

EVLM project¹

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1. Project main objectives

Project EVLM - European Virtual Laboratory on Mathematics (basic information in (1)) is an approved project in the Leonardo da Vinci scheme of the professional vocational training for the period 2006-2008, and it is aimed at development, storage and dissemination of basic source e-learning materials and consultation topics in different parts of Mathematics. Many different electronic learning and teaching materials in various forms, formats and in many national languages appear frequently in large numbers on WWW. Therefore there has arisen an urgent need for an authoritative catalogue of available quality source materials for educational purposes in mathematics. A platform that will provide not only information on and links to existing e-learning materials, but also a centralised database of short compound materials bringing brief explanations and pieces of basic information and description of the most widely used mathematical concepts is extremely timely. Expert consultancy on how to use the aforementioned materials, on-line or personal help to interested users and the capacity to translate relevant materials to/from English will be necessary in order to disseminate existing e-learning resources and increase their overall usage and utilisation.

Teaching of Mathematics is still predominantly provided in a traditional way throughout the Europe, in spite of the fact that exceptionally this subject is one of the most important ones in connection to the fast development of ICT (which relies heavily on the development of Mathematics itself). Many new and quality e-learning educational materials developed at the institutional level at many European educational institutions that are appearing on internet are often not properly disseminated and commonly utilised, though these could be well shared by other institutions. European Virtual Laboratory of Mathematics – EVLM (2), operating on a transnational level in the form of a network of National Centres of Mathematics (3) located at the partner institutions and working within the common framework will serve as a platform to disseminate relevant information and a virtual database enabling not only the share of all available resources but also training materials on how to create and use them.

2. Project aims

Consultancy service in mathematics (in electronic or personal form) is the most specific aim of the project leading to the upgrade of the overall level of mathematical knowledge, and to the enhancement of competencies in Mathematics within the indicated target groups, which are secondary and university bachelor, master or PhD. students, secondary school teachers, trainers, university lecturers, researchers or scientists who are interested in improving their knowledge and understanding of mathematics.

The aim is also to promote e-learning in Mathematics, to provide solutions for different target groups and help for teachers and trainers to enhance their skills in using the most advanced educational tools and environments and to develop and authorise their own electronic learning materials that might be shared by others through the EVLM Portal. The access to advanced and supportive materials for all interested party is the way of supporting life-long learning activities and also any other potential final users, as e.g. private individuals as home self-learners, interested in self-education who, for personal or any other reasons, may not normally

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engage with formal education including disabled people and people perceiving discrimination due to social and gender stereotypes.

Project is aimed at promoting the most advanced and innovative ways of teaching, electronic based courses and e-learning materials, and to support the process of innovation in mathematics education. National Centres of Mathematics will combine innovative and traditional educational methods in the form of "blended solution", e-learning and ICT utilisation in addition to the personal individual approach and guidance of the consultant - teacher in the role of tutor serving individual students or teams of cooperating colleagues.

3. Project materials

National Centres of Mathematics will provide consultancy at a national level in national languages, which will eliminate or reduce constraints and obstacles that might appear when using e-learning materials available mostly in English. The on-going development of educational resources in national languages, for a range of scientific disciplines, is of a great importance in the European context. Nevertheless, all information will be provided also in English on the transnational level at the Central portal. Free on-line access to the Central portal and autonomous separate National portals for all interested party will be permanently available, providing service of a consultancy request delivery. Individual help and care will be provided in the National Centres of Mathematics on a personal level, and it will aim to create a friendly environment for the encouragement of all users. Use of the most advanced ICT under the guidance of experienced tutors providing personal help and advice at hand can greatly reduce the stress, experienced by some people in a competitive university environment.

Point of Inflection

If a graph of a continuous function possesses a tangent line at a point where its concavity changes from upward to downward or vice versa, then the point is called the point of inflection. Graph of the function crosses the tangent line at a point of inflection. Since a point of inflection occurs at the point of change of graph concavity, at this point the sign of the function second derivative (if it exists) must be changing. To locate possible points of inflection, values of x for which $f''(x) = 0$, or $f''(x)$ does not exist must be determined.

