specific systems. As standards take hold, the market for content will take on more of the properties of a typical consumer market with lower costs and increased choice.

Con: The cost of converting existing content to a learning object approach may be significant.

#### 4. Some Examples of RLOs in EVLM Central Database

In the EVLM database there are available several forms of reusable learning objects. Majority of the current study material is developed as educational text with illustrative figures and solved examples. These are stored as PDF, JSP, HTML or XML document files with embedded MathML coding of formulas and expressions. Presentation of these materials is possible in the most commonly used web-browsers, e.g. MS Internet Explorer or Mozilla Firefox, using the standard plug-in as Acrobat reader, Java Plug-in technology included as part of the Java Runtime Environment, Standard Edition (Java SE), which establishes a connection between popular browsers and the Java platform. This connection enables applets on Web sites to be run within a browser on the desktop.

MathML is a new way of encoding mathematics using XML developed under the auspices of the W3C (World Wide Web Consortium), the group that sets the basic standards that define the Web. A growing number of software packages including browsers, editors, computer algebra programs and publishing software use MathML to communicate. Unlike other ways of putting math in a web page, such as images and PDFs, MathML provides ways to directly encode various interactivity properties of an equation, which makes it an ideal choice for dynamic math on the web. When someone visits a Web page that contains math, Internet Explorer "sees" a document written in HTML containing MathML "islands" for each equation. Some information in the header of the document tells Internet Explorer that MathPlayer software is to be used to display and print the MathML islands. Internet Explorer loads and executes the Math Player software on an as-needed basis. Whenever Internet Explorer sees MathML, it gives it to Math Player to display on the screen (or print it). Equations can be copied to the clipboard as MathML and then pasted into any application that understands MathML or into a web editor. Drag-and-drop works similarly. Among the applications that understand MathML are the popular computer algebra systems, Mathematica and Maple. In the near future eventually all mathematical and scientific software applications will support MathML. One of the advantages of embedding math in a web page using MathML is that it makes it possible for the maths to be spoken. This was always a goal of the W3C in order to make maths easier to understand by the visually impaired. Math Player 2 begins to realize that goal by providing the ability to speak the math in a web page. The user can do this by choosing *Speak Expression* on Math Player's right-click menu or, more importantly, via a screen reader application such as Window-Eyes or JAWS. In the current release of Math Player, the math-to-speech capability is fairly natural, as the main goal of math-to-speech technology is in teaching normally sighted students how mathematics is spoken.

Embedding maths in a web page using MathML also makes it possible for web searches to include the mathematics on pages, not just text.

Many Windows applications use Microsoft's HTML engine, MSHTML, to display formatted content. This includes email clients, alternative web browsers, weblog (RSS) clients, instant messaging clients, help engines, and so on. Because Math Player works by MathML-enabling MSHTML, MathML support is available in such applications literally for free. The only thing missing is a way to get MathML content into the HTML.

