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COMPARATIVE STUDY OF SEVEN METHODS FOR ESTIMATING THE WEIBULL DISTRIBUTION PARAMETERS FOR WIND SPEED IN BRATISLAVA - MLYNSKÁ DOLINA

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Abstract. In this paper the Weibull distribution is used to describe the wind speed data collected in Bratislava-Mlynská dolina in the period of the years 2005-2009. The parameters are estimated using seven methods: the maximum likelihood, method of moments, empirical method, power density method, least squares method and weighted least squares method with two weight factors. Their performance is compared using the root mean square error and coefficient of determination. It was observed that the weighted least squares method perform the best, followed by the maximum likelihood method and the method of moments.

Keywords: Weibull distribution**,** parameter estimation, wind speed

Mathematics Subject Classification: Primary 6207, Secondary 62P30, 62P12.

1 Introduction

Wind power belongs to among promising energy sources with minimal environmental impact and huge energetic potential. However, attention has to be paid to the optimal locality selection to optimize cost/benefit ratio (Morgan et al., 2011).
Many conditions have to be satisfied in the process of locality selection, but one of the most important is a suitable wind speed spectrum. The literature contains several distributions that can be applied to calculate the wind speed distribution (Morgan et al., 2011; Celik, 2003; Carta et al., 2009; Lun and Lam, 2000; Gryning, 2016). By far the most widely-used distribution for the characterization of average wind speeds is the two-parameter Weibull distribution. Most likely it is a consequence of relative simplicity and flexibility of Weibull distribution, as well as its good ability to fit measured data. The way the distribution parameters are estimated is at least of the same importance as the correct distribution choice. There are several methods for estimating the Weibull distribution parameters (Justus et al., 1978; Stevens et al., 1979; Conradsen et al., 1984; Seguro and Lambert, 2000; Dorvlo, 2002; Wu et al., 2006; Akdağ and Dinler, 2009). In the recent literature

these methods are compared several times and in different ways (Kantar and Senoglu, 2008; Chang, 2011; Kantar et al., 2011; Costa Rocha et al., 2012).
This paper uses the two-parameter Weibull distribution to describe the wind speed in Bratislava-Mlynská dolina for given wind speed data that cover the period of years 2005 - 2009. The presented paper therefore examines seven different methods for estimating Weibull distribution parameters. Namely: the maximum likelihood method (MLM), the method of moments (MOM), the empirical method (EM), the power density method (PDM), the least squares method (LSM) and the weighted least squares method (WLSM) with two weight factors. The comparison of these methods is based on the monthly, seasonal and yearly wind speed data. Finally the root mean square error ($RMSE$) and the coefficient of determination (R^2) are used to measure the performance of the estimating methods. All the numerical computations are performed in Matlab 7.1.

2 Methodology

2.1 Description of wind speed data

The wind speed data processed in the presented paper were measured at the Meteorological observatory Bratislava-Mlynská dolina, situated on the campus of the Faculty of Mathematics, Physics and Iformatics, Comenius University in Bratislava, within time frame January 2005 to December 2009. The sampling site Mlynská dolina is situated at the SW boundary of Bratislava (N 48°09'08", E 17°04'13", 195 m.a.s.l). It is situated approximately 700 m to the north from the Danube river, on the university campus. The experimental setup is installed on the roof of the university building. The data are collected 7 m above the roof and 25 m above ground. The wind speed has been measured by universal anemograph METRA, working on Pitot tube principle. Data are recorded on the paper tape and manually transcripted once a day. Hourly averages are evaluated, rounded to the nearest integer and stored in the electronic form. The sample size to be researched is 43 824.

2.2 Methods for estimating the Weibull distribution parameters

The probability density function of the two-parameter Weibull distribution is given by

$$f(v) = \frac{k}{c^k} v^{k-1} \exp\left(-\left(\frac{v}{c}\right)^k \right) \tag{1}$$

and the cumulative distribution function is given by

$$F(v) = 1 - \exp\left(-\left(\frac{v}{c}\right)^k \right), \tag{2}$$

where $v > 0$, $k > 0$ and $c > 0$. Here v is the wind speed, k is the dimensionless shape parameter and c is the scale parameter in units of the wind speed. The mean wind speed $E(V)$ and the variance of the wind speed $Var(V)$ are given by

$$E(V) = c\Gamma\left(1 + \frac{1}{k}\right), \tag{3}$$

$$Var(V) = c^2\left[\Gamma\left(1+\frac{2}{k}\right)-\Gamma^2\left(1+\frac{1}{k}\right)\right], \tag{4}$$

where $\Gamma(a), a>0$, is the gamma function.

