PARALLEL ILLUMINATION IN GEOGEBRA

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Abstract. The paper introduces several parallel illumination problems solved using geometric GeoGebra software. The solution of the problem both in 3D and in 2D can be identified in GeoGebra at the same time. As a result it provides students better imagination of the problem submission and furthermore its solution.

Keywords: descriptive geometry, parallel illumination, GeoGebra, Monge projection shadow, moveable

Mathematics Subject Classification: Primary 97B50, 68U05; Secondary 51N05

1 Introduction

The descriptive geometry is very demanding in terms of spatial imagination. Students have serious difficulties with solving spatial problems by using different projecting methods. They are often able to memorize the individual constructs, but they are not able to imagine the constructs in space. If the solution of the problem is non-standard, the students are uncertain of the solution. GeoGebra can additionally be used for parallel illumination.

Geometric software GeoGebra provides a better idea of the exercise and its solution as it displays spatial exercises both in space and their projection onto the plane. Therefore it is very helpful to the students. GeoGebra is also dynamic software, which means it allows to shift certain elements and to study various solutions of the problems (including non-standard problems). The dynamical applets facilitate better comprehension of the issues.

GeoGebra workspace can be divided into one, two or three parts. One part is for a 3D view and two others for a 2D view. We use one of them for the “Check Boxes” that allow to show and hide one or more parts of the constructions. All examples introduced in the paper are solved in 3D and in 2D together. The solution is displayed in Monge projection on the plane. All applets are dynamic and linked. If some points are shifted in 3D, subsequently the same points have been shifted in 2D. Moveable points are always marked blue. If we decide to change placement of the points, we move them by cursor.

Points in Monge projection in a 2D window are given by using coordinates of the same points in a 3D view. However, the constructions in 2D and in 3D windows need to be done separately.

2 Some Exercises solved in GeoGebra

2.1 The Parallel Illumination of Quadrilateral Prism

First problem to be solved is the illumination of the quadrilateral prism. The initial submission sets the prism with one base in the horizontal plane. Vertices *A*, *B*, *C*, *D* are chosen on the horizontal plane and the vertex *E* on the upper base. Their location is random, therefore these vertices are moveable. The base and the height of prism can be modified. These points are marked blue in the picture. Other vertices are dependent on the moveable vertices (these points are marked red). The direction of the illumination is also given by two points in 3D and these points are moveable (blue). The shape of the prism and the direction of the illumination can be modified (see Fig. 1).

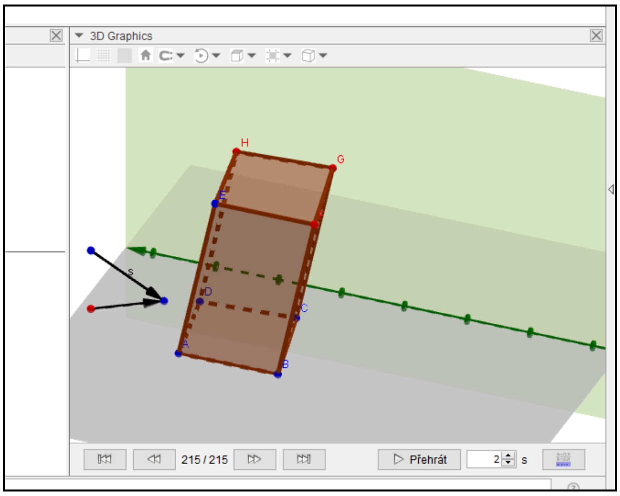

Fig. 1

Vertices A_1, A_2, B_1, B_2, …, H_1, H_2 of the prism and points of direction in Monge projection in 2D window are given by using coordinates of the points *A, B, C, …, H* in a 3D view. If the blue vertices in a 3D window have been shifted, then the points in a 2D window have also moved. The folding line is set by two points (see Fig. 2.)

