Slovak University of Technology in Bratislava
Faculty of Mechanical Engineering
17th Conference on Applied Mathematics **APLIMAT 2018**

Proceedings

GEOGEBRA AND LOGARITHMIC SPIRAL IN EDUCATIONAL PROCESS

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Abstract. In this paper, we introduce a usage of dynamical geometry software Geogebra in teaching Mathematics and in teachers training education. For this purpose, we concerned some specific curves like spirals, especially logarithmic spiral. In software environment, we demonstrate some properties of this curve and its application in metal engineering. Some didactical remarks are included in the paper, too.

Keywords: curve, Geogebra, spirangle, spiral, logarithmic spiral, education

Mathematics subject classification: Primary 97D40; Secondary 97D40, 97D50, 97C20

The National Curriculum for Upper Secondary Education in Slovakia (ISCED 3A) classified the mathematical standards in Slovak high schools. One of the major topics in ISCED 3A is Geometry. The main role of education in Geometry is oriented to a development of geometric imagination of the students as one of the most important ability for their future profession. From this reason, various technologies and software products enter in the learning process. Pedagogical software like dynamical geometry software (DGS) has the most significant impact on its qualitative change. These software develop creative thinking, mathematical skills and lead to activity.

In this paper we introduce a usage of dynamical geometry software Geogebra in teaching Geometry. For this purpose, we pay an attention to algebraic curves - spirals, especially logarithmic spiral. In software environment we demonstrate some properties of this curve and its application in metal engineering. Didactical remarks are included in the paper, too.

1 Introduction

The use of ICT in teaching mathematics has its merit. Digital technologies offer a wide range of other, modern learning, knowledge transfer and skills. They are a driving force that gradually transforms the traditional school and its educational process. Various technologies and software products enter in the learning process, but pedagogical software like dynamical geometry software (DGS) has the most significant impact on its qualitative change. [1, 2, 3] They allow the development of teaching in the spirit of the principles of didactical constructivism. It means to teach mathematics and geometry with a constructivist approach, emphasizing the pupil's activity, in a creative environment, a good working atmosphere in the classroom, looking for different representations and detracting the formalism in the knowledge of students. [4]

In this paper, we will focus on usage of the dynamical geometry Geogebra in an investigation of properties of specific curves like spirals. In software environment, we demonstrate the attributes of the logarithmic spiral and we also suggest it as motivating factor in teaching. A few didactical remarks will be noted, too.

2 Spirals in generally

There exists many kinds of the spirals like easily constructed spirangles (Fig. 1) or continuous curves, whose constructions are more complicated. [5, 6] Some spirals are termed by the famous Mathemati-

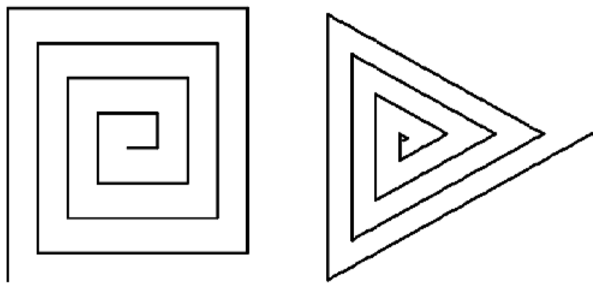

Fig. 1. Spirangles - figures related to spirals.

cians like Archimedes, Fermat, Galileo, etc.

Well-known are also spirals such as parabolic spiral, logarithmic spiral or sine spirals. [7, 8]. In (Fig. 2) are the curves constructed in Geogebra as geometrical loci of points. In general, spirals are planar curves generated by a continuous motion of a point on a line with prescribed velocity. The line rotates around a fixed center.

This free explanation indicates that the spirals can be well-defined by equations in polar coordinate system. In the following we present properties of the logarithmic spiral via software Geogebra.