Let $V_1,V_2,\ldots,V_n$ be a random sample of size n from the Weibull distribution with parameters k and c and let $v_1,v_2,\ldots,v_n$ be a realization of a random sample. Let $V_{(1)}<V_{(2)}<\ldots<V_{(n)}$ be the order statistics of $V_1,V_2,\ldots,V_n$ and let $v_{(1)}<v_{(2)}<\ldots<v_{(n)}$ be the observed ordered observations.

There are several methods described in the literature for estimating the Weibull distribution parameters. This paper presents seven methods for estimating the parameters k and c.

Maximum likelihood method

The likelihood function of the Weibull distribution is given by

$$L(k,c)=\prod_{i=1}^{n}\frac{k}{c^k}v_i^{k-1}\exp\left(-\left(\frac{v_i}{c}\right)^k\right). \tag{5}$$

By differentiating the logarithm of the function (5) with respect to k and c, respectively, and equating them to zero, we obtain

$$\begin{aligned}\frac{\partial \ln L(k,c)}{\partial k}&=\frac{n}{k}-n\ln c-\frac{\sum_{i=1}^{n}v_i^k\ln v_i-\ln c\sum_{i=1}^{n}v_i^k}{c^k}+\sum_{i=1}^{n}\ln v_i=0,\\ \frac{\partial \ln L(k,c)}{\partial c}&=-\frac{nk}{c}+\frac{k}{c^{k+1}}\sum_{i=1}^{n}v_i^k=0.\end{aligned} \tag{6}$$

The parameters k and c can be estimated by the following equations (Carta et al., 2009)

$$\frac{1}{k}-\frac{\sum_{i=1}^{n}v_i^k\ln v_i}{\sum_{i=1}^{n}v_i^k}+\frac{1}{n}\sum_{i=1}^{n}\ln v_i=0, \tag{7}$$

$$c=\left(\frac{1}{n}\sum_{i=1}^{n}v_i^k\right)^{1/k}. \tag{8}$$

The estimate $\hat{k}$ of the parameter k can be obtained by solving (7) with respect to k. The estimate $\hat{c}$ of the parameter c can be obtained using equation (8). The equation (7) can be solved iteratively with respect to k, after which equation (8) can be solved analytically. The Newton method was used for the numerical computations.

Method of moments

The estimates of the parameters k and c can be obtained by equating the moments of the Weibull distribution with the corresponding sample moments. The parameters k and c can be estimated by the following equations (Carta et al., 2009)

$$c\Gamma\left(1+\frac{1}{k}\right)=\overline{v}, \tag{9}$$

$$c^2\Gamma\left(1+\frac{2}{k}\right)=\frac{1}{n}\sum_{i=1}^{n}v_i^2\,, \tag{10}$$

where $\bar{v}=\frac{1}{n}\sum_{i=1}^{n}v_i$ is the sample mean wind speed. By dividing (10) by the square of (9) one obtains

$$\frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)}=\frac{\frac{1}{n}\sum_{i=1}^{n}v_i^2}{\bar{v}^2}. \tag{11}$$

The estimate $\hat{k}$ of the parameter k can be obtained by solving (11) with respect to k. This equation does not have an analytical solution and can be solved iteratively with respect to k. The estimate $\hat{c}$ of the parameter c can be obtained using equation (9).

Empirical method

The empirical method can be used as a special case of the MOM. The parameter k can be estimated by following equation (Morgan et al., 2011; Akdağ and Dinler, 2009)

$$\hat{k}=\left(\frac{\bar{v}}{s_v}\right)^{1.086}, \tag{12}$$

where $s_v=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(v_i-\bar{v}\right)^2}$ is the sample standard deviation. The parameter c can be estimated using (9).

Power density method

The energy pattern factor is defined as (Akdağ and Dinler, 2009)

$$E_{pf}=\frac{\overline{v^3}}{\bar{v}^3}, \tag{13}$$

where $\overline{v^3}=\frac{1}{n}\sum_{i=1}^{n}v_i^3$ is the sample mean of the wind speed cubes. The parameter k can be estimated by the following equation

$$\hat{k}=1+\frac{3.69}{\left(E_{pf}\right)^2}, \tag{14}$$

and the parameter c can be estimated using (9).

