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STOCHASTIC EXTRAPOLATION OF MORTALITY RATES IN THE CZECH REPUBLIC WITH AN IMPACT ON PROBABILITY AND NUMBER OF SURVIVING

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Abstract. The aim of this paper is to compare the results of stochastic (autoregressive integrated moving averages) and deterministic (linear regression) extrapolations of logs of age-and-sex-specific mortality rates in the Czech Republic with impact on selected characteristics of mortality tables – probability and number of surviving. Research shows that selected ARIMA models provide more precise extrapolation of the analysed characteristics mainly in advanced ages and they are much less biased. Deterministic models are useful mainly in lower ages and up to 65 years.

Keywords: stochastic modelling, mortality rates, probability of surviving, projection

Mathematics Subject Classification: Primary 90C30; Secondary 62H12.

1 Introduction

Intensity of mortality (denoted $\mu_{x,t}$) at the advanced ages (65 years and above, which is mostly acknowledged as retirement age in developed populations) becomes important topic for demographers and for analysts in the field of health, pension insurance and for public policy planning. The countries of Central and Eastern Europe are aging as it is in the case of developed Western European countries. Extensions of length of human life in Central and Eastern Europe is influenced mainly by improvement in medicine and in healthcare, which has been running since the early 90s of the last century. At first, there was a significant improvement in a care of live born persons and infants, which caused the decrease of infant mortality and mortality rates during childhood. Later, mortality began to improve even at the advanced ages. Among reasons for this evolution could be included higher level of health care, standard of living, information technologies and scientific progress, better housing conditions or functional health, pension and social systems. Another equally important reason could be more interest in a healthy lifestyle of the people and better environment in most countries (especially in industrial cities with a heavy load transport). This slow prolongation of human life means for our future, that the proportion of persons at higher ages will continue to increase, while the proportion of young people is decreasing.

1.1 Current state and literature review

According to Finkelstein [11], “mortality rates of human populations in developed countries are declining with time”. With increasing life expectancy (denoted $e_{x,t}$) improving long-term care and sustaining the pension system are becoming an important issue. Besides, establishing methodologically sound longitudinal data sets is necessary to examine the phenomena (Andel [1]). For the purposes of actuarial demography and life insurance, it is not sufficient to construct deterministic models only, but it is necessary to combine them and to compare the results also which probabilistic and stochastic ones. Finding optimal form of the model for age-and-sex specific mortality rates (denoted $m_{x,t}$) (respectively probability of surviving (denoted $p_{x,t}$) after recalculation according to mortality table algorithm) is a complicated process, because it is necessary to test not only statistical significance of the parameters, but also the significance of the whole models and to respect the results of the diagnostics tests, which include tests of autocorrelation, heteroskedasticity and normality of a non-systematic (residual) component (ε_t). “The actuarial and demographic literature has introduced a myriad of (deterministic and stochastic) models to forecast mortality rates of single populations” (Antonion, Bardoutsos and Ouburg [2]). Work of Gompertz [14] played a key role in shaping the emerging statistical science. Gompertz model provided a powerful stimulus to examine the patterns of death (“law of human mortality”) across the life course not only in humans but also in a wide range of other organisms (Makeham [22], Kirkwood [17]). Since that many of models have been developed. Lee and Carter [19] published a new statistical method for forecasting US mortality in 1992. Since that it has been applied on many real data of the populations. For example, Li and Lee [21] applied the Lee-Carter model to a group of populations, allowing each its own age pattern and level of mortality but imposing shared rates of change by age. However, there are more models used. For example, Gogola [13] used stochastic mortality modelling approach – Cairns, Blake and Dowd model that is well suited at very high ages to calculate mortality rates of age categories from 85 to 115 for selected countries. Antonion, Bardoutsos and Ouburg [2] presented in their paper a Bayesian analysis of two related multi-population mortality models of log-bilinear type, designed for two or more populations. Godunov [12] compared several models applied on particular populations and found that „the most appropriate models of smoothing mortality curve are Kannistö-Thatcher (United Kingdom) Martinell (Sweden) and Kannistö (Canada). On the other side, the least suitable models are Coale-Kisker (Singapore), Gompertz-Makeham and modified Gompertz-Makeham (Czech Republic, Slovakia, Germany).” Mortality rates in the Czech Republic was calculated for example by Jindrová and Slavíček [15]. They applied Lee-Carter model on Czech population in the period of 1950–2009 and predicted the development of age-and-sex specific mortality rates and consequently life expectancy for period 2010–2029. Followed Arltová, Langhamrová and Langhamrová [3], who used the same data base, but the life expectancy was predicted up to the year 2050, because unlike Jindrová and Slavíček [15] they tried to find and use the cointegration relationship between male’s and female’s parameters (denoted k_t) of the Lee-Carter model. This long-term relationship was later used for a better and more significant prediction of life expectancy until 2050. In the broader context, Fiala and Langhamrová [10] analysed of the development of the sex-and-age structure of the Czech population of productive age based on the latest population projection of the Czech Statistical Office (CZSO). Probabilistic projections of age-specific mortality and fertility rates were done for example by Ševčíková et al. [23] to apply probabilistic population projections on United Nations (UN) countries. This important research is best suited to international comparison, because it is constructed with the same approach for all populations.