Second derivative of a function

$f(x) = \sqrt{2x^2+4}$

$f'(x) = ?$

$$\frac{d}{dx} \sqrt{2x^2+4} = \frac{1}{2\sqrt{2x^2+4}} \cdot 4x = \frac{2x}{\sqrt{2x^2+4}}$$

$f''(x) = ?$

$$\frac{d}{dx} \left(\frac{2x}{\sqrt{2x^2+4}} \right) = \frac{2 \cdot \sqrt{2x^2+4} - 2x \cdot \frac{2x}{\sqrt{2x^2+4}}}{(2x^2+4)^{3/2}} = \frac{2\sqrt{2x^2+4} - \frac{4x^2}{\sqrt{2x^2+4}}}{(2x^2+4)^{3/2}} = \frac{\frac{2(2x^2+4) - 4x^2}{\sqrt{2x^2+4}}}{(2x^2+4)^{3/2}} = \frac{4 - 2x^2}{(2x^2+4)^{5/2}}$$

Locate points of inflection

$f''(x) = 0$

Evaluate

$(x0 \rightarrow 0.), (x0 \rightarrow 0.), (x0 \rightarrow 0.), (x0 \rightarrow 0.), (x0 \rightarrow 0.),$
 $> (x0 \rightarrow 0.278414 + 1.21981 I), (x0 \rightarrow 1.12728 + 0.542867 I),$
 $> (x0 \rightarrow -1.25118), (x0 \rightarrow -0.780099 - 0.978213 I),$
 $> (x0 \rightarrow 0.278414 - 1.21981 I), (x0 \rightarrow -0.780099 + 0.978213 I),$
 $> (x0 \rightarrow 1.12728 - 0.542867 I)$

Fig. 1: Example of a short consultation topic on concept of a point of inflection

The most widely used pieces of electronic learning materials are expected to be short consultation topics, presented on-line at the portals and ready for use in the form of webMathematica applets (jsp. files) enabling on-line calculations and manipulations. These live mathematical materials will be supported by the webMathematica calculation server. Most frequently asked mathematical topics will be stored as FACTs – Frequently Asked Consultation Topics. Examples of such materials are presented in the following figures.

First material in fig. 1. shows the determination of a point of inflection of an elementary function. Few pieces of basic information are given, and example function is executed, resulting in the plot of the function graph, and its tangent line in the examined point of inflection. Own input data can be inserted interactively, while all necessary calculations are done on request, and new graph is plotted.