Least squares method

The cumulative distribution function (2) can be linearized as follows

$$\ln\left(-\ln\left(1-F(v)\right)\right)=k\ln(v)-k\ln(c). \tag{15}$$

Let $Y=\ln\left(-\ln\left(1-F(v)\right)\right)$, $X=\ln v$, $b=k$ and $a=-k\ln c$. Then the equation (15) can be rewritten as

$$Y=b\,X+a. \tag{16}$$

The parameters in equation (15) can be estimated using the least squares method. The estimates of the regression parameters a and b minimize the function

$$Q(a,b)=\sum_{i=1}^{n}(Y_i-a-b\ln v_{(i)})^2. \qquad (17)$$

The parameters k and c can be estimated by the following equations

$$\hat{k}=\frac{n\sum_{i=1}^{n}\ln v_{(i)}\ln\left[-\ln\left(1-\hat{F}\left(v_{(i)}\right)\right)\right]-\sum_{i=1}^{n}\ln v_{(i)}\sum_{i=1}^{n}\ln\left[-\ln\left(1-\hat{F}\left(v_{(i)}\right)\right)\right]}{n\sum_{i=1}^{n}\ln^2 v_{(i)}-\left(\sum_{i=1}^{n}\ln v_{(i)}\right)^2}, \qquad (18)$$

$$\hat{c}=\exp\left(-\frac{\sum_{i=1}^{n}\ln\left[-\ln\left(1-\hat{F}\left(v_{(i)}\right)\right)\right]-\hat{k}\sum_{i=1}^{n}\ln v_{(i)}}{\hat{k}\,n}\right). \qquad (19)$$

To estimate the values of the cumulative distribution function, the mean rank is used

$$\hat{F}\left(v_{(i)}\right)=\frac{i}{n+1}, \qquad (20)$$

where i denotes the i^{th} smallest value of $v_{(1)},v_{(2)},\ldots,v_{(n)}$, $i=1,2,\ldots,n$.

Weighted least squares method

The estimates of the regression parameters a and b minimize the function

$$Q(a,b)=\sum_{i=1}^{n}w_i(Y_i-a-b\ln v_{(i)})^2, \qquad (21)$$

where w_i is the weight factor, $i=1,2,...,n$. Two weight factors are used in this paper. The first weight factor was proposed in Bergman (1986)

$$w_i=\left[\left(1-\hat{F}(v_{(i)})\right)\ln\left(1-\hat{F}(v_{(i)})\right)\right]^2, \quad i=1,2,...,n. \qquad (22)$$

and the second weight factor was proposed in Faucher and Tyson (1988)

$$w_i=3.3\,\hat{F}(v_{(i)})-27.5\left[1-\left(1-\hat{F}(v_{(i)})\right)^{0.025}\right], \quad i=1,2,...,n. \qquad (23)$$

The parameters k and c can be estimated by the following equations

$$\hat{k}=\frac{\sum_{i=1}^{n}w_i\sum_{i=1}^{n}w_i\ln v_{(i)}\ln\left[-\ln\left(1-\hat{F}\left(v_{(i)}\right)\right)\right]-\sum_{i=1}^{n}w_i\ln v_{(i)}\sum_{i=1}^{n}w_i\ln\left[-\ln\left(1-\hat{F}\left(v_{(i)}\right)\right)\right]}{\sum_{i=1}^{n}w_i\sum_{i=1}^{n}w_i\ln^2 v_{(i)}-\left(\sum_{i=1}^{n}w_i\ln v_{(i)}\right)^2}, \qquad (24)$$

$$\hat{c}=\exp\left(-\frac{\sum_{i=1}^{n}w_i\ln\left[-\ln\left(1-\hat{F}\left(v_i\right)\right)\right]-\hat{k}\sum_{i=1}^{n}w_i\ln v_i}{\hat{k}\sum_{i=1}^{n}w_i}\right). \qquad (25)$$

2.3 Statistical analysis

The wind speed data were divided into subsets with respect to the months, four seasons and years, respectively. The months were divided as follows: winter (December, January, February), spring (March, April, May), summer (June, July, August) and autumn (September, October, November). The estimates of the Weibull distribution parameters were calculated for each month, season, year and the whole monitored period of 5 years, respectively.

To find the best method to estimate the Weibull distribution parameters the coefficient of determination (R^2) and the root mean square error ($RMSE$) were used. These parameters can be calculated using

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (v_i - x_i)^2}{\sum_{i=1}^{n} (v_i - \bar{v})^2}, \tag{26}$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (v_i - x_i)^2}, \tag{27}$$

where n is the number of the wind speed data, x_i is the i^{th} predicted data calculated using the Weibull distribution, v_i is the i^{th} ordered observed wind speed.