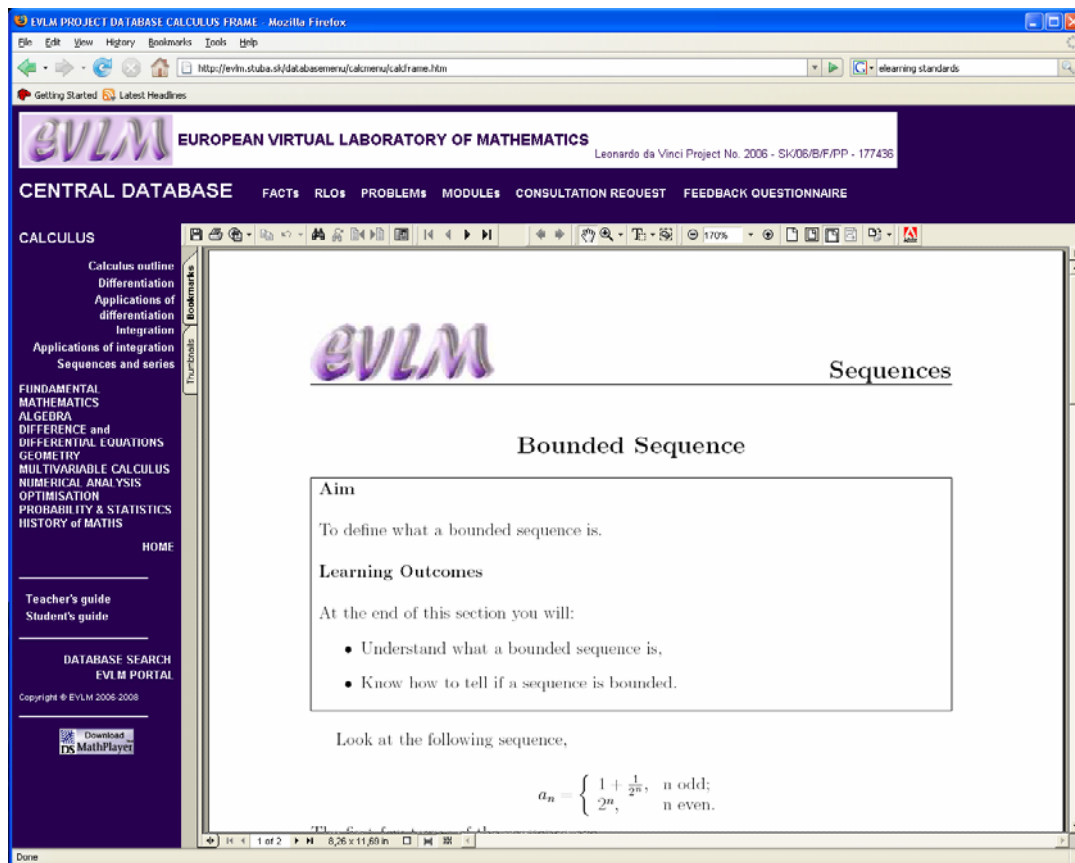


Fig. 2: Example of a reusable learning object – Bounded Sequences, PDF file

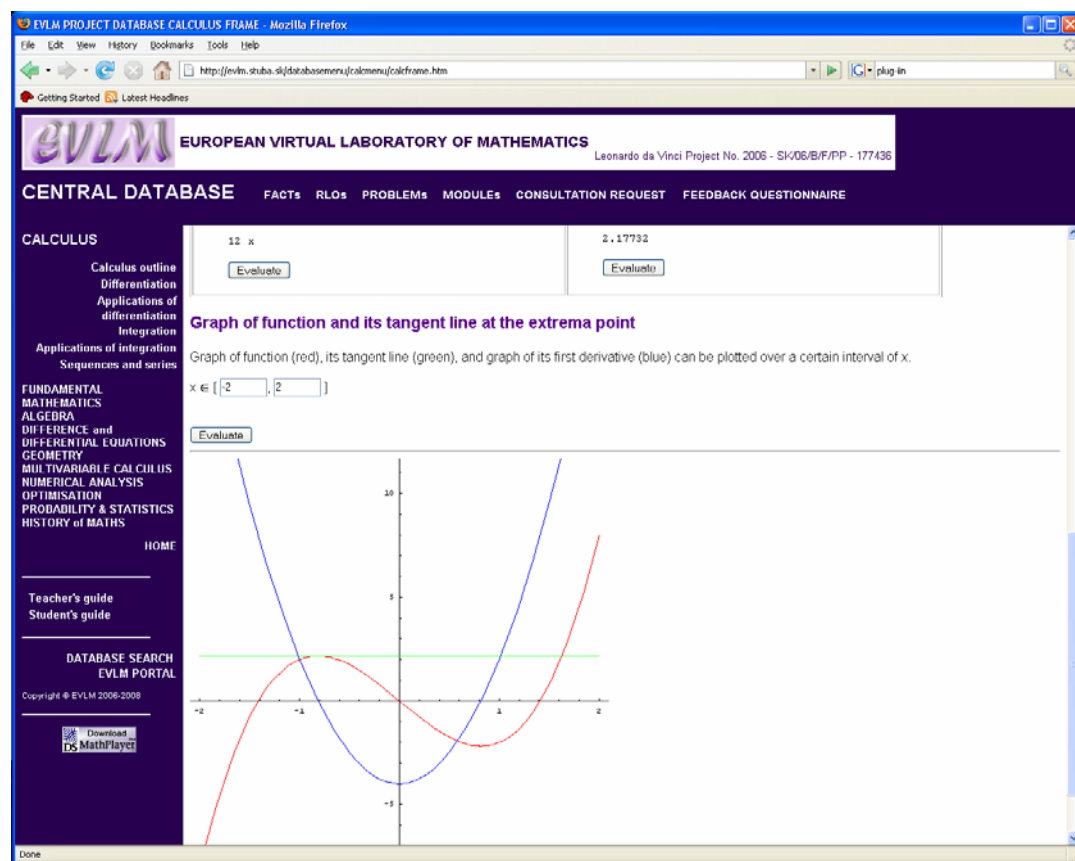


Fig. 3: Example of a reusable learning object – Function Extrema, JSP file

**Point of Inflection**

If a graph of a continuous function possesses a tangent line at a point where its concavity changes from upward to downward or vice versa, then the point is called the point of inflection. Graph of the function crosses the tangent line at a point of inflection. Since a point of inflection occurs at the point of change of graph concavity, at this point the sign of the function second derivative (if it exists) must be changing.