1.2 Problem formulation and goal of the paper

Finding a suitable model for extrapolation of $\ln(m_{x,t})$ to the future raises a problem that there are about 2 × 101 or more models in advance populations[1], because each gender and age[2] is necessary to model. Therefore, the aim of the paper is to compare selection of ARIMA models towards the linear deterministic models based on the results of the mortality projection that are recalculated on the probabilities of surviving ($p_{x,t}$) of x-year old persons and number of surviving persons (tabular $l_{x,t}$), where the base at age of 0 years is set on 100 000 persons. Paper shows that process of user selection of partial appropriate models is although slower to process, but the results of projections are more credible in comparison with linear deterministic models (see also paper by Šimpach and Langhamrová [26]).

2 Materials and methods

For the description of mortality development is most often used an indicator known as life expectancy ($e_{x,t}$), (as a key output of the mortality tables), but for analytical purposes may be used other characteristics of mortality, such as e.g. age-and-sex specific mortality rates ($m_{x,t}$), probability of surviving ($p_{x,t}$) and tabular number of surviving ($l_{x,t}$). These characteristics are obtained as an output indicator from mortality tables. The calculation is carried out in several steps. First, we calculate the age-and-sex specific mortality rates as

$$m_{x,t} = \frac{M_{x,t}}{\bar{S}_{x,t}}, \tag{1}$$

where $M_{x,t}$ is the number of deaths x years old and $\bar{S}_{x,t}$ is the mid-year number of living persons (exposure to risk). Between the mortality rate and the intensity of mortality (which is considered in similar studies and analysis, see e.g. paper by Bogue, Anderton, Arriaga [4], Fiala [9] or Dotlačilová, Šimpach and Langhamrová [8]) is valid the followed relationship

$$m_{x,t} \approx \mu\left(x + \frac{1}{2}\right). \tag{2}$$

Next, we will calculate the probability of death for 0-year-old person as

$$q_{0,t} = \frac{M_{0,t}}{\alpha N_t^v + (1-\alpha) N_{t-1}^v} \quad \text{for } x = 0, \tag{3}$$

where $M_{0,t}$ is the number of deaths at the age 0, α is the proportion of lower elementary file of died persons from Lexis [20] diagram (approximately 0.85–0.91) and N_t^v, respectively N_{t-1}^v is the number of live births in year t, respectively in year t-1 (see Keyfitz [16]). The probability of dying is given by formula

[1] As stated in subchapter 1.1, Gogola [13] used stochastic mortality modelling approach for persons in very high ages to calculate mortality rates of age categories from 85 to 115 years. It is understandable that at the age of 115, almost nobody lives from an empirical point of view the intensity of mortality at this age must be adjusted.

[2] Age range is usually surveyed by statistical offices in the intervals 0–100+ completed years of life.

$$q_{x,t} = 1 - p_{x,t}, \tag{4}$$

and is valid for age $x > 0$. The calculation of the probability of surviving is given by formula

$$p_{0,t} = 1 - q_{0,t} \text{ for age } x = 0, \tag{5}$$

and by formula

$$p_{x,t} = e^{-m_{x,t}} \text{ for age } x > 0. \tag{6}$$

Next part of the calculation relates to tabular (i.e. imaginary) population. First, we select the initial number of live births in tabular population: $l_{0,t}$ = 100 000. Based on knowledge of the probability of surviving, we can calculate the number of surviving in the further exact ages by

$$l_{x+1,t} = l_{x,t}\, p_{x,t}, \tag{7}$$

where $l_{x,t}$ is the number of surviving at the exact age x from the default file of 100 000 live births of tabular population.

Extrapolation approaches applied on the mortality development, which are based on deterministic principles are still used in a lot of countries, including the Czech Republic. We can mention e.g. study by Fiala [9] or Koschin et al. [18]. Especially the study by Fiala [9] provides information that $\ln(m_{x,t})$ is for each age x approximately linear and can be modelled by linear function. According to this evidence we can conclude that if we find the appropriate constant (intercept) (β_0) and slope (β_1) of the regression function

$$\ln(m_{x,t}) = \beta_0 + \beta_1 t + \varepsilon_t, \tag{8}$$

we can easily perform extrapolation. This approach is simple, however, because the empirical data of the Czech Republic is quite variable, and the nature of individual time series of mortality is volatile, residuals of these regressions could be autocorrelated and the slope of the regression line (especially in the lowest and highest age groups) may not be optimal for achieving the desired results. Another approach which was used e.g. by Arltová, Langhamrová and Langhamrová [3], Šimpach [24], [25] or Šimpach, Pechrová [27] is based on Box and Jenkins [5] methodology, where the linear regression model (8) is replaced by ARIMA models, which can be generally written as

$$\ln m_{x,t} = \beta_0 + \sum_{i=1}^{p} \phi_i \ln m_{x,t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}. \tag{9}$$

where ϕ_i (i = 1, ..., p), and θ_j (j = 1, ..., q) are the parameters of AR and MA process, β_0 represents the constant and ε_t is a pure white noise error term, where $E(\varepsilon_t) = 0$, $D(\varepsilon_t) = \sigma^2$, $cov(\varepsilon_t\, ;\, \varepsilon_t') = 0$ and $\varepsilon_t \approx$ N (normal) distribution.