The coefficient of determination R^2 ranges from 0 to 1. The ideal value of R^2 is equal to 1. The root mean square error $RMSE$ ranges from 0 to infinity. The ideal value of $RMSE$ is close to zero. The best method for estimating the Weibull distribution parameters can be determined according to the highest values of R^2 and the lowest values of $RMSE$. R^2 and $RMSE$ were calculated for each month, season, year and the whole monitored period of 5 years, respectively.

4 Results and discussion

Tables 1, 2 and 3 show the estimates of the Weibull distribution parameters and the results of the statistical analysis of all of the examined methods.

The comparison of seasonal Weibull probability density distributions with the observed seasonal probability density distributions of the wind speed are illustrated in Figure 1. The comparison of the Weibull probability density distributions with the observed probability density distribution of the wind speed for the whole monitored period of 5 years is illustrated in Figure 2. The comparison of the yearly Weibull probability density distributions for the years 2005-2009 with the observed yearly probability density distributions of the wind speed is illustrated in Figure 3.

The comparison of the results in Table 1 and Figure 1 shows that the seasonal value of $RMSE$ ranges from 0.00198 to 0.00939. The seasonal value of R^2 ranges from 0.81982 to 0.99559 which means that all the presented methods fit the data successfully. Each approach has R^2 value better than 0.96777 in spring, summer, autumn and better than 0.81982 in winter. The WLSMa seems to have the best performance for all seasons in terms of R^2 and $RMSE$. The second best method is the WLSMb, except spring, when the second best performance can be assigned to the LSM in terms of R^2 and $RMSE$.

	Winter				Spring			
Method	$\hat{k}$	$\hat{c}$	$RMSE$	R^2	$\hat{k}$	$\hat{c}$	$RMSE$	R^2
MLM	1.7169	12.6399	0.00878	0.84264	1.9022	12.0985	0.00427	0.96971
MOM	1.7502	12.6802	0.00902	0.83388	1.9073	12.1044	0.00429	0.96946
EM	1.7745	12.6700	0.00921	0.82671	1.9309	12.1088	0.00440	0.96777
PDM	1.7969	12.6981	0.00939	0.81982	1.9235	12.1075	0.00437	0.96835
LSM	1.6193	12.7254	0.00804	0.86787	1.8687	12.1287	0.00414	0.97158
WLSMa	1.6119	13.0680	0.00791	0.87232	1.8765	12.2167	0.00411	0.97198
WLSMb	1.6244	12.8472	0.00803	0.86823	1.8829	12.1384	0.00417	0.97113
	Summer				**Autumn**			
Method	$\hat{k}$	$\hat{c}$	$RMSE$	R^2	$\hat{k}$	$\hat{c}$	$RMSE$	R^2
MLM	1.9502	10.4870	0.00219	0.99457	1.8100	11.1920	0.00313	0.98530
MOM	1.9323	10.4670	0.00213	0.99487	1.8065	11.1848	0.00309	0.98561
EM	1.9557	10.4703	0.00216	0.99470	1.8307	11.1915	0.00335	0.98309
PDM	1.9233	10.4657	0.00214	0.99482	1.8233	11.1895	0.00327	0.98392
LSM	2.0365	10.4303	0.00271	0.99172	1.8123	11.1768	0.00315	0.98503
WLSMa	1.9303	10.3838	0.00198	0.99559	1.7457	11.2540	0.00262	0.98965
WLSMb	1.9478	10.4060	0.00201	0.99543	1.7640	11.2115	0.00273	0.98879

Tab. 1. The estimates of the Weibull distribution parameters and the results of the statistical analysis for four seasons of composite year.