3 Logarithmic spiral

This spiral is a curve whose polar equation is

$$\rho = a \cdot e^{b \cdot \varphi},$$

where ρ is radial distance from pole, φ is a counterclockwise polar angle from the positive semiaxis $x-$axis, $a, b \in R$, $a > 0$.

The point construction of the logarithmic spiral is in [7]. This synthetic approach can be replaced by the using software potential.

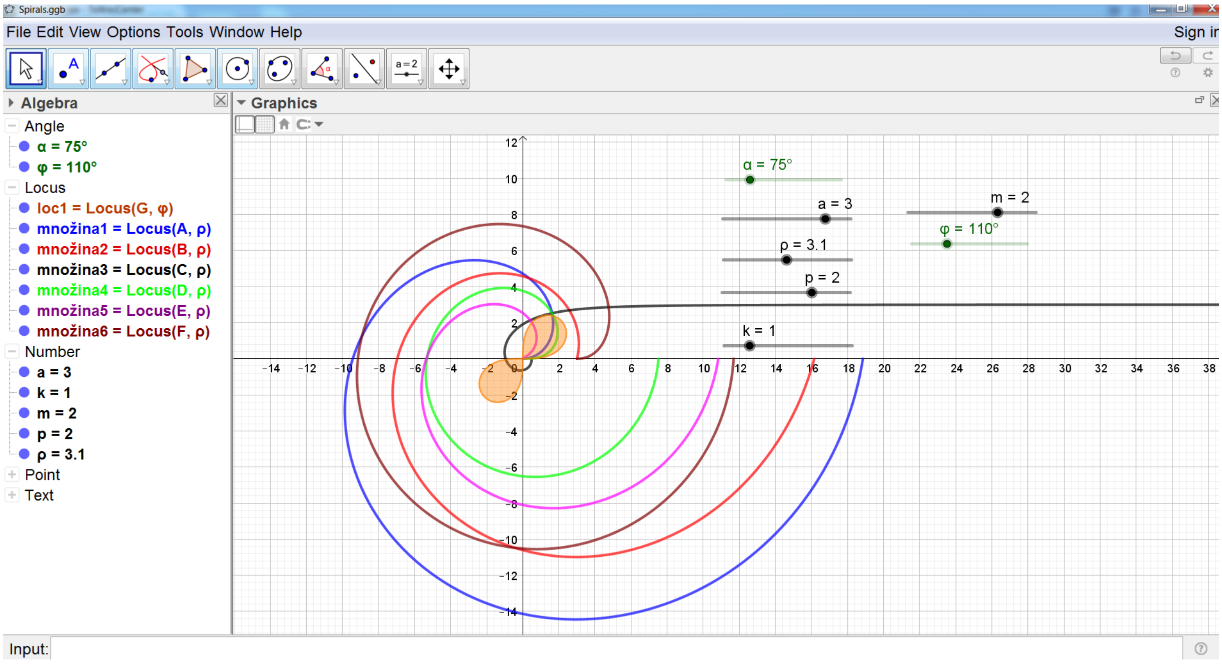

Fig. 2. A few spirals in Geogebra - Archimedes' (blue), Fermat's (green), Gallileo's (pink), Lemniscate of Bernoulli (orange; special case of sine spiral), logarithmic (red), hyperbolic (black), parabolic (brown).

In Geogebra graphic window we define the sliders a, b and $\varphi \in [0, 2\pi]$. In the next we re-define Cartesian coordinates of an arbitrary point M to polar coordinates and we re-write $M = (a \cdot e^{b \cdot \varphi}; \varphi)$. The point M lies on the logarithmic spiral. The tool *Trace on* allows us to visualize few its positions for φ. The tool *Locus* draws the implicit curve continuously (Fig. 3). The presented construction in