**Function**

$$f(x) = 2x^3 - 4$$

To locate possible points of inflection, those values of  $x$  must be determined, for which the function second derivative is zero, or it does not exist.

**The first derivative of the function**

$$f'(x) = ?$$

$6x^2$

**The second derivative of the function**

$$f''(x) = ?$$

$12x$

**Located points of inflection**

$$f''(x) = 0$$

$(x0 \rightarrow 0.1)$

**Tangent line equation**

Tangent line at the point of inflection can be determined by equation

$$y = f(x0) + f'(x0)(x - x0)$$

$x0 = 0$

Insert one from the critical points calculated above.

$$y =$$

$-4$

Fig. 4: Example of a reusable learning object – Point of Inflection, JSP file

**The Meaning of the Derivative**

Move the slider  $h$  to reduce its value. You will see that as  $h$  gets closer to zero the red chord gets closer to becoming a tangent. The derivative of the function is the slope of the tangent at the point where  $h$  is zero.

$\frac{f(x+h) - f(x)}{h}$  = Slope of the red chord

$f(x+h)$

$f(x)$

$h$

$h = 2$

Created with [GeoGebra](#)

Fig. 5: Example of a reusable learning object - the Meaning of the derivative, JSP file



Examples of reusable learning objects that can be found in the EVLM Central database are illustrated in Figures 2 - 5. The first presented example in Figure 2 is PDF file, where prior to the instructional text there are presented the aim and learning outcomes. JSP files, which are webMathematica applets are presented in Figures 3 and 4. Concepts of extrema points and point of inflection of function in one variable are explained and calculated by webMathematica engine for a particular function that can be inserted interactively, on-line. This way, students can practise mechanisms of identification of extrema points or points of inflection, and enhance the concept understanding by its geometric interpretation and representation provided in the plotted graph of the respective inserted function and tangent line to the function graph at the indicated point.

Concept of the function derivative and its geometric meaning is graphically illustrated in the last example presented in the Figure 5. Here the JSP file was created by the GeoGebra free-ware software is illustrated. The presented file is a Java applet, in which you can interactively create animation of the limit process hidden in the function derivative definition.

## 5. Conclusions

The use of learning object as a part of the design and development process of online learning settings provides a number of advantages. Until now, there has been relatively little activity undertaken to ensure that online learning settings are being designed in way that promote flexible use of the learning resources. This paper has discussed the use of learning objects with sound instructional design principles to define the object-based lesson model, and then highlighted the practical experiences of storing, using and assembling learning objects of particular interest is the RLOs from which multitude of different object-based lesson can be made.

E-learning technologies allow information to be presented in a more accessible, customized, learner-centred manner. Combining text, theory, and application with computer software, simulations, audio, video, and real-time discussion, all offered over the Internet, has made education more accessible than ever before. Students control the times and the pace of learning and determine the best method suited to their learning style. Teachers present the information, provide feedback, and serve in an advisory manner.

Research in cognitive psychology indicates that our brains store knowledge using both words and images. Instruction that targets and engages both of these systems of representation has been shown to significantly increase students' comprehension and retention. Explicitly engaging students in the creation and usage of non-linguistic representations has even been shown to stimulate and increase activity in the brain. Manipulatives are concrete or symbolic artefacts that students interact with while learning new topics. They are powerful instructional aids because they enable active, hands-on exploration of abstract concepts. Research has shown that computer-based manipulatives are even more effective than ones involving physical objects, in part because they can dynamically link multiple representations together.

A powerful XML-based mark-up language for publishing mathematics on the Web, MathML makes it possible to develop Web-based applications for displaying, searching, indexing, archiving, and evaluating mathematical content. MathML is about encoding the structure of mathematical expressions so that they can be displayed, manipulated and shared over the World Wide Web. A carefully encoded MathML expression can be evaluated in a computer algebra system, rendered in a Web browser, edited in the word processor, and printed on the laser printer. Mathematical software vendors are adding MathML support at a rapid pace, and MathML is fast becoming the lingua franca of scientific publication on the Web.

While much of these information may seem at times like techno-mumbo-jumbo and engineering speak, like learning any new language or way of doing something, it will seem hard at first but one will learn it quickly. Better start now and lead the pack than miss this vibrant opportunity to change and improve how all of us work and learn each day (Wayne Hodgins and Marcia Conner: Everything you ever wanted to know about learning standards but were afraid to ask.)

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