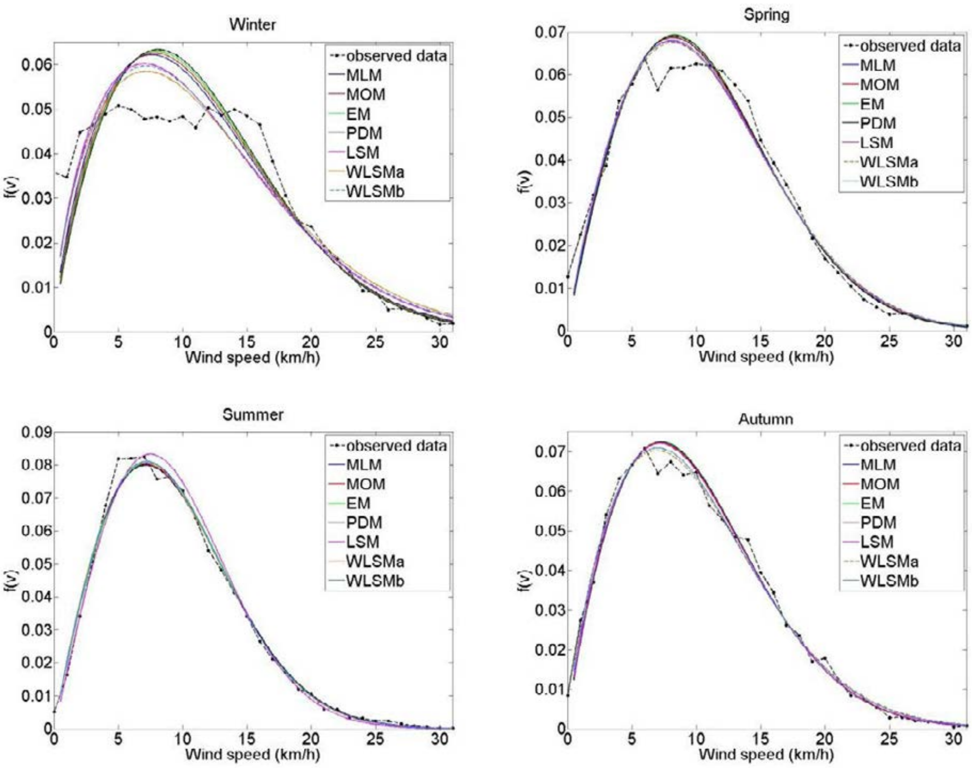

Fig. 1. Comparison of Weibull probability density distributions for four seasons of composite year

The analyses based on Table 2 and Figure 2 further show that the value of $RMSE$ for the whole monitored period of 5 years ranges from 0.00302 to 0.00360 and the value of R^2 ranges from 0.97851 to 0.98482. The WLSMa performs better than the other methods in terms of R^2 and $RMSE$. The second best method is the WLSMb.

From Table 2 and Figure 3 comparison shows that the yearly value of $RMSE$ ranges from 0.00223 to 0.00828 and the yearly value of R^2 ranges from 0.88641 to 0.99230. The WLSMa performs better than any other method for all the years in terms of R^2 and $RMSE$. The second best method is the WLSMb.

	2005				2006			
Method	$\hat{k}$	$\hat{c}$	$RMSE$	R^2	$\hat{k}$	$\hat{c}$	$RMSE$	R^2
MLM	1.8516	11.7633	0.00321	0.98358	1.8153	11.3147	0.00253	0.99047
MOM	1.8442	11.7510	0.00316	0.98412	1.8071	11.3032	0.00249	0.99081
EM	1.8682	11.7570	0.00335	0.98209	1.8313	11.3100	0.00264	0.98962
PDM	1.8608	11.7552	0.00329	0.98277	1.8111	11.3044	0.00251	0.99065
LSM	1.8692	11.7280	0.00337	0.98188	1.8378	11.2898	0.00270	0.98919
WLSMa	1.7727	11.8314	0.00282	0.98730	1.7780	11.3352	0.00239	0.99153
WLSMb	1.8017	11.7756	0.00290	0.98656	1.7951	11.2991	0.00243	0.99119
	2007				**2008**			
Method	$\hat{k}$	$\hat{c}$	$RMSE$	R^2	$\hat{k}$	$\hat{c}$	$RMSE$	R^2
MLM	1.8008	11.5603	0.00625	0.93528	1.7284	11.4348	0.00802	0.89352
MOM	1.8127	11.5720	0.00634	0.93339	1.7375	11.4436	0.00809	0.89168
EM	1.8369	11.5787	0.00654	0.92902	1.7618	11.4528	0.00828	0.88641
PDM	1.8437	11.5805	0.00660	0.92771	1.7525	11.4494	0.00821	0.88845
LSM	1.7458	11.6021	0.00584	0.94341	1.6929	11.4630	0.00774	0.90063
WLSMa	1.7419	11.7171	0.00578	0.94458	1.6783	11.5986	0.00763	0.90349
WLSMb	1.7433	11.6462	0.00581	0.94404	1.6859	11.5023	0.00769	0.90202
	2009				**2005-2009**			
Method	$\hat{k}$	$\hat{c}$	$RMSE$	R^2	$\hat{k}$	$\hat{c}$	$RMSE$	R^2
MLM	1.9024	11.8970	0.00251	0.99027	1.8183	11.5958	0.00340	0.98077
MOM	1.8876	11.8769	0.00239	0.99114	1.8165	11.5912	0.00339	0.98091
EM	1.9113	11.8816	0.00257	0.98975	1.8408	11.5979	0.00360	0.97851
PDM	1.8928	11.8780	0.00243	0.99087	1.8311	11.5953	0.00351	0.97952
LSM	1.9581	11.8375	0.00304	0.98572	1.8165	11.5867	0.00339	0.98089
WLSMa	1.8445	11.8798	0.00223	0.99230	1.7649	11.6729	0.00302	0.98482
WLSMb	1.8715	11.8586	0.00230	0.99182	1.7811	11.6189	0.00313	0.98377