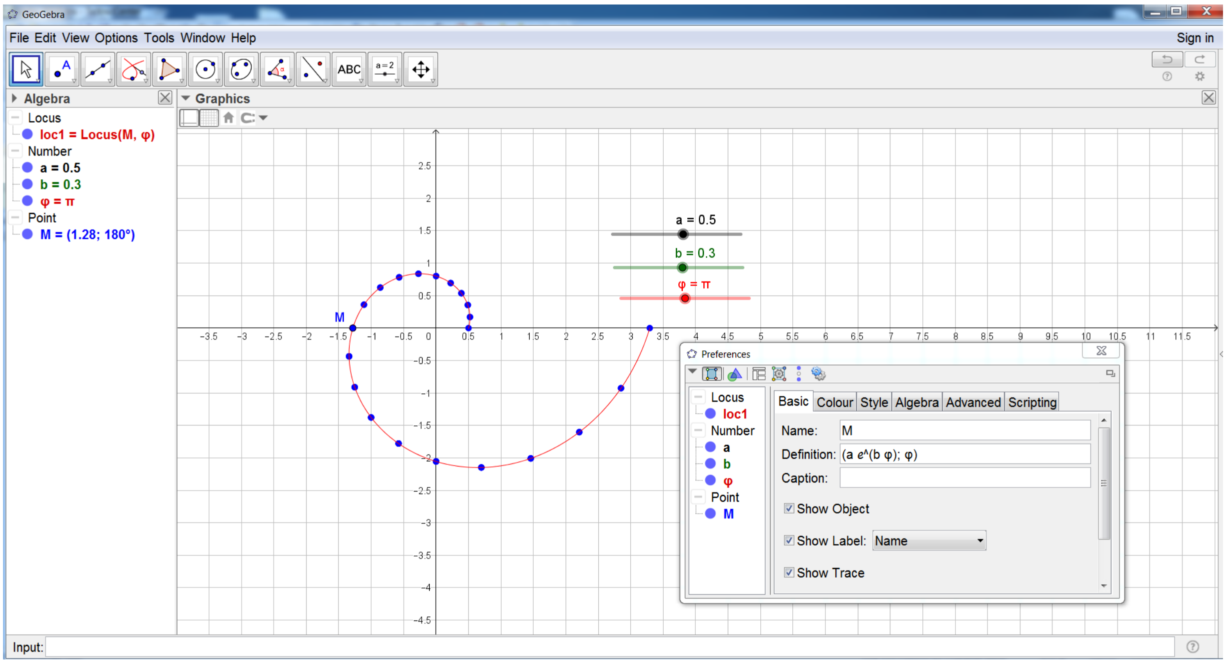

Fig. 3. Logarithmic spiral for $a = 0.5$, $b = 0.3$.

Geogebra software allows a user (student, teacher, ...) to change the parameters a, b, φ. It can be observed:

- the parameter a determines a position of the "starting" point $(a, 0)$ for $\varphi = 0$. The software allows us to set $a \leq 0$, what is in the contrary to the definition of the spiral. In both cases, if $a \neq 0$ we obtain the counter clockwise spiral. This situation explains the loading of the polar coordinate system. If $a < 0$, the starting point lies on the negative semiaxis o_x and the rotation of his radius vector around the pole O is counter clockwise, too.
- The change of the value b "opens" the spiral. For $b = 0$ it is a circle with the center O and the radius $r = a$.
- The value φ affects the position of the point M on the curve. It carries out one full circumvolution for $\varphi \in [0, 2\pi]$.

We must note that the logarithmic spiral has another remarkable attribute.
If we construct the tangent line t at the point M, we observe that the angle $\mu = \angle\left(\vec{OM}, t\right)$ is constant and it holds that

$$\tan \mu = \frac{1}{b}.$$

Remark. The tangent line at the point M is parallel to a hypotenuse RQ of triangle QRO, where $|OQ| = 1$, $|OR| = \frac{1}{b}$. The logical reasoning of this construction is in [7].
Logarithmic spiral has many applications in computer graphics like a modeling spirals in animal