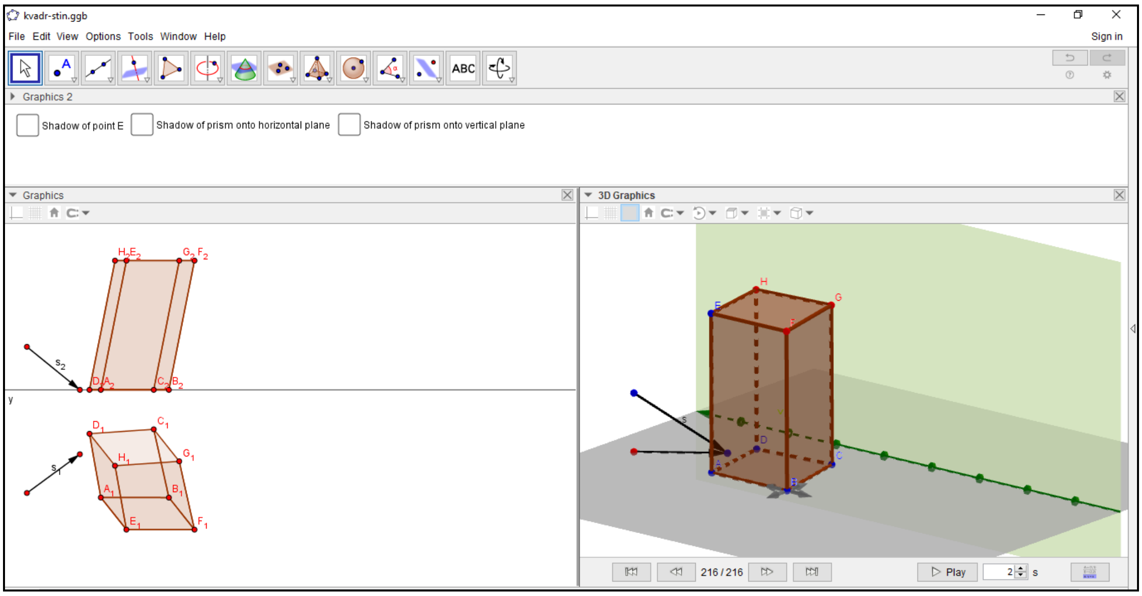

Fig. 2

Three check boxes are placed into one 2D window. The first check box shows and hides construction of the shadow of the point E. It is evident that the shadow of point E on the vertical and horizontal plane are vertical and horizontal trace points E'', E' of parallel line intersecting point E with direction of the illumination. See Fig. 3.

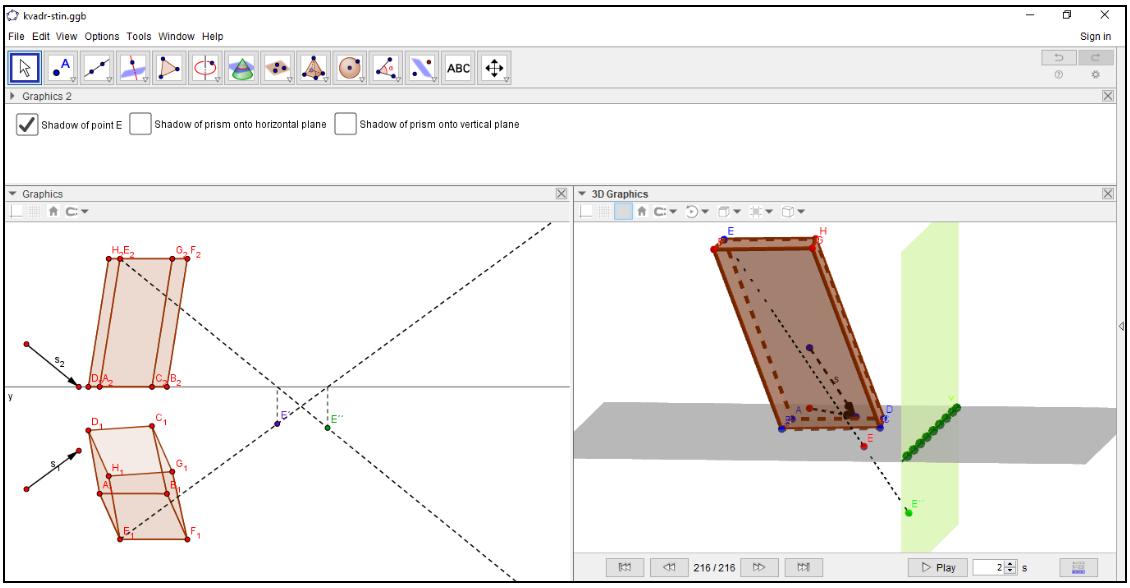

Fig. 3

The second check box “Shadow of the prism onto the horizontal plane” shows and hides the complete construction of the shadow in a 3D and a 2D window. The shadow of the prism on the horizontal plane is marked blue. We aim to construct shadows of all vertices and then to connect them. The lower base of the prism is on the horizontal plane, therefore we can create shadows of the vertices of the upper base only. However, we can also place vertices of the lower base to other points above and below on the horizontal plane.

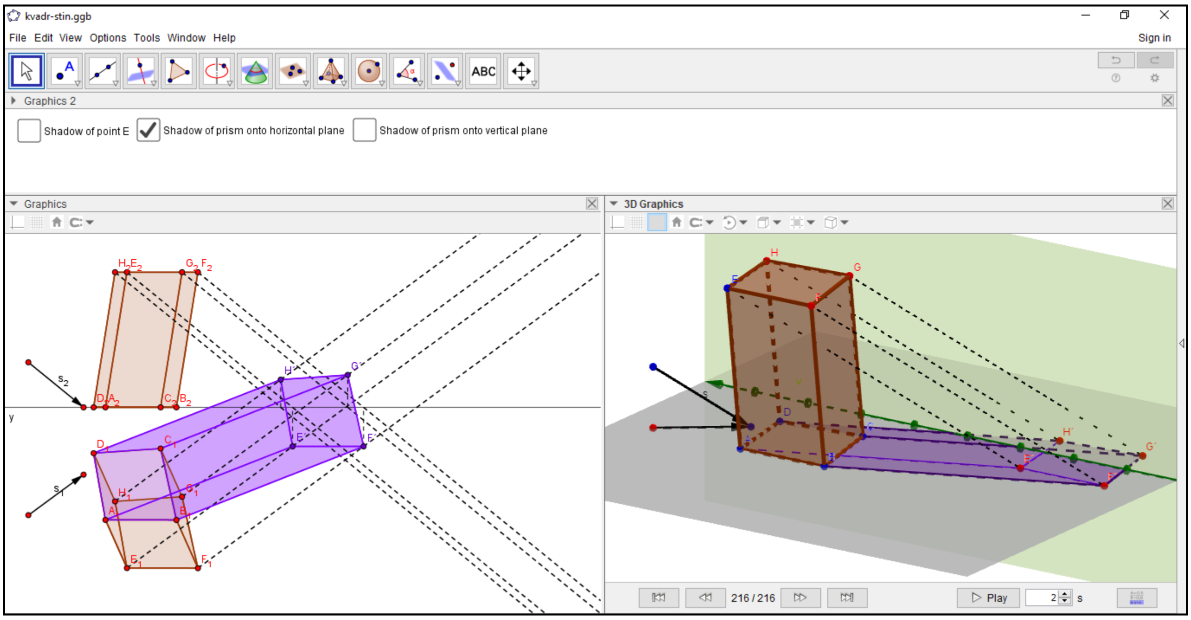

Fig. 4

The third check box displays the illumination of the prism on the vertical plane. The shadow is marked green, and we have to create the shadow of all vertices of the lower and upper base of the prism. In a 3D window the shadow in the vertical plane is displayed only above the folding line for clarity (see Fig. 5.).

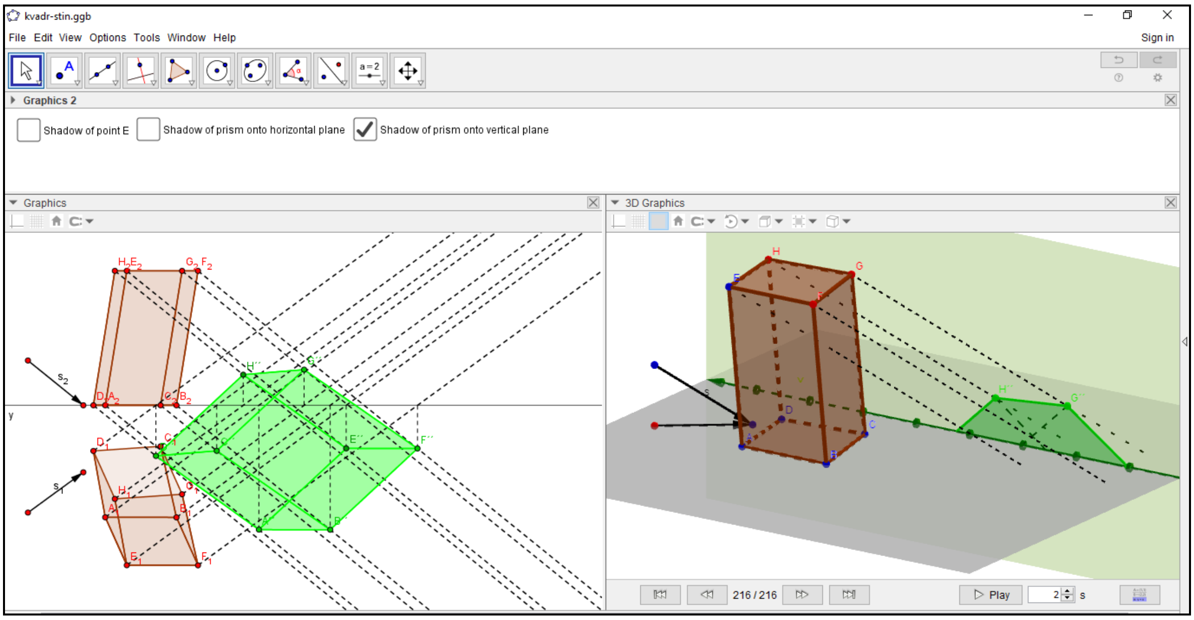

Fig. 5

If the second and third check boxes are displayed together, it is noticeable that the shadows on the vertical and horizontal plane intersect on the folding line (see Fig. 6).

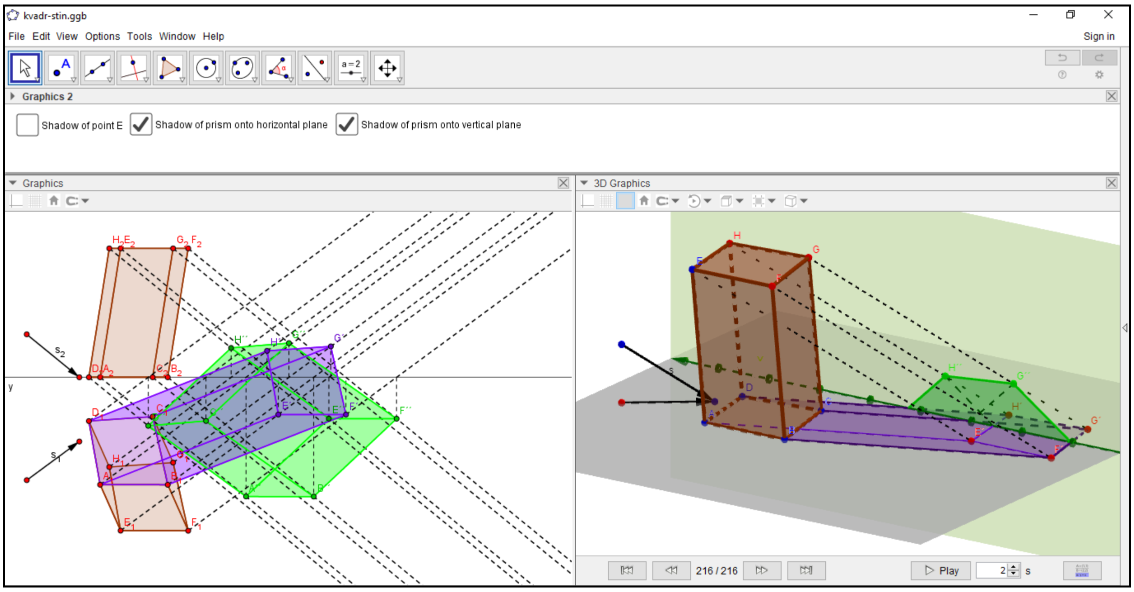

Fig. 6

2.2 The illumination of the Pyramid and the Line

The base of pyramid *ABCDV* is set on the horizontal plane. Vertices and the apex are moveable. Line *p* is given by two points that are also moveable. Direction of the illumination is also given by two points. The moveable points are marked blue and the dependent points are marked red (see Fig. 7.).

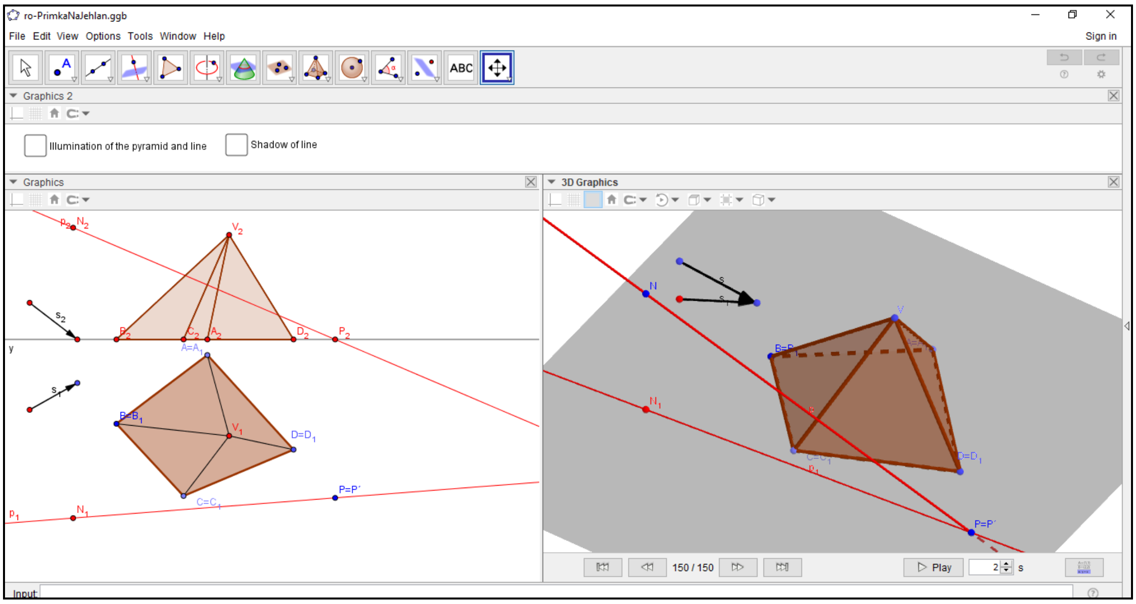

Fig. 7

The first check box displays the illumination of the pyramid and the line on the horizontal plane. The shadow on the vertical plane is not constructed for simplicity and clarity. The shadow of line *p* is line *p´* and the shadow of apex *V* is point *V´* (see Fig. 8.). Vertices *A, B, C, D* are on the horizontal plane, their shadows are the same points, however, moving of the vertices above or under the horizontal plane is possible. Subsequently the shadows of the vertices are to be found.

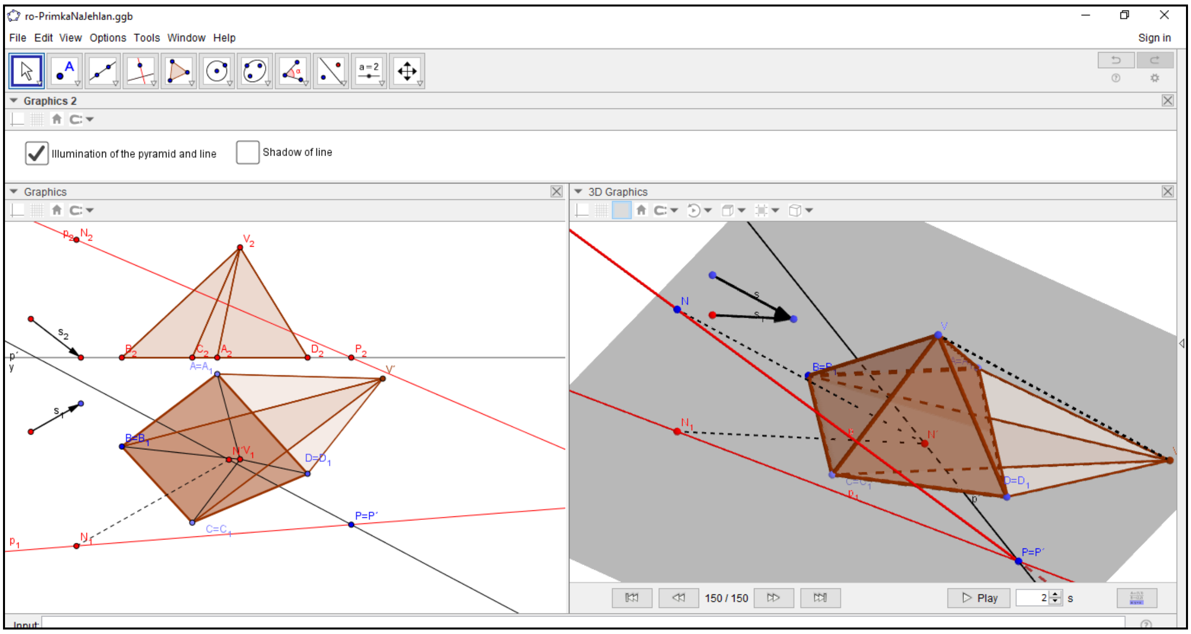

Fig. 8

Following the previous constructions, the shadow of line *p*, which is projected on the pyramid (polyline) is to be found. The shadow *p´* intersects the shadows of edges *AV´, BV´, CV´, DV´* in the points 1´, 2´, …, 4´ or on the base edges in the points 1*, 4* (depending on the position of the pyramid and the line). Points 1´, …, 4´ represent the shadows of points 1*, 2*, 3*, 4* displayed on the shadow line *p* projected on the pyramid (see Fig. 9.).

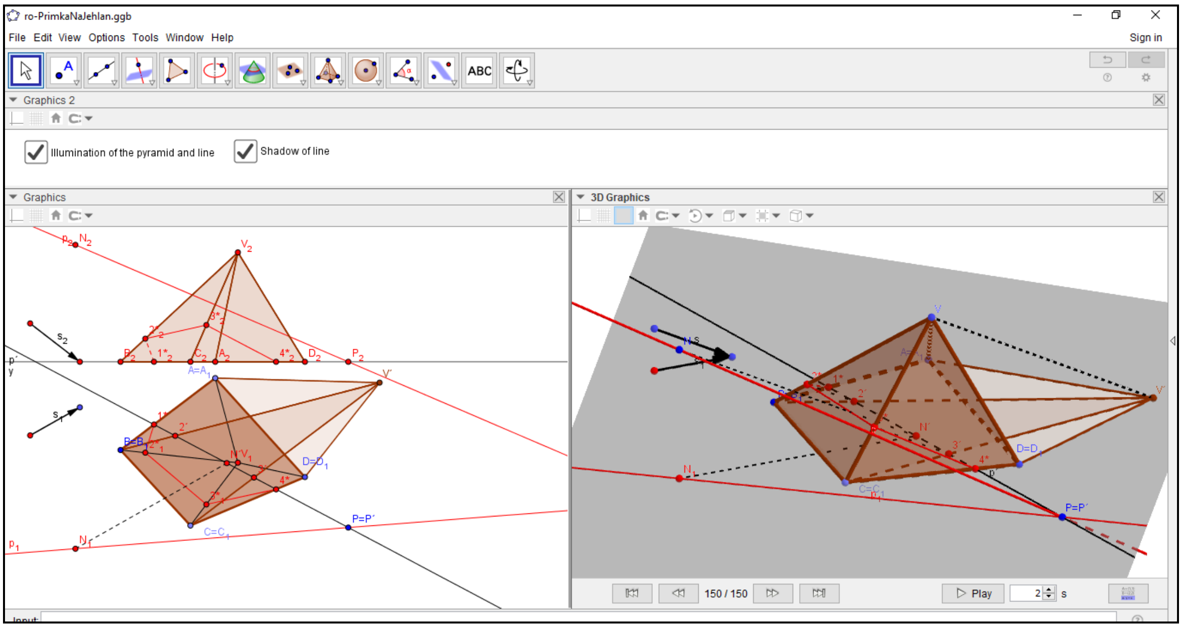

Fig. 9

When the points *P* or *N* of the line or the points on the direction of the illumination are shifted, the shadow of line *p* on the pyramid has also been changed. GeoGebra indicates all the possible positions of the shadow.

Line *p* can intersect the pyramid. The intersection points *X*, *Y* are marked green. The shadow of the line ends in the above mentioned points. Unfortunately, it cannot be displayed in GeoGebra (see Fig. 10.). Therefore the students need to be made aware of the deficiency.

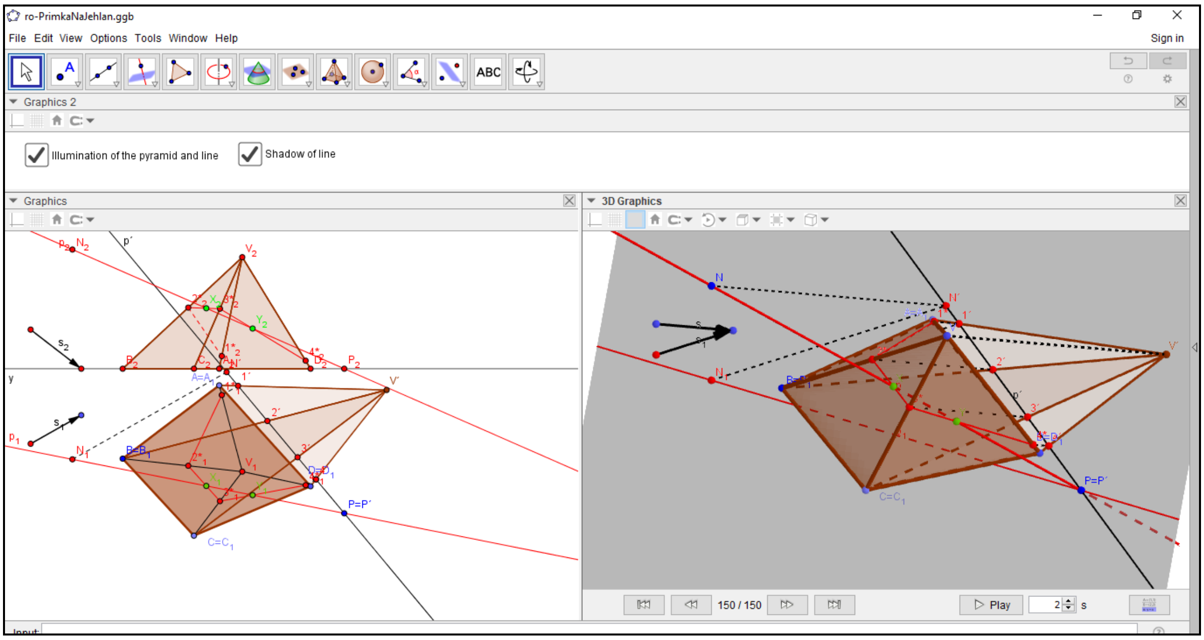

Fig. 10

2.3 The Illumination of the Hollow Cylinder

In this exercise the shadow of the hollow cylinder thrown inside is aimed to be found. Another task is to learn what curve the shadow creates.

The cylinder has the first base on the horizontal plane and the cylinder is perpendicular. An oblique cylinder cannot be created in GeoGebra. We choose the centre and the radius of the lower circle and the height of the cylinder. The radius and height can be changed. A tool slider can be used for this. The direction of the illumination is given by two moveable points (see Fig. 11.).

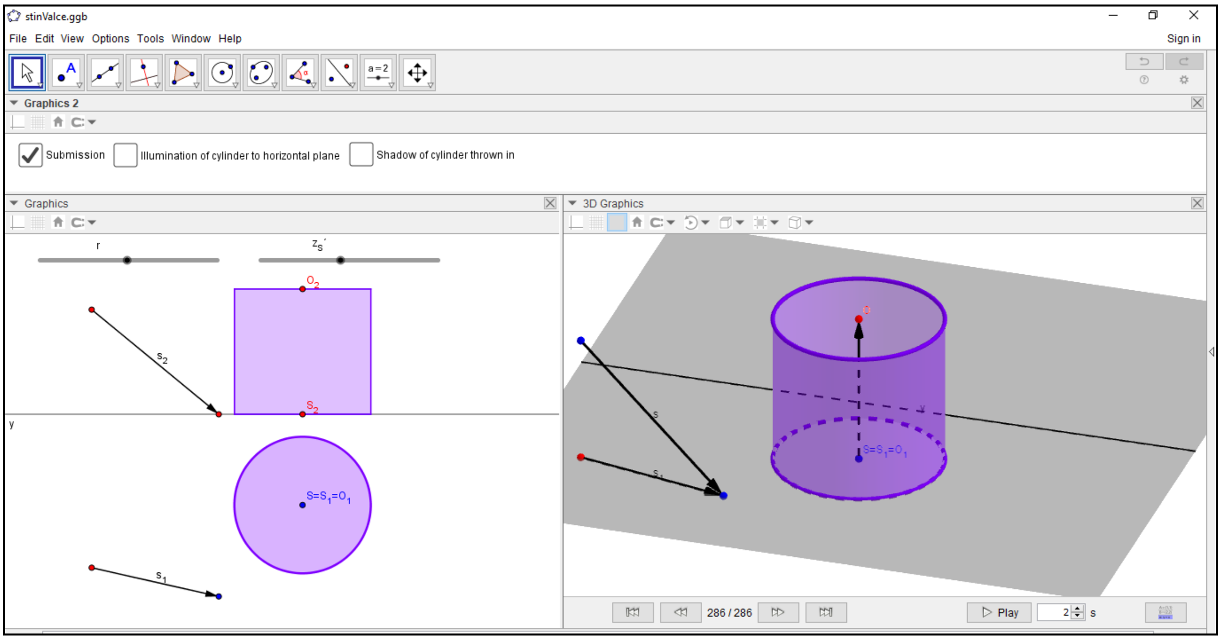

Fig. 11

The second checkbox displays the shadow of the cylinder on the horizontal plane. The shadow on the vertical plane is not constructed in this exercise. At first the shadow of the centre of the upper circle is to be found. Then the shadow of the whole circle is the identical circle because the plane of the circle is parallel with the horizontal plane (see Fig. 12.).

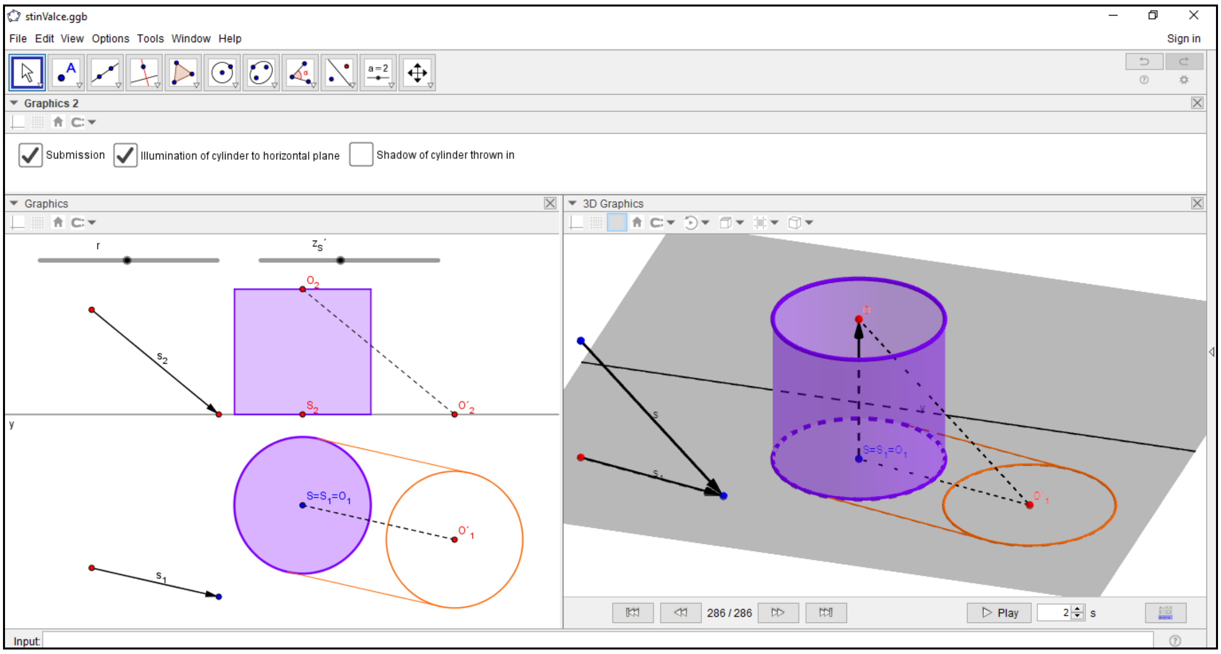

Fig. 12

The third check box displays the shadow of the upper circle thrown into the cylinder. We choose a few points 2´, 3´, …, 7´ on the inner circular arc. These points are shadows of the points in the part of the upper circle, which also create shadows 2*, …, 7* thrown in. Points 1´ and 7´ are the shadows of points 1, 7 of the circle in which the shadow thrown in begins and ends (see Fig. 13.). When the points 1, 2*, 3*, …, 7 are connected by the curve, the GeoGebra names this curve an ellipse.

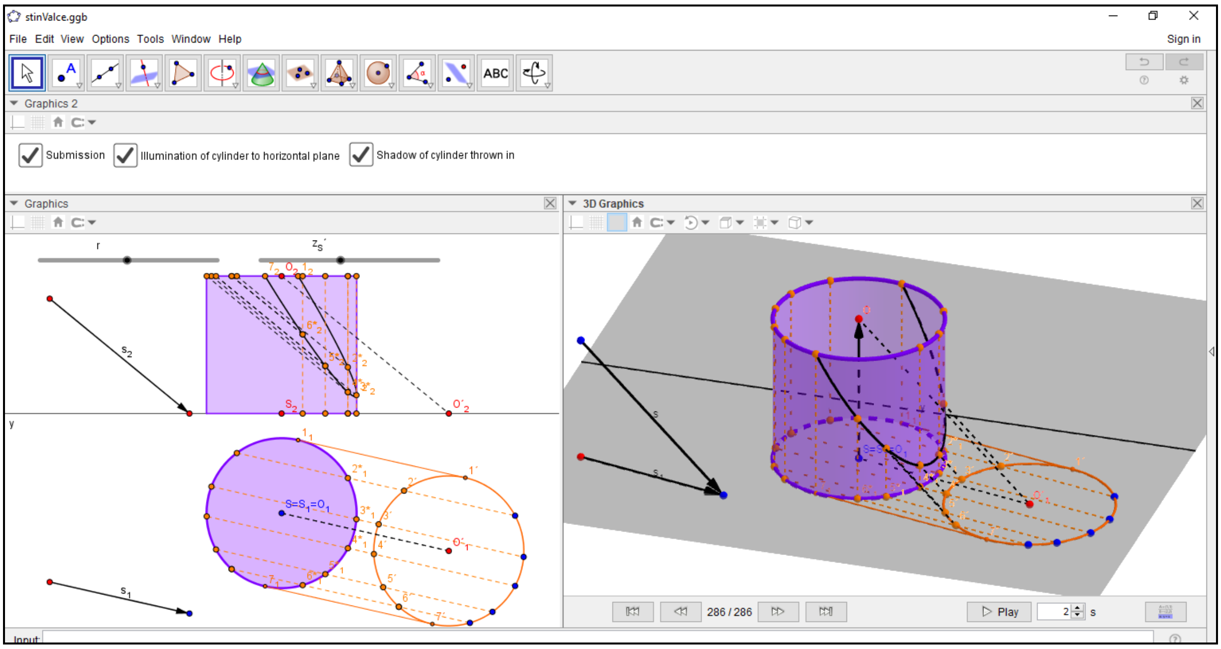

Fig. 13

If the direction of the illumination or the cylinder are moved, the shadow of the upper circle can intersect the lower circle. The intersection points are marked green (see Fig. 14.). Then the shadow thrown in intersects the lower circle in these points and the upper circle has the shadow on the lower circle (yellow on the figure). However, the ellipse cannot be ended in the green points in GeoGebra.