Followed by this procedure it is obvious, that in the first step we estimate 101 regression models for males (age 0–100+ years of life) and 101 models for females with use of database of logarithms of age-specific mortality rates. Then, these models are used for extrapolation up to the year 2065, (in the same way as it performed e.g. CZSO [7]). In the second step there are calculated another 2 × 101 models using ARIMA approach with optimally selected parameters. Resulting forms are used to extrapolate the logarithms of age-specific mortality rates, also up to the year 2065. In the third step, there are the empirical and predicted mortality rates recalculated to probability ($p_{x,t}$) and number of surviving ($l_{x,t}$) using an algorithm of mortality tables. For mutual comparison, the differences were calculated as

$$diff\left(p_{x,t}\right) = p_{x,t}^{arima} - p_{x,t}^{regression}, \tag{10}$$

respectively

$$diff\left(l_{x,t}\right) = l_{x,t}^{arima} - l_{x,t}^{regression}, \tag{11}$$

and these results are subsequently presented in charts. The deviations are commented, and the advantages and disadvantages of both approaches are subsequently discussed.

3 Results

The empirical values of male's and female's age-and-sex specific mortality rates are shown in Fig. 1, logarithmic values in Fig. 2. Due to the nature of the data and a small number of living people, high variability is evident in the highest age groups.

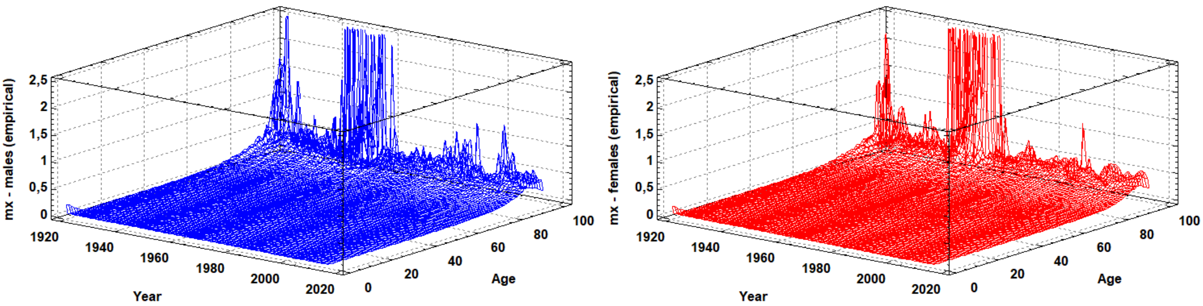

Fig. 1. Empirical age-and-sex specific mortality rates in the Czech Republic from 1920 to 2016, males – left chart, females – right chart.
Data source: CZSO [6], author's calculation and illustration.

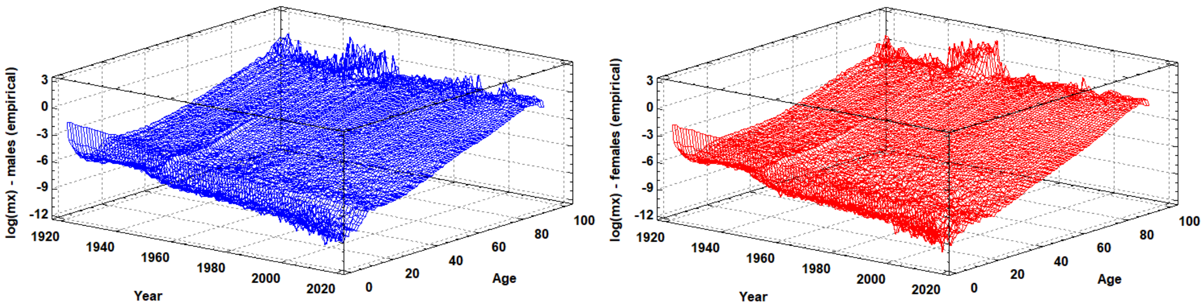

Fig. 2. Logs of age-and-sex specific mortality rates in the Czech Republic from 1920 to 2016, males – left chart, females – right chart.
Data source: CZSO [6], author's calculation and illustration.

From the mathematical nature of the indicator $m_{x,t}$ is clear, that its maximum value can be 1. Higher values presented in the graph are a systematic error which can be eliminated by smoothing in the highest ages. In this article, smoothing will not be performed to prevent data in front of external interference. Results will show how the used methodology can respond to variation of the Czech data.