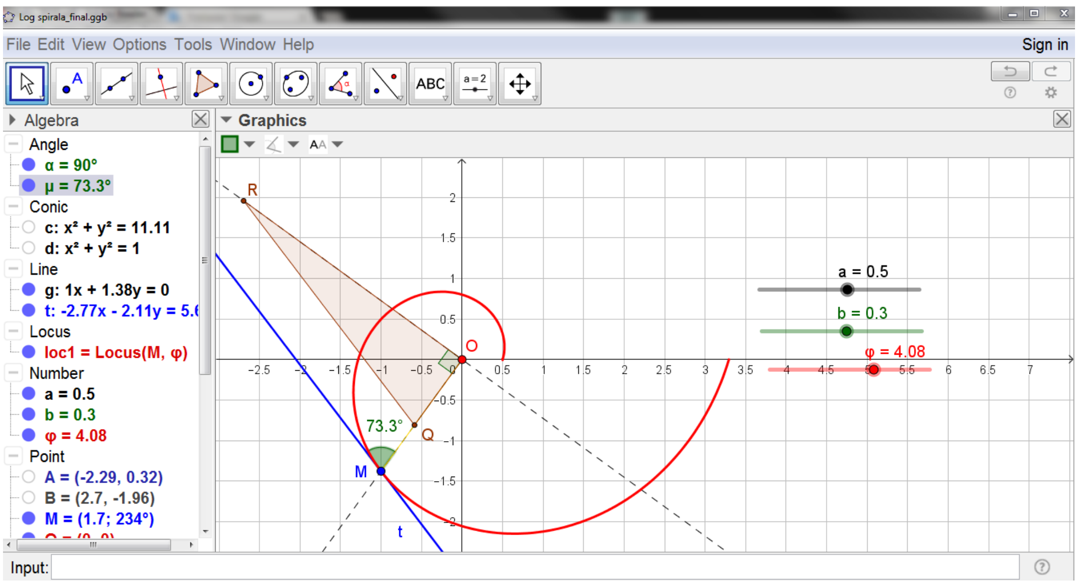

Fig. 4. Tangent line t at the point M of the logarithmic spiral for $a = 0.5$, $b = 0.3$.

structures [9] or in metal engineering, e.g. in the gear design of circular-saw blade (Fig. 5). [10, 11]
The attributes of logarithmic spiral mentioned above can be implemented in educational process as students' projects.
In Fig. 6 one can see a sample of design of the circular-saw blade. Its construction suits to the drawing of the logarithmic spiral like locus of points M for $\varphi \in \left[0, \frac{\pi}{4}\right]$. The n multiple repetitions of the arc

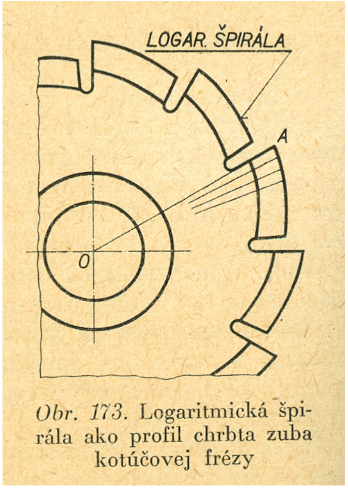

Fig. 5. Gear design of circular-saw blade. Source [10].

of spiral are constructed via the command *Sequence*in the form

$$Sequence\left(Rotate\left(M, \frac{2k\pi}{n}, O\right), k, 1, n, 1\right),$$

which produces the list of images of the point M.
The construction of the equidistant curve to the logarithmic spiral is based on the construction of the tangent line at M and the command *Sequence*, too.

