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LINES ON COMPLETE INTERSECTION OF TWO QUADRICS IN $\mathbb{P}^4$

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Abstract. In this paper we investigate a complete intersection of two quadrics in the projective 4-space over an algebraically closed field. This quartic surface contains one singular line. We determine all the lines on these surfaces and the arrangement of these lines.

Keywords: projective space, quadric, quartic, singular line, Plücker coordinates

Mathematics subject classification: Primary 14N10, 14Q10

1 Introduction

In the paper [2] there is the classification of all irreducible non-normal complete intersections of two quadrics that are not cones. Let $\mathbb{P}^r_K$ be the projective r-space over an algebraically closed field K of arbitrary characteristic. The authors of [2] proved the next theorem.

Theorem 1.1 *Let $X \subset \mathbb{P}^r_K$ be a complete intersection defined by two quadrics Q_1 and Q_2 in $S = K[x_0, \dots, x_r]$. Suppose that X is irreducible, non-normal and not a cone.*
If $\mathrm{char} K \neq 2$, then X is transformed into one of the following projectively non-equivalent cases:

(1) $r = 3$ *and* $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_3^2 + x_0x_2 = 0$

(2) $r = 3$ *and* $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_3^2 + x_0x_2 + x_0x_3 = 0$

(3) $r = 4$ *and* $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_3^2 + x_2x_4 = 0$

(4) $r = 4$ *and* $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_1x_3 + x_2x_4 = 0$

(5) $r = 4$ *and* $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_1^2 + x_0x_4 + x_2x_3 = 0$

(6) $r = 5$ *and* $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_0x_4 + x_1x_5 + x_2x_3 = 0.$

If $\mathrm{char} K = 2$, then X is transformed into one of the cases $(1), \dots, (6)$ and in addition:

(7) $r = 3$ *and* $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_1^2 + x_0x_1 + x_2x_3 = 0$

(8) $r = 4$ *and* $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_4^2 + x_0x_4 + x_2x_3 = 0$

(9) $r = 4$ *and* $Q_1 : x_0^2 + x_1x_2 + x_0x_2 = 0,\ Q_2 : x_1x_4 + x_2x_3 = 0.$

We were motivated by this results. In this paper we specialize in the cases $(3), (4), (5)$ from Theorem 1.1. We work in the projective 4-space $\mathbb{P}^4_C$ over the complex numbers C. The intersection of two quadrics Q_1, Q_2 (hypersurfaces of 2^{nd} degree) is a quartic surface $X = Q_1 \cap Q_2$ (surface of 4^{th} degree).

Let $r = 4$, $X \subset \mathbb{P}^4_K$ be as in Theorem 1.1 and $\mathrm{Sing}(X)$ is a set of singular points of $X = Q_1 \cap Q_2$. Then $\mathrm{Sing}(X)$ is a line and the following is true (see [2]):

(3) If $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_3^2 + x_2x_4 = 0$ then $\mathrm{Sing}(X) : x_0 = x_2 = x_3 = 0.$

(4) If $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_1x_3 + x_2x_4 = 0$ then $\mathrm{Sing}(X) : x_0 = x_1 = x_2 = 0.$

(5) If $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_1^2 + x_0x_4 + x_2x_3 = 0$ then $\mathrm{Sing}(X) : x_0 = x_1 = x_2 = 0.$

The 3^{rd} author (P. Schenzel) of [2] asked if there are more lines on X than $\mathrm{Sing}(X)$. By using two different methods we determined the lines on some cubic and quartic surfaces in $\mathbb{P}^3$ (see [3] and [5]). Here we want to apply one of these methods to add some properties of surfaces X given in the paper [2]. In this article we study the number of lines on X and the arrangement of these lines. The main result is in the following theorem.

Theorem 1.2 (Main Theorem) *Let $X \subset \mathbb{P}^4_K$ be as in Theorem 1.1. Then for $X = Q_1 \cap Q_2$ it holds*

a) *If $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_3^2 + x_2x_4 = 0$ then X contains the line $\mathrm{Sing}(X)$ given by points $(0, 1, 0, 0, 0), (0, 0, 0, 0, 1)$ and no more lines.*

b) *If $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_1x_3 + x_2x_4 = 0$ then X contains infinitely many lines:*

- *the singular line $\mathrm{Sing}(X)$ given by points $(0, 0, 0, 1, 0), (0, 0, 0, 0, 1)$,*
- *two lines given by points $(0, 1, 0, 0, 0), (0, 0, 0, 0, 1)$ and $(0, 0, 1, 0, 0)$, $(0, 0, 0, 1, 0)$ which intersect the singular line,*
- *the family of lines $m(u, v)$ given by points $(0, 0, 0, u^2, v^2), (-uv, -v^2, u^2, 0, 0)$, $u, v \neq 0$. Every line from this family intersects the singular line and every two lines $m(u, v)$ and $m(-u, v)$ given by (u, v) and $(-u, v)$ have a common point on the singular line. The lines of this family have no other common point.*

c) *If $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_1^2 + x_0x_4 + x_2x_3 = 0$ then X contains infinitely many lines:*

- *the singular line $\mathrm{Sing}(X)$ given by points $(0, 0, 0, 1, 0), (0, 0, 0, 0, 1)$,*
- *the family of lines $n(u, v)$ given by points $(0, 0, 0, u, v), (-uv^2, -u^2v, v^3, 0, u^3)$, $v \neq 0$. Every line from this family intersects the singular line and any two lines of this family are skew - they have no common point.*

We use the Plücker coordinates of the lines to prove this theorem (see also [1] and [4]).

2 Plücker coordinates of the lines

Definition 2.1 *Let the line $l \subset \mathbb{P}^4$ is given by points $(x), (y)$, with $(x) = (x_0, x_1, x_2, x_3, x_4)$, $(y) = (y_0, y_1, y_2, y_3, y_4)$. Then the Plücker coordinates of the line l are $(p_{01}, p_{02}, p_{03}, p_{04}, p_{12}, p_{13}, p_{14}, p_{23}, p_{24}, p_{34})$, where*

$$p_{ij} = x_i y_j - y_i x_j \; (0 \leq i < j \leq 4).$$

By means of the Plücker coordinates we map the lines of $\mathbb{P}^4$ to the points of $\mathbb{P}^9$. Every line $l \subset \mathbb{P}^4$ determines a point $(p_{ij}) \in \mathbb{P}^9$. Note that not every point $(p_{ij}) \in \mathbb{P}^9$ determines a line in $\mathbb{P}^4$.

Proposition 2.2 *The point $(p_{ij}) = (p_{01}, p_{02}, p_{03}, p_{04}, p_{12}, p_{13}, p_{14}, p_{23}, p_{24}, p_{34})$ of $\mathbb{P}^9$ corresponds to a line of $\mathbb{P}^4$ if it holds*

$$\begin{aligned} p_{01}p_{23} - p_{02}p_{13} + p_{03}p_{12} &= 0 \\ p_{01}p_{24} - p_{02}p_{14} + p_{04}p_{12} &= 0 \\ p_{01}p_{34} - p_{03}p_{14} + p_{04}p_{13} &= 0 \\ p_{02}p_{34} - p_{03}p_{24} + p_{04}p_{23} &= 0 \\ p_{12}p_{34} - p_{13}p_{24} + p_{14}p_{23} &= 0. \end{aligned} \tag{1}$$

Proof. Let $l = (x)(y),\ (x) = (x_0, x_1, x_2, x_3, x_4),\ (y) = (y_0, y_1, y_2, y_3, y_4)$ be a line of $\mathbb{P}^4$. For the matrix

$$\begin{pmatrix} x_0 & x_1 & x_2 & x_3 & x_4 \\ y_0 & y_1 & y_2 & y_3 & y_4 \\ x_0 & x_1 & x_2 & x_3 & x_4 \\ y_0 & y_1 & y_2 & y_3 & y_4 \end{pmatrix}$$

all the determinants 4×4 are equal to zero. These 5 determinants can be simplified to the equations (1). ■

Remark 2.3 *The points $(p_{ij}) \in \mathbb{P}^9$, which coordinates satisfy conditions* (1)*, are lying on the intersection of* 5 *hypesurfaces of* 2^{nd} *degree in* $\mathbb{P}^9$.

We can determine the arrangement of two lines from the Plücker coordinates of these lines.

Proposition 2.4 *Let $(p_{ij}),\ (q_{ij})$ be the Plücker coordinates of two lines $p,\ q$ of $\mathbb{P}^4$. The lines $p,\ q$ are intersecting lines if and only if their Plücker coordinates satisfy the following conditions:*

$$\begin{aligned} p_{01}q_{23} - p_{02}q_{13} + p_{03}q_{12} + p_{12}q_{03} - p_{13}q_{02} + p_{23}q_{01} &= 0 \\ p_{01}q_{24} - p_{02}q_{14} + p_{04}q_{12} + p_{12}q_{04} - p_{14}q_{02} + p_{24}q_{01} &= 0 \\ p_{01}q_{34} - p_{03}q_{14} + p_{04}q_{13} + p_{13}q_{04} - p_{14}q_{03} + p_{34}q_{01} &= 0 \\ p_{02}q_{34} - p_{03}q_{24} + p_{04}q_{23} + p_{23}q_{04} - p_{24}q_{03} + p_{34}q_{02} &= 0 \\ p_{12}q_{34} - p_{13}q_{24} + p_{14}q_{23} + p_{23}q_{14} - p_{24}q_{13} + p_{34}q_{12} &= 0. \end{aligned} \tag{2}$$

Proof. Let $p = AB,\ q = CD$ be two lines with $A = (a_0, a_1, a_2, a_3, a_4), B = (b_0, b_1, b_2, b_3, b_4)$, $C = (c_0, c_1, c_2, c_3, c_4),\ D = (d_0, d_1, d_2, d_3, d_4)$ and M is 4×5 matrix

$$M = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 & a_4 \\ b_0 & b_1 & b_2 & b_3 & b_4 \\ c_0 & c_1 & c_2 & c_3 & c_4 \\ d_0 & d_1 & d_2 & d_3 & d_4 \end{pmatrix}.$$

The lines $p,\ q$ are

- identical lines if $rank(M) = 2$
- intersecting lines if $rank(M) = 3$
- skew lines if $rank(M) = 4$.

The lines $p,\ q$ are intersecting lines if and only if $rank(M) = 3$, i. e. determinant of every 4×4 matrix is equal to zero. There are 5 determinants of 4×4 matrix. If we substitute $p_{ij} = a_i b_j - b_i a_j$ and $q_{ij} = c_i d_j - d_i c_j$ $(0 \leq i < j \leq 4)$ into these determinants we get the equations (2). ■

Now we are going to describe an algorithm for computing all the lines on a hypersurface. We use the Plücker coordinates of the lines.

Proposition 2.5 *Let $l \subset \mathbb{P}^4$ be a line with the Plücker coordinates (p_{ij}) and $Q(F) \subset \mathbb{P}^4$ be a hypersurface with the defining polynom F. The condition for a line l to lie on $Q(F)$ can be expressed by algebraic equations with p_{ij} and coefficients of the polynom F.*

Proof. We will follow [4], § 6, section 4. Let $(x),\ (y)$ be two different points of the line l. For an arbitrary point (u) of the line l we have

$$(u) = (x)f(y) - (y)f(x),$$

where f is a linear form with the coefficients $\alpha_0,\ \alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4$, i. e. $f(x) = \sum\limits_{i=0}^{4} \alpha_i x_i$ and $f(y) = \sum\limits_{i=0}^{4} \alpha_i y_i$. For the coordinates of $(u) \in l$ we get

$$u_i = \sum_{j=0}^{4} \alpha_j p_{ij}, \quad \text{with } p_{ij} = x_i y_j - y_i x_j,$$

and therefore
$$\begin{pmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} 0 & p_{01} & p_{02} & p_{03} & p_{04} \\ -p_{01} & 0 & p_{12} & p_{13} & p_{14} \\ -p_{02} & -p_{12} & 0 & p_{23} & p_{24} \\ -p_{03} & -p_{13} & -p_{23} & 0 & p_{34} \\ -p_{04} & -p_{14} & -p_{24} & -p_{34} & 0 \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix}.$$

By substituting this into the defining equation $F = 0$ of the hypersurface $Q(F)$ we get the equation $F(u_0, u_1, u_2, u_3, u_4) = F(\alpha_i) = 0$. The line l is on the hypersurface $Q(F)$ if and only if all the coefficients of monoms in α_i are zero. Thus we get a system of equations in p_{ij}. By solving this system of equations we get the Plücker coordinates of all lines on the hypersurface $Q(F)$. ■

Remark 2.6 *For a quadric hypersurface in $\mathbb{P}^4$ we get 15 equations of 2^{nd} degree. To find all the lines on an intersection of two quadrics we have to solve a system of 30 equations of 2^{nd} degree. Though it seems too difficult, one can be able in some situations to find the solutions "manually" since many of the coefficients equals to zero and the system becomes to be much simplier one (see e.g. Example 2 in [5] or the proof of the part a) of Theorem 1.2 in this paper). In case of more complicated systems we can use computer algebra software to solve this system (e.g. Singular or Reduce and Cali).*

3 Proof of the Main Theorem 1.2

Now we are going to prove Theorem 1.2, the main result of the paper. We apply the algorithm from Proposition 2.5 to find all the lines on the quartic surfaces $X = Q_1 \cap Q_2$. Then we determine the arrangement of these lines.

Proof. Let $X \subset \mathbb{P}^4_C$ be a complete intersection defined by two quadrics Q_1 and Q_2 in the projective 4-space $\mathbb{P}^4_C$ over the complex numbers C.

a) $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_3^2 + x_2x_4 = 0$
We substitute $u_i,\ i = 0, \dots, 4$ from the proof of Proposition 2.5 into the defining equations of Q_1 and Q_2. For the line lying on the hypersurface it holds that all the coefficients of monoms in α_i are zero:

$$\begin{array}{llll} Q_1: & p_{01} = p_{02} = p_{12} = 0 & Q_2: & p_{23} = p_{24} = p_{34} = 0 \\ & p_{03}^2 + p_{13}p_{23} = 0 & & p_{13}^2 + p_{12}p_{14} = 0 \\ & 2p_{03}p_{04} + p_{13}p_{24} + p_{14}p_{23} = 0 & & 2p_{03}p_{13} + p_{02}p_{14} + p_{04}p_{12} = 0 \\ & p_{04}^2 + p_{14}p_{24} = 0 & & p_{03}^2 + p_{02}p_{04} = 0 \end{array}$$

From this conditions we can calculate the lines on $X = Q_1 \cap Q_2$. One can see that all p_{ij} vanish (equals to zero) except of p_{14}, so there is only one solution of this system of equations:

$$(0, 0, 0, 0, 0, 0, 1, 0, 0, 0).$$

These are the Plücker coordinates of the singular line $\mathrm{Sing}(X)$ on X. It is given as intersection of three hyperplanes $x_0 = x_2 = x_3 = 0$ or by two points $(0, 1, 0, 0, 0)$, $(0, 0, 0, 0, 1)$ in $\mathbb{P}^4$.

b) $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_1x_3 + x_2x_4 = 0$
The system of equations from Proposition 2.5 is

$$\begin{array}{ll} p_{01} = p_{02} = p_{12} = 0 & p_{13}p_{14} + p_{14}p_{24} = 0 \\ p_{03}^2 + p_{13}p_{23} = 0 & p_{13}p_{23} + p_{23}p_{24} = 0 \\ 2p_{03}p_{04} + p_{13}p_{24} + p_{14}p_{23} = 0 & p_{14}p_{23} + p_{24}^2 = 0 \\ p_{04}^2 + p_{14}p_{24} = 0 & p_{23}p_{34} = 0 \\ p_{03}p_{13} + p_{04}p_{23} = 0 & p_{13}p_{34} - p_{24}p_{34} = 0 \\ p_{03}p_{14} + p_{04}p_{24} = 0 & p_{14}p_{34} = 0 \\ p_{13}^2 + p_{14}p_{23} = 0 & \end{array}$$

Solutions of this system give us the Plücker coordinates of all lines on X. Here are the results:

$$\begin{array}{l} r : (0, 0, 0, 0, 0, 0, 0, 0, 0, 1), \\ m : (0, 0, u^3v, uv^3, 0, u^2v^2, v^4, -u^4, -u^2v^2, 0),\ \text{with } (u, v) \neq (0, 0). \end{array}$$

- The line r with Plücker coordinates $(0, 0, 0, 0, 0, 0, 0, 0, 0, 1)$ is a singular line $\mathrm{Sing}(X)$. It is given by equations $x_0 = x_1 = x_2 = 0$ or by two points $(0, 0, 0, 1, 0),\ (0, 0, 0, 0, 1)$.
- The Plücker coordinates $(0, 0, u^3v, uv^3, 0, u^2v^2, v^4, -u^4, -u^2v^2, 0)$, with $(u, v) \neq (0, 0)$, determine lines $m(u, v)$ given by two points $(0, 0, 0, u^2, v^2),\ (-uv, -v^2, u^2, 0, 0)$.

So there are infinitely many lines on $X = Q_1 \cap Q_2$.

Now we want to find the arrangement of these lines. It is clear that all of the lines $m(u,v)$ intersect the line r in the point $(0,0,0,u^2,v^2)$. The equations (2) are satisfied for the Plücker coordinates of the lines r and m.

Two lines m given by (u,v) and (u_1,v_1) intersect if and only if the equations (2) are satisfied. So we get the following equations

$$\begin{array}{l} -u^3vv_1^4 + uv^3u_1^2v_1^2 + u^2v^2u_1v_1^3 - v^4u_1^3v_1 = 0 \\ u^3vu_1^2v_1^2 - uv^3u_1^4 - u^4u_1v_1^3 + u^2v^2u_1^3v_1 = 0 \\ 2u^2v^2u_1^2v_1^2 - v^4u_1^4 - u^4v_1^4 = 0 \end{array}$$

and from this system we have the arrangement of these lines.

- For $u = 0$ we have the line with Plücker coordinates

$$(0,0,0,0,0,0,1,0,0,0)$$

  that does not intersect any line $m(u,v)$. This line is given by two points $(0,1,0,0,0)$, $(0,0,0,0,1)$.
- For $v = 0$ we have the line with Plücker coordinates

$$(0,0,0,0,0,0,0,1,0,0)$$

  that does not intersect any line $m(u,v)$. This line is given by two points $(0,0,1,0,0)$, $(0,0,0,1,0)$.
- For $u \neq 0,\ v \neq 0$ the line $m(u,v)$ intersects only the line $m(-u,v)$ in the point $(0,0,0,u^2,v^2)$ on the line $r = \mathrm{Sing}(X)$. These intersecting lines can be given by two points

$$\begin{array}{l} m(u,v):\ (0,0,0,u^2,v^2),\ (-uv,-v^2,u^2,0,0) \\ m(-u,v):\ (0,0,0,u^2,v^2),\ (uv,-v^2,u^2,0,0). \end{array}$$

We got the following result: For the points $A \in \mathrm{Sing}(X)$ it holds that there are two lines different from $\mathrm{Sing}(X)$ through every point A and lying on X except the points $(0,0,0,1,0)$ and $(0,0,0,0,1)$. For each of these two points there is exactly one line different from $\mathrm{Sing}(X)$ through them and lying on X (see Fig. 1).

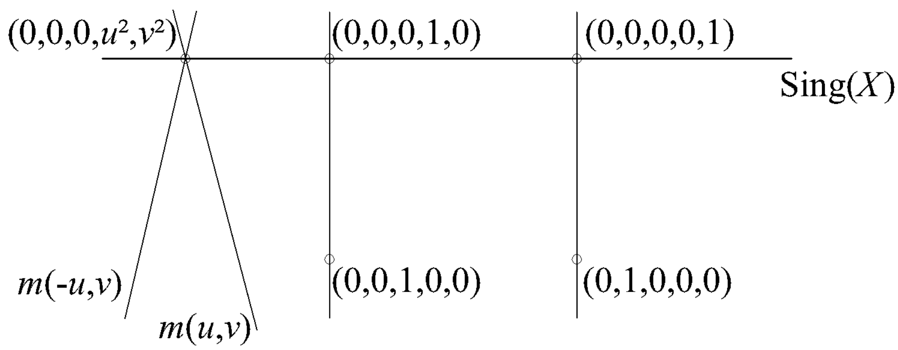

Fig. 1. The arrangement of lines on $X = Q_1 \cap Q_2$, $u,v \neq 0$.

Remark 3.1 *The lines which do not meet in the Figure are mutually skew.*

c) $Q_1 : x_0^2 + x_1x_2 = 0,\ Q_2 : x_1^2 + x_0x_4 + x_2x_3 = 0$
We get the Plücker coordinates of all the lines on $X = Q_1 \cap Q_2$ from the following system of equations (see Proposition 2.5)

$$
\begin{array}{ll}
p_{01} = p_{02} = p_{12} = 0 & p_{04}p_{14} + p_{13}p_{24} = 0 \\
p_{03}^2 + p_{13}p_{23} = 0 & p_{23}^2 + p_{03}p_{24} = 0 \\
2p_{03}p_{04} + p_{13}p_{24} + p_{14}p_{23} = 0 & p_{04}p_{24} + p_{23}p_{24} = 0 \\
p_{04}^2 + p_{14}p_{24} = 0 & p_{13}^2 - p_{03}p_{34} = 0 \\
p_{03}p_{04} + p_{03}p_{23} = 0 & 2p_{13}p_{14} - p_{04}p_{34} + p_{23}p_{34} = 0 \\
p_{04}^2 + p_{03}p_{24} = 0 & p_{14}^2 + p_{24}p_{34} = 0 \\
p_{03}p_{14} + p_{13}p_{23} = 0 &
\end{array}
$$

and the solution is

$$n : (0, 0, u^2v^2, uv^3, 0, u^3v, u^2v^2, -uv^3, -v^4, u^4),\ \text{with } (u, v) \neq (0, 0).$$

These Plücker coordinates determine infinitely many lines given by two points

$$n(u, v) :\ (0, 0, 0, u, v),\ (-uv^2, -u^2v, v^3, 0, u^3).$$

For $v = 0$ we get the singular line which is given by equations $x_0 = x_1 = x_2 = 0$ or by two points $(0, 0, 0, 1, 0),\ (0, 0, 0, 0, 1)$.

We get the arrangement of these lines from the equations (2) for two lines n given by (u, v) and (u_1, v_1):

$$
\begin{array}{l}
-2u^2v^2u_1^2v_1^2 + uv^3u_1^3v_1 + u^3vu_1v_1^3 = 0 \\
u^2v^2v_1^4 - 2uv^3u_1v_1^3 + v^4u_1^2v_1^2 = 0 \\
u^3vv_1^4 - u^2v^2u_1v_1^3 - uv^3u_1^2v_1^2 + v^4u_1^3v_1 = 0.
\end{array}
$$

From these conditions we have that the singular line $(0, 0, 0, 0, 0, 0, 0, 0, 0, 1)$ intersects every line $(0, 0, u^2v^2, uv^3, 0, u^3v, u^2v^2, -uv^3, -v^4, u^4)$ with $v \neq 0$. The point of intersection is $(0, 0, 0, u, v)$.

We got the following result: For the points $A \in \mathrm{Sing}(X)$ it holds that there is exactly one line different from $\mathrm{Sing}(X)$ through every point A and lying on X except the point $(0, 0, 0, 1, 0)$. For the point $(0, 0, 0, 1, 0)$ there is no line on X different from $\mathrm{Sing}(X)$ through this point (see Fig. 2).

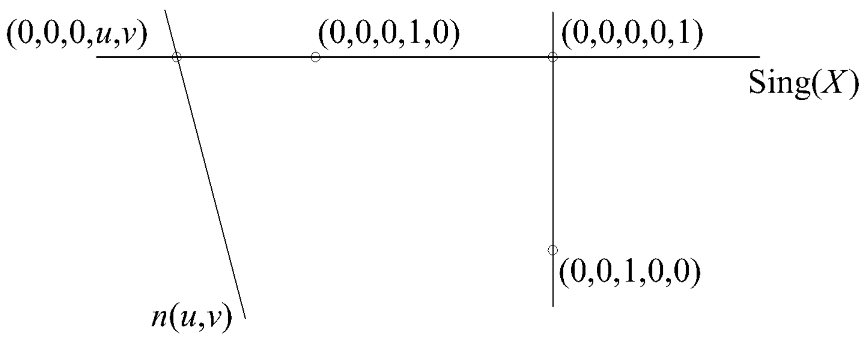

Fig. 2. The arrangement of lines on $X = Q_1 \cap Q_2$, $u, v \neq 0$.

■

4 Conclusion

We determined some properties of complete intersections of two quadrics in $\mathbb{P}^4$ over an algebraically closed field which can be taken as an addition to the results of the paper [2]. We applied the method which uses the Plücker coordinates of the lines. This method is universal but sometimes complicated calculation requires to use some algebra software.

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