According to mortality tables algorithm we recalculated $m_{x,t}$ using formulas (3), (4), (5) and (6) into $p_{x,t}$, which are presented in Fig. 3. The root of tabular number of 0-year-old persons is set on 100 000 every year and using by formula (7) we estimated tabular number of surviving. Results are presented in Fig. 4, from which it is clearly visible the change towards better mortality development.

3.1 Modelling

Optimized forms of ARIMA models for logarithms of age-and-sex specific mortality rates are written in Tab. 1 and 2 in Annex of this paper. We respected the residual diagnostic of these models. Forecast was calculated for all 101 males' and 101 females' time series and graphic presentation is shown in Fig. 5 (top charts).

Estimates of intercepts (β_0) and slopes (β_1) of the regression lines for logarithms of age-and-sex specific mortality rates were calculated as well. Extrapolation is presented in Fig. 5 (bottom charts). For better comparison of the results, Fig. 6 also shows the values $e^{\ln(m_{x,t})}$, i.e. classical mortality rates usable for mortality tables algorithm.

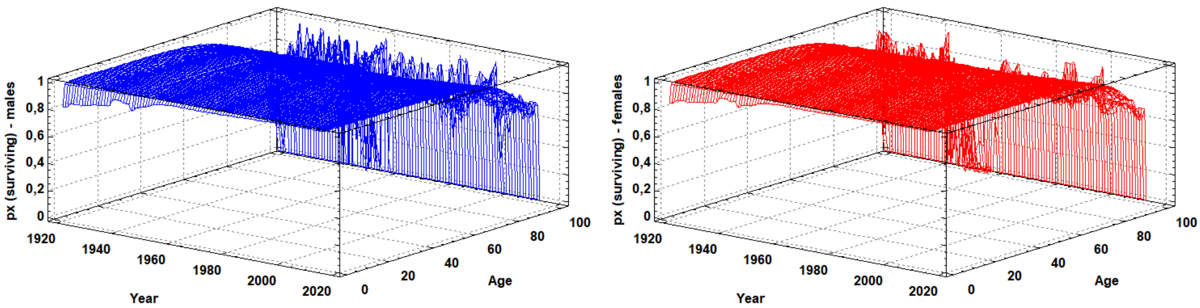

Fig. 3. Calculated age-and-sex specific probability of surviving in the Czech Republic from 1920 to 2016, males – left chart, females – right chart.
Data source: CZSO [6], author's calculation and illustration.

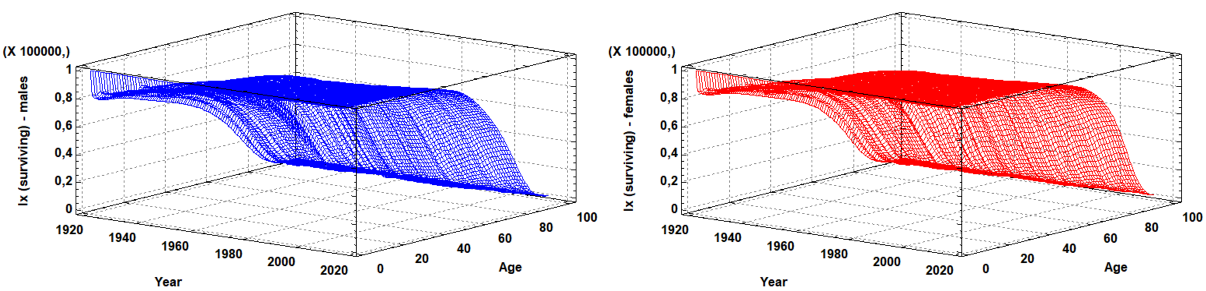

Fig. 4. Calculated age-and-sex specific tabular number of surviving in the Czech Republic from 1920 to 2016, males – left chart, females – right chart.
Data source: CZSO [6], author's calculation and illustration.