Tab. 2. The estimates of the Weibull distribution parameters and the results of the statistical analysis for individual years and for years 2005-2009.

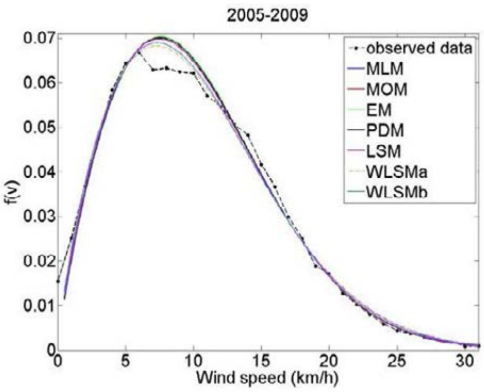

Fig. 2. Comparison of Weibull probability density distributions for the period 2005-2009.

January					February			
Method	$\hat{k}$	$\hat{c}$	*RMSE*	R^2	$\hat{k}$	$\hat{c}$	*RMSE*	R^2
MLM	1.6900	13.3453	0.00903	0.81423	1.7669	12.2243	0.01584	0.56197
MOM	1.7331	13.4025	0.00930	0.80283	1.8089	12.2746	0.01616	0.54401
EM	1.7573	13.4134	0.00947	0.79540	1.8329	12.2819	0.01636	0.53275
PDM	1.7842	13.4244	0.00967	0.78671	1.8694	12.2915	0.01666	0.51514
LSM	1.5740	13.4581	0.00825	0.84487	1.6234	12.3657	0.01467	0.62399
WLSMa	1.5644	13.9283	0.00805	0.85222	1.6594	12.6871	0.01486	0.61416
WLSMb	1.5821	13.6107	0.00823	0.84575	1.6630	12.4637	0.01493	0.61069
March					**April**			
Method	$\hat{k}$	$\hat{c}$	*RMSE*	R^2	$\hat{k}$	$\hat{c}$	*RMSE*	R^2
MLM	1.7926	13.3675	0.00675	0.89492	1.9366	11.2466	0.00598	0.94583
MOM	1.8152	13.3979	0.00684	0.89182	1.9433	11.2532	0.00603	0.94495
EM	1.8392	13.4056	0.00698	0.88756	1.9664	11.2564	0.00624	0.94114
PDM	1.8472	13.4080	0.00703	0.88604	1.9788	11.2579	0.00635	0.93890
LSM	1.7032	13.4657	0.00637	0.90632	1.8755	11.2831	0.00552	0.95384
WLSMa	1.7619	13.6143	0.00649	0.90264	1.8443	11.4240	0.00528	0.95778
WLSMb	1.7479	13.4989	0.00648	0.90294	1.8709	11.3296	0.00546	0.95495
May					**June**			
Method	$\hat{k}$	$\hat{c}$	*RMSE*	R^2	$\hat{k}$	$\hat{c}$	*RMSE*	R^2
MLM	2.1266	11.6513	0.00532	0.96344	1.9643	10.0331	0.00358	0.98717
MOM	2.1258	11.6523	0.00532	0.96348	1.9448	10.0127	0.00357	0.98725
EM	2.1466	11.6526	0.00538	0.96258	1.9678	10.0155	0.00354	0.98744
PDM	2.1487	11.6526	0.00539	0.96248	1.9196	10.0091	0.00366	0.98661
LSM	2.0886	11.6708	0.00523	0.96464	2.0764	9.9750	0.00397	0.98426
WLSMa	2.0605	11.7785	0.00515	0.96578	1.9710	9.9094	0.00337	0.98865
WLSMb	2.0850	11.6944	0.00520	0.96499	1.9933	9.9249	0.00341	0.98838
July					**August**			
Method	$\hat{k}$	$\hat{c}$	*RMSE*	R^2	$\hat{k}$	$\hat{c}$	*RMSE*	R^2
MLM	1.9115	10.9645	0.00286	0.98969	1.9994	10.4444	0.00312	0.98931
MOM	1.8932	10.9424	0.00279	0.99015	1.9869	10.4305	0.00306	0.98975
EM	1.9168	10.9466	0.00285	0.98979	2.0096	10.4327	0.00316	0.98903
PDM	1.8926	10.9422	0.00279	0.99015	1.9886	10.4307	0.00306	0.98971
LSM	1.9827	10.9057	0.00320	0.98705	2.0516	10.4086	0.00347	0.98680
WLSMa	1.8796	10.8608	0.00270	0.99079	1.9474	10.4005	0.00300	0.99011
WLSMb	1.8892	10.8964	0.00273	0.99059	1.9782	10.3997	0.00301	0.99008
September					**October**			
Method	$\hat{k}$	$\hat{c}$	*RMSE*	R^2	$\hat{k}$	$\hat{c}$	*RMSE*	R^2
MLM	1.9773	9.8233	0.00326	0.98897	1.7302	10.8425	0.00475	0.96516
MOM	1.9747	9.8203	0.00324	0.98912	1.7319	10.8387	0.00477	0.96484
EM	1.9975	9.8226	0.00347	0.98748	1.7560	10.8476	0.00508	0.96024
PDM	1.9975	9.8226	0.00347	0.98747	1.7573	10.8480	0.00509	0.95998
LSM	1.9570	9.8309	0.00307	0.99019	1.7179	10.8313	0.00461	0.96717
WLSMa	1.9043	9.8800	0.00280	0.99186	1.6448	10.9560	0.00392	0.97630
WLSMb	1.9248	9.8479	0.00287	0.99145	1.6616	10.8925	0.00405	0.97475
November					**December**			
Method	$\hat{k}$	$\hat{c}$	*RMSE*	R^2	$\hat{k}$	$\hat{c}$	*RMSE*	R^2
MLM	1.9012	12.9339	0.00519	0.94647	1.7220	12.3016	0.00656	0.91430
MOM	1.9086	12.9415	0.00523	0.94561	1.7423	12.3228	0.00674	0.90963
EM	1.9320	12.9461	0.00539	0.94218	1.7665	12.3325	0.00696	0.90338
PDM	1.9434	12.9481	0.00548	0.94033	1.7821	12.3383	0.00712	0.89909
LSM	1.8469	12.9660	0.00488	0.95267	1.6575	12.3427	0.00600	0.92829