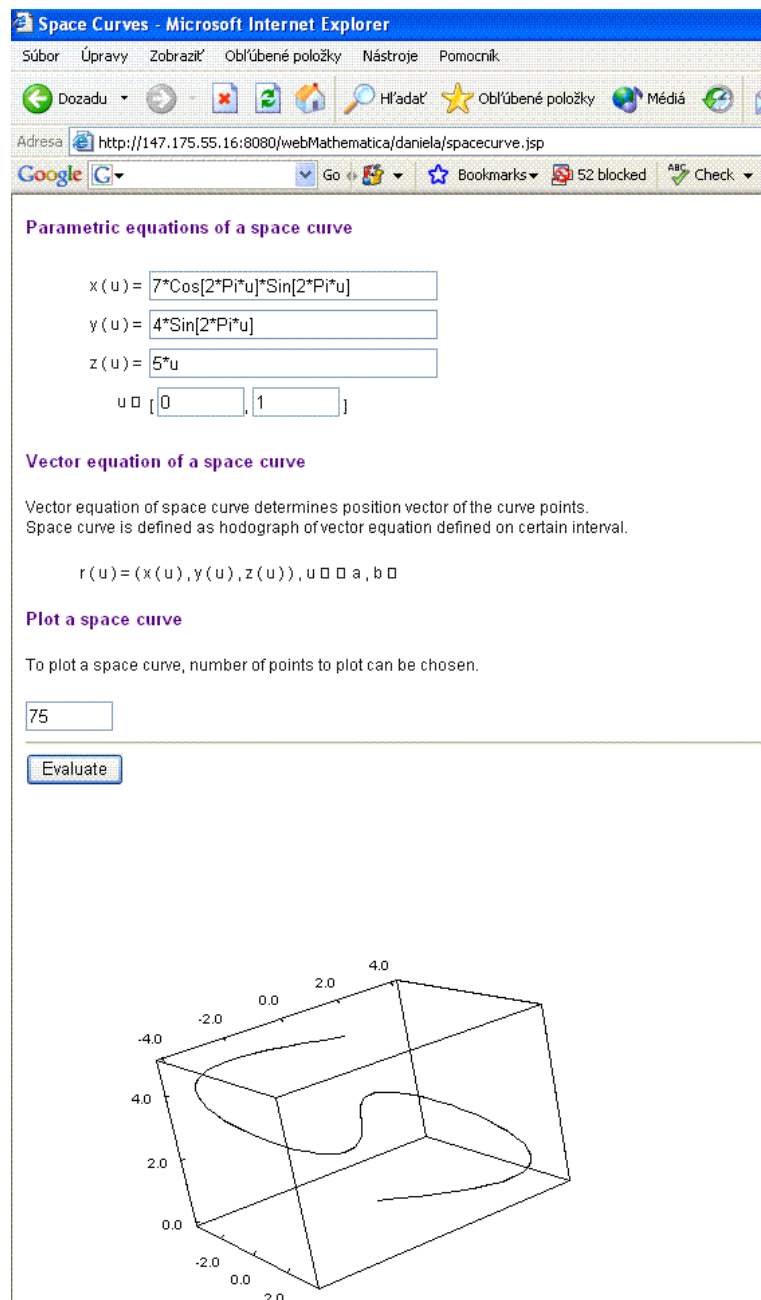


Fig. 2: Example of a short consultation topic on concept of a space curve

The next example in fig. 2. provides basic information on the concept of a space curve. There are presented parametric equations of a space curve and its equation in parametric vector form. Material can be used interactively, inserting own input data, curve parametric equations and number of curve points to plot. After evaluating the requested calculations, curve image is plotted on-line in a live Java application, which can be interactively manipulated by revolution in the space, in order to achieve curve views in different directions and angles.

The third example in fig. 3. brings the educational text presented on web in the form of an xml. file explaining the concept of a limit of number sequence. Basic theoretical information is given, including several solved examples, where limit of a sequence is calculated and illustrations of the geometric interpretation is given.

$a_1 = \frac{1}{2}(1 - \cos \pi) = 1, a_2 = \frac{1}{2}(1 - \cos 2\pi) = 0, \dots$

teda $a_n = 1$ pre nepárne n , $a_n = 0$ pre párne n .

Postupnosť, ktorá má limitu, sa nazýva **konvergentná**.

Postupnosť, ktorá limitu nemá, sa nazýva **divergentná**.

Ak má postupnosť $\{a_n\}$ limitu a , hovoríme, že postupnosť konverguje k číslu a .

Vlastnosť členov konvergentnej postupnosti "nachádzať sa v blízkosti čísla, ktoré je označené ako limita postupnosti" platí pre takmer všetky členy konvergentnej postupnosti až na ich konečný počet. Definíciu limity postupnosti môžeme preto vysloviť aj inak.

Definícia 2.

Číslo a sa nazýva limita postupnosti $\{a_n\}$, ak pre každé číslo $\varepsilon > 0$ a pre takmer všetky členy tejto postupnosti platí $|a_n - a| < \varepsilon$.

Počet tých členov postupnosti, pre ktoré neplatí $|a_n - a| < \varepsilon$, závisí od voľby čísla ε . Ak číslo ε znižujeme, počet takýchto členov postupnosti sa zväčšuje, pri každom ε je však tento počet konečný.

Definíciu limity postupnosti preto môžeme sformulovať aj nasledovne:

Definícia 3.

Číslo a sa nazýva limita postupnosti $\{a_n\}$, ak v každom okolí $O_\varepsilon(a)$ ležia takmer všetky členy tejto postupnosti.

Geometricky môžeme konvergenciu postupnosti, teda existenciu jej limity $\lim_{n \rightarrow \infty} a_n = a$, interpretovať na grafe postupnosti tak, že body A_n grafu (až na konečný počet) sa nachádzajú v páse určenom rovnobežkami so súradnicovou osou x prechádzajúcimi bodmi $[0, a - \varepsilon]$ a $[0, a + \varepsilon]$ na súradnicovej osi y .

Obr. 5.12: Limita postupnosti

Fig. 3: Example of an educational text on concept of a sequence limit

The last example in fig. 4. brings the short consultation topic explaining the concept of a surface determined by its creative law, from a basic curve subdued to a generating principle in the form of a one-parametric system of geometric transformations. Vector equations of the basic figure and the ones of the entire surface patch are presented, and Live 3D view, which can be manipulated by movements of mouse, is available. Own shape parameters influencing the surface patch can be inserted into the surface patch parametric equations, domain of surface patch definition as a range of two surface patch curvilinear parameters, and number of points to be plotted in direction of both curvilinear parameters. Perspective view of the created surface patch is provided interactively, which can be interactively rotated in the space by simple mouse movements.



Surface parametric equations

$$x(u, v) = -y[u] \cdot \sin[k \text{ Pi } v]$$

$$y(u, v) = y[u] \cdot \cos[k \text{ Pi } v] \cdot \cos[l \text{ Pi } v] - z[u] \cdot \sin[l \text{ Pi } v] + d \cdot \cos[l \text{ Pi } v] - d$$

$$z(u, v) = y[u] \cdot \cos[k \text{ Pi } v] \cdot \sin[l \text{ Pi } v] + z[u] \cdot \cos[l \text{ Pi } v] + d \cdot \sin[l \text{ Pi } v]$$

$$u \in [0, 1]$$

$$v \in [0, 1]$$

Number of points to plot 20 x 150

Evaluate

ParametricPlot3D

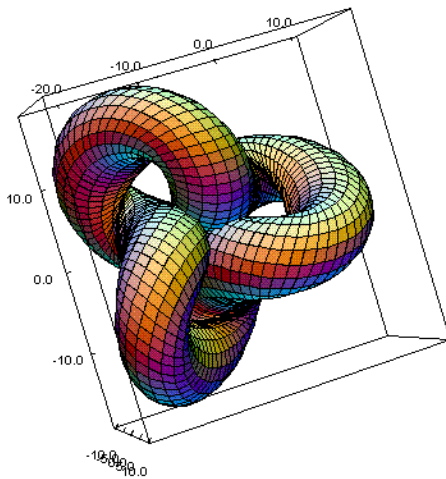


Fig. 4: Example of a surface patch modelling

4. Conclusions

A radical change of the education of mathematics has to be introduced, in order to enhance the role of mathematics in the general education and to stress its importance as a scientific field. The main idea and aims of the presented project is to prevent the decreasing level of mathematical knowledge among secondary school graduates and consequences of this unfavourable development resulting in difficulties with the graduates' further study at the technical universities. These facts remarkably influence numbers of students applying for the technical study programmes, and successful overall study results of the university freshmen. This general European trend is in a sharp contrast to the continuously increasing needs of the qualified graduates from the technical universities in various technical study programmes not only in the Europe, but also in other world countries.

Europe, which aspires to become one of the most advanced knowledge based societies in the world, cannot afford any depreciation in the established level of education and prestige of the most famous educational institutions. New process of reconciliation of the basic knowledge and skills must be started as soon as possible.

Usage of new communication and information media and technology is essential in this process. Educators, teachers, trainers and tutors need to be aware of the existence of new educational methods and they must be competent and skilful in using available resources, as well as being able to develop their own educational materials at the most advanced level. They have to be trained first, and EVLM project provides a solution for enhancing their competencies.

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