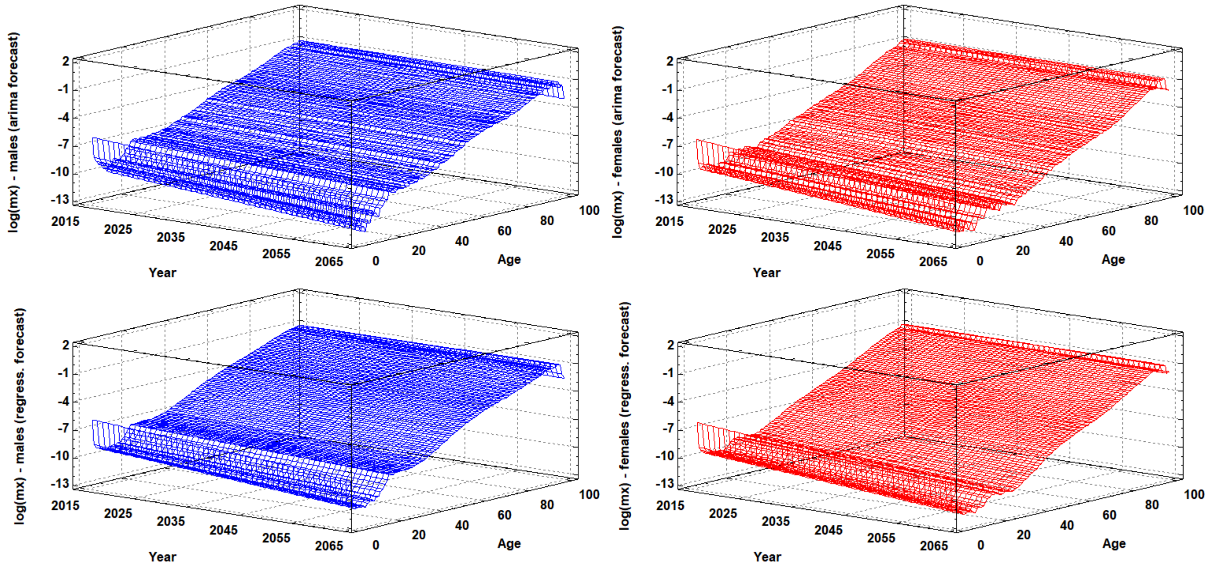

Fig. 5. Forecasted logs of age-and-sex specific mortality rates in the Czech Republic from 2016 to 2065 by selected ARIMA models (top) and by linear regression models (bottom), males – left chart, females – right chart. Source: author's calculation and illustration.

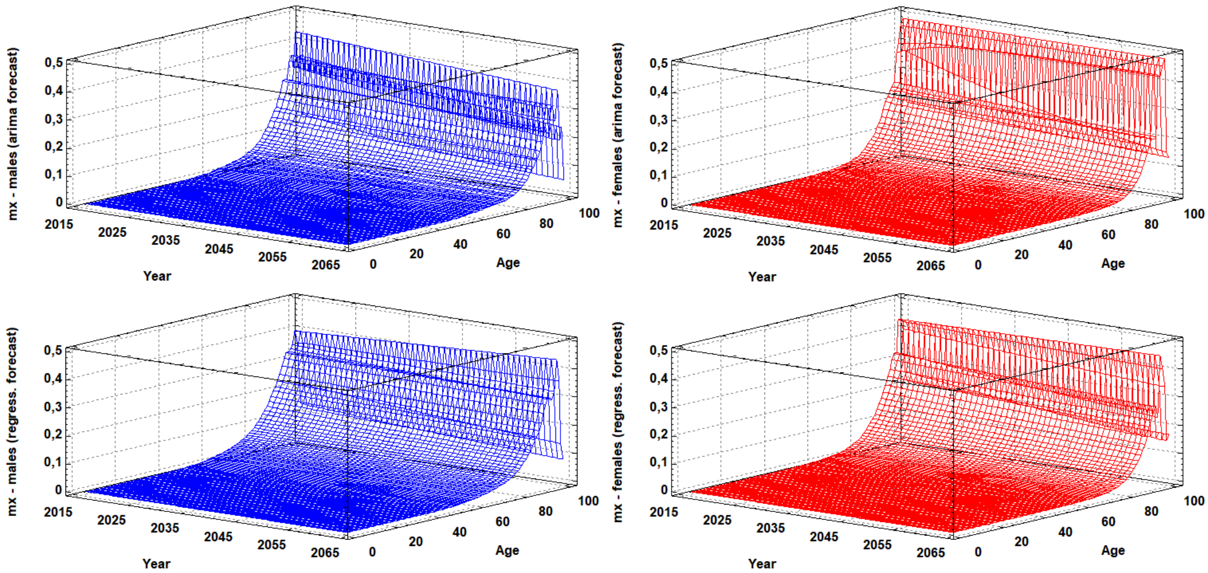

Fig. 6. Forecasted age-and-sex specific mortality rates in the Czech Republic from 2016 to 2065 by selected ARIMA models (top) and by linear regression models (bottom), males – left chart, females – right chart. Source: author's calculation and illustration.

It is clear that results by ARIMA models show higher variability in forecast, but it does not mean that they are wrong. On the contrary, they respond better to the past development of the time series and do not act so "artificially". In the case of regression approach and male population is obvious an error in the form of increasing development of mortality in the highest age groups. This error is due to the high variability in empirical data, and the parameters of linear regression were therefore deflected.

Forecast of probability ($p_{x,t}$) and tabular number ($l_{x,t}$) of surviving up to the year 2065 was calculated on the basis of mortality tables algorithm and results are displayed in Fig. 7 and 8.