WLSMa	1.8115	13.1461	0.00467	0.95662	1.6294	12.5881	0.00572	0.93488
WLSMb	1.8373	13.0319	0.00480	0.95424	1.6404	12.4448	0.00583	0.93228

Tab. 3. The estimates of the Weibull distribution parameters and the results of the statistical analysis for composite months for years 2005-2009.

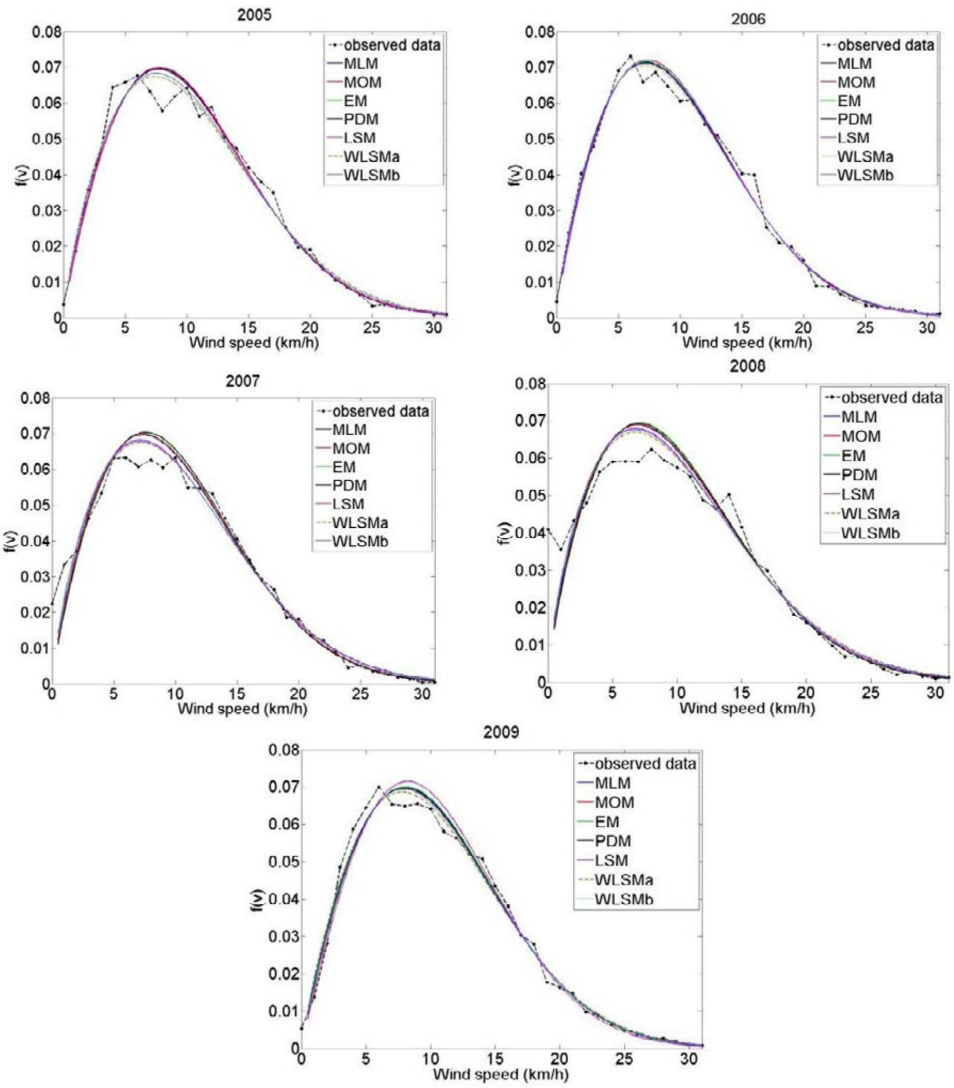

Fig. 3. Comparison of Weibull probability density distributions for individuals years.

Analyzing the results in Table 3 month by month one can find that the WLSMa performs better than other methods in January and in April - December in terms of R^2 and $RMSE$. In February and March the LSM performs best of all methods. The second best method is the WLSMb, except for February, when the second best method is the WLSMa in terms of R^2 and $RMSE$.
The comparison shows that all seven methods are applicable for estimating the Weibull distribution parameters for all seasons and for the whole monitored period of 5 years with a sufficient level of performance.
The comparison shows that the WLSMa and the WLSMb perform better than the other methods in terms of R^2 and $RMSE$ in the majority of the cases.

The MLM and the MOM give comparable values of R^2 and $RMSE$. These two methods perform well in terms of R^2 and $RMSE$. However, these methods perform worse than the WLSMa and the WLSMb.
The EM and the PDM are comparable methods in terms of R^2 and $RMSE$. These two methods give the largest values of $RMSE$ and the smallest values of R^2 in the majority of the cases. These methods perform worse than the others methods in terms of R^2 and $RMSE$.

5 Conclusion