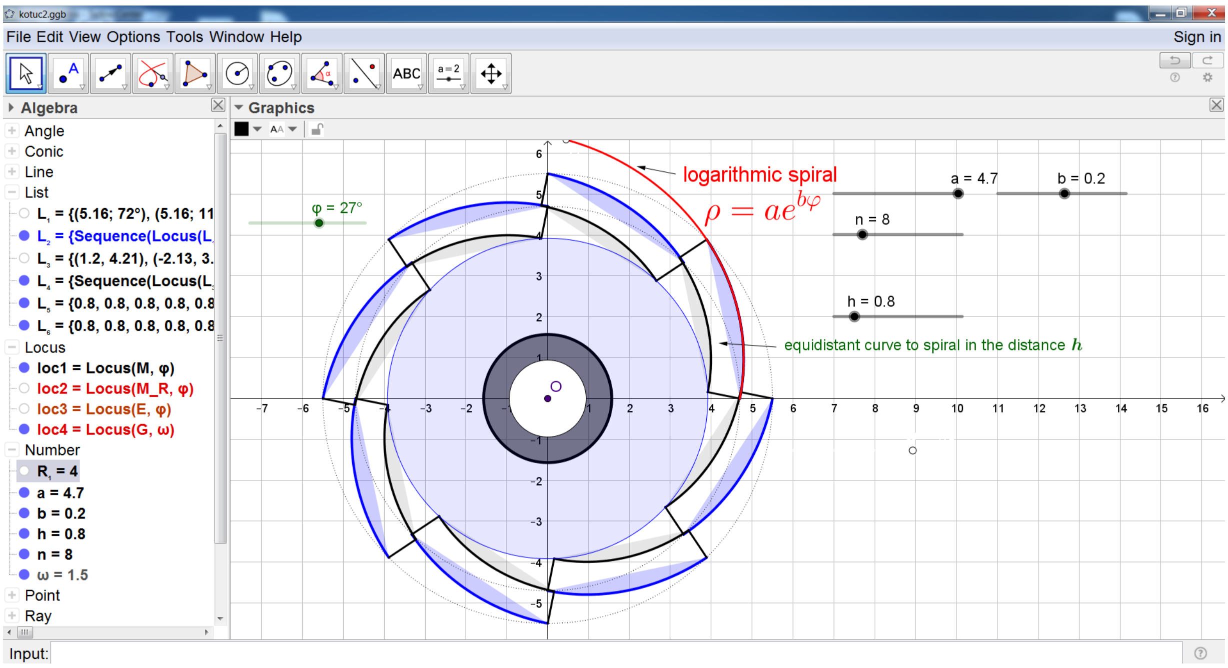

Fig. 6. Project of the gear of the circular-saw blade in GeoGebra.

4 Discussion

The National Curriculum for Upper Secondary Education in Slovakia (ISCED 3A) specifies the mathematical standards in Slovak high schools. One of the major topics in ISCED 3A is Geometry. The

geometrical curriculum is focused on the teaching of backgrounds of planimetry and stereometry. The content of the curriculum was reduced in recent years, e.g. conic sections are no longer a part of the high school curriculum. From this perspective, it seems that the investigation of the properties of other algebraic curves, such as spirals, is counterproductive.
On the other hand, there is a psychological point of view. The theory of didactical constructivism states that the exploration and discovery have a great importance for students to motivate them to learn Mathematics. Especially, if it is related to an interesting topic with impact on a practice.
The main role of education in Geometry is oriented to a development of geometric imagination of the students as one of the most important abilities for their future profession as engineers, architects, structural engineers, etc. Learning supported with ICT motivates the pupils in many ways. Static models are replaced by dynamic ones, thus developing creative thinking, mathematical skills, leading students to act and develop the constructive thinking. Technology brings to students and their teacher's great opportunity to individualize learning, to eliminate formalism and to present examples, which do not belong to the standard topic in curriculum and therefore represent a challenge.

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Acknowledgment

This paper was supported by project KEGA No. 016UKF-4/2016 MINE Slovak Republic titled *Implementation of Constructivist Approaches to Mathematics Teaching with a Focus on Active Acquisition of Knowledge by Pupils in the Context of Bilingual Education*.

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