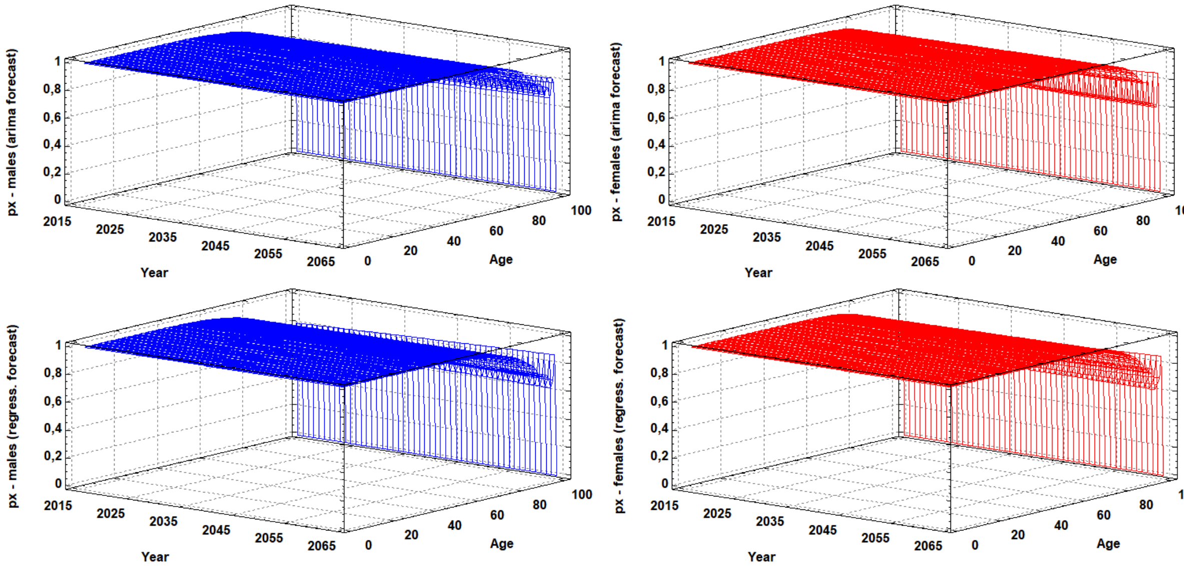

Fig. 7. Forecasted age-and-sex specific probability of surviving in the Czech Republic from 2016 to 2065 by selected ARIMA models (top) and by linear regression models (bottom), males – left chart, females – right chart. Source: author's calculation and illustration.

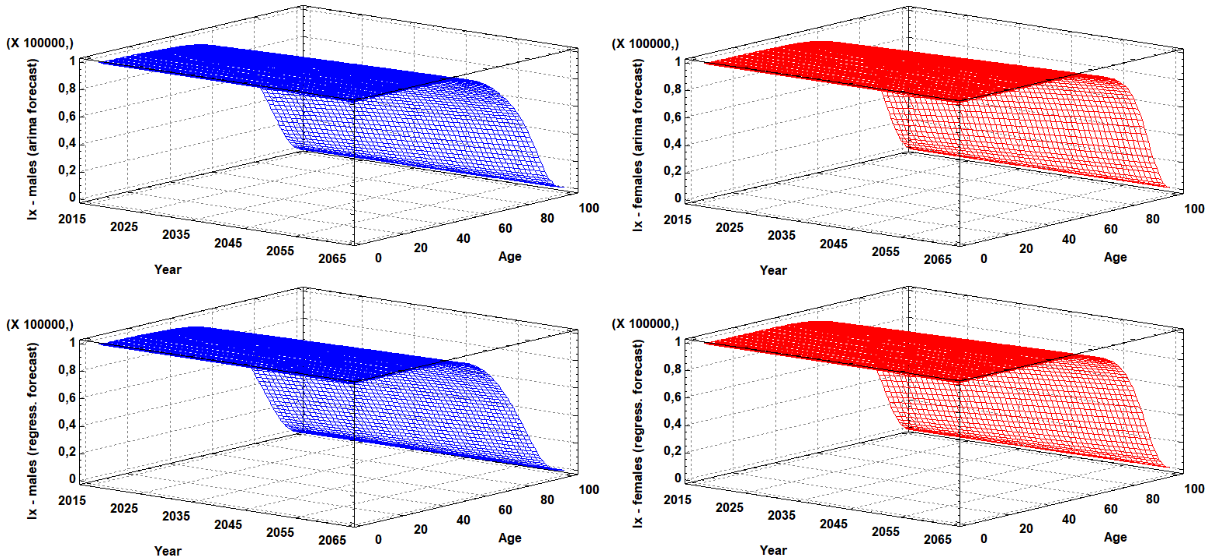

Fig. 8. Forecasted age-and-sex specific number of surviving in the Czech Republic from 2016 to 2065 by selected ARIMA models (top) and by linear regression models (bottom), males – left chart, females – right chart. Source: author's calculation and illustration.

Because the differences between the individual results may not be apparent from the figures, calculations of the differences between ARIMA models and linear regressions were made using formulas (10) and (11). The differences that are presented in Fig. 9 and 10 indicate more optimistic results in case of ARIMA models, especially at the age of 40–60 years. In later ages (65 and more), the higher probability of surviving is reflected in the overestimation of the table number of surviving persons, which is approximately 15,000 in case of male population and 8,000 in case of female population at the end of forecast in 2065. These values were observed in the mode of the distribution of $l_{x,t}$ differences. Increased variability in the case of probability of surviving in the highest age groups (95 and more) is caused by an error in linear regression models that deviated the prediction.

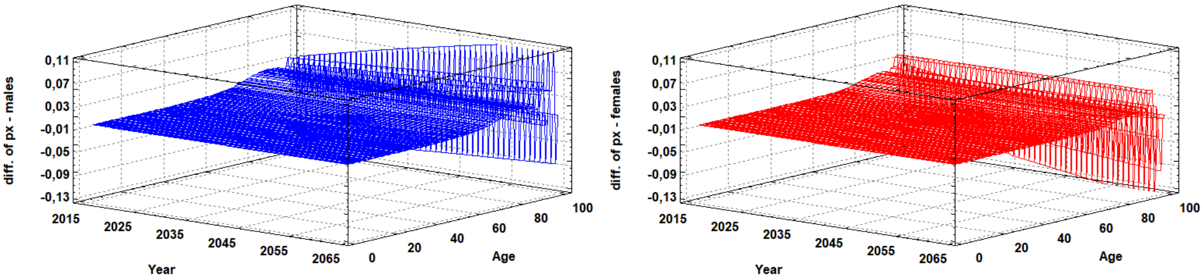

Fig. 9. Differences of forecasted age-and-sex specific probability of surviving in the Czech Republic from 2016 to 2065, males – left chart, females – right chart.
Source: author's calculation and illustration.

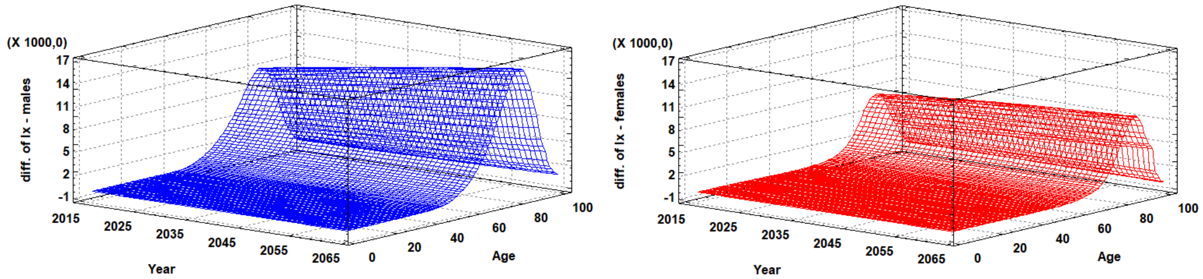

Fig. 10. Differences of forecasted age-and-sex specific number of surviving in the Czech Republic from 2016 to 2065, males – left chart, females – right chart.
Source: author's calculation and illustration.

4 Conclusion

The aim of this paper was to compare the results of stochastic (autoregressive integrated moving averages) and deterministic (linear regression) extrapolations of logs of age-and-sex-specific mortality rates in the Czech Republic with impact on selected characteristics of mortality tables – probability ($p_{x,t}$) and tabular number ($l_{x,t}$) of surviving. We estimated 2×101 ARIMA models with respect to diagnostic tests of residues (autocorrelation, heteroskedasticity and normality) and another 2×101 models based on linear regression approach. Consequently, we calculated the forecasts up to the year 2065. Differences depending on used approach are greater in the case of male population. It is mainly caused by great variability in the empirical dataset at the highest ages. This problem could be solved by smoothing of mortality rates by some of the existing models (see e.g. comparing study by Godunov [12] or Gogola [13]), but that would bring a prolongation of the procedure. Our research showed that selected ARIMA models provided more precise extrapolation of the analysed characteristics mainly in advanced ages and they are much less biased. Deterministic models are useful mainly in lower ages and up to 65 years.

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Annex

Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model
0	(0,2,1) c	13	(0,1,1) c	26	(0,1,1) c	39	(0,1,1) c	52	(0,1,1) c	65	(0,1,1) c	78	(0,1,1) c	91	(0,1,2) c
1	(0,1,1) c	14	(0,1,1) c	27	(0,1,1) c	40	(2,2,1)	53	(0,1,1) c	66	(0,1,1) c	79	(1,1,1) c	92	(2,1,0) c
2	(2,2,1)	15	(0,1,1) c	28	(0,1,1) c	41	(0,1,1) c	54	(0,1,1) c	67	(0,1,1) c	80	(0,1,1) c	93	(0,1,1) c
3	(0,1,1) c	16	(0,1,1) c	29	(0,1,1) c	42	(0,1,1) c	55	(0,1,1) c	68	(0,1,1) c	81	(0,1,1) c	94	(0,1,2) c
4	(0,1,1) c	17	(0,1,1) c	30	(0,1,1) c	43	(0,1,1) c	56	(2,1,0) c	69	(0,1,1) c	82	(0,1,1) c	95	(1,1,1) c
5	(0,1,1) c	18	(0,1,1) c	31	(2,2,1)	44	(0,1,1) c	57	(0,1,1) c	70	(0,1,1) c	83	(1,1,1) c	96	(0,1,2) c
6	(0,1,1) c	19	(0,1,1) c	32	(0,1,1) c	45	(0,1,1) c	58	(0,1,1) c	71	(0,1,1) c	84	(0,1,1) c	97	(0,1,1) c
7	(0,1,1) c	20	(0,1,2) c	33	(0,1,1) c	46	(0,1,1) c	59	(0,1,1) c	72	(0,1,1) c	85	(0,1,1) c	98	(0,1,1) c
8	(0,1,1) c	21	(0,1,1) c	34	(0,1,1) c	47	(2,1,0) c	60	(0,1,1) c	73	(1,1,0) c	86	(0,1,1) c	99	(0,1,2) c
9	(0,1,1) c	22	(0,1,1) c	35	(0,1,1) c	48	(1,1,0) c	61	(0,1,1) c	74	(1,1,0) c	87	(0,1,1) c	100+	(0,1,1) c
10	(0,1,1) c	23	(0,1,1) c	36	(0,1,1) c	49	(0,1,1) c	62	(0,1,1) c	75	(2,1,0) c	88	(0,1,1) c		
11	(0,1,1) c	24	(0,1,1) c	37	(0,1,1) c	50	(2,1,0) c	63	(0,1,1) c	76	(0,1,1) c	89	(0,1,2) c		
12	(0,1,1) c	25	(2,1,0) c	38	(0,1,1) c	51	(1,1,0) c	64	(0,1,1) c	77	(0,1,1) c	90	(2,1,0) c		

Tab. 1. ARIMA (p, d, q) models with or without constant (c) for logs of male’s age-specific mortality rates in range of 0–100+ years in the Czech Republic.
Source: author’s calculation and illustration.

Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model
0	(0,2,1) c	13	(0,1,1) c	26	(0,1,1) c	39	(0,1,1) c	52	(0,1,1) c	65	(1,1,0) c	78	(2,1,2) c	91	(1,1,1) c
1	(0,1,1) c	14	(1,1,1) c	27	(0,1,1) c	40	(2,1,0) c	53	(0,1,1) c	66	(1,1,0) c	79	(0,1,1) c	92	(2,1,0) c
2	(1,1,1) c	15	(0,1,1) c	28	(0,1,1) c	41	(1,1,0) c	54	(0,1,1) c	67	(0,1,1) c	80	(0,1,1) c	93	(0,1,1) c
3	(2,1,0) c	16	(2,1,0) c	29	(0,1,1) c	42	(0,1,1) c	55	(0,1,1) c	68	(0,1,1) c	81	(0,1,1) c	94	(2,1,1) c
4	(2,1,0) c	17	(0,1,1) c	30	(0,1,1) c	43	(0,1,1) c	56	(0,1,1) c	69	(0,1,1) c	82	(0,1,1) c	95	(1,0,0) c
5	(0,1,1) c	18	(0,1,2) c	31	(0,1,1) c	44	(0,1,1) c	57	(0,1,1) c	70	(0,1,1) c	83	(0,1,1) c	96	(1,0,0) c
6	(1,1,1) c	19	(0,1,1) c	32	(0,1,1) c	45	(0,1,1) c	58	(2,1,0) c	71	(1,1,0) c	84	(0,1,1) c	97	(0,0,2) c
7	(0,1,1) c	20	(0,1,1) c	33	(0,1,1) c	46	(0,1,1) c	59	(1,1,1) c	72	(2,1,0) c	85	(0,1,1) c	98	(0,0,2) c
8	(2,1,0) c	21	(0,1,1) c	34	(0,1,1) c	47	(0,1,1) c	60	(0,1,1) c	73	(1,1,0) c	86	(0,1,1) c	99	(1,1,0) c
9	(2,1,0) c	22	(2,1,0) c	35	(2,1,0) c	48	(0,1,1) c	61	(0,1,2) c	74	(2,1,0) c	87	(0,1,1) c	100	(0,1,1) c
10	(0,1,1) c	23	(0,1,1) c	36	(0,1,1) c	49	(1,1,1) c	62	(1,1,0) c	75	(2,1,0) c	88	(0,1,1) c		
11	(0,1,1) c	24	(1,1,0) c	37	(0,1,1) c	50	(0,1,1) c	63	(1,1,0) c	76	(0,1,1) c	89	(1,1,1) c		
12	(1,1,1) c	25	(0,1,1) c	38	(0,1,1) c	51	(0,1,1) c	64	(0,1,1) c	77	(0,1,1) c	90	(2,1,0) c		

Tab. 2. ARIMA (*p, d, q*) models with or without constant (*c*) for logs of female's age-specific mortality rates in range of 0–100+ years in the Czech Republic.
Source: author's calculation and illustration.

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