In this paper the wind speed data from Bratislava-Mlynská dolina were fitted using the two-parameter Weibull distribution. The seven methods: the maximum likelihood method, the method of moments, the empirical method, the power density method, the least squares method and the weighted least squares method with two weight factors were used for estimating the Weibull distribution parameters. The coefficient of determination and the root mean square error were used to evaluate the performance of the considered methods. The following results can be concluded.
All seven methods perform well and are applicable for estimating the Weibull distribution parameters for all seasons, years and for the whole monitored period of 5 years as is indicate by high values of R^2 and low values of $RMSE$.
The EM and the PDM perform poorer compared to the other methods. Both methods are comparable in terms of R^2 and $RMSE$.
The MLM and the MOM that use numerical methods for estimating the Weibull distribution parameters perform a little worse than the WLSMa and the WLSMb in terms of R^2 and $RMSE$. These two methods are comparable.
The LSM perform worse than the WLSM, MLM and the MOM in terms of R^2 and $RMSE$. The study showed that the weight factor has an effect on the performance of the parameter estimation.
Statistical analysis showed that the values of R^2 and $RMSE$ are similar for the WLSMa and the WLSMb for the months, seasons, years and the whole monitored period in the majority of the cases. It can be concluded that these two methods have the best performance in estimating the Weibull distribution parameters. The WLSMa ranked first followed by the WLSMb in the majority of the cases. These methods provide the best performance in fitting the wind speed data measured in Bratislava – Mlynská dolina by Weibull distribution and are recommended. Another useful feature of these methods is the relative simplicity of the required numerical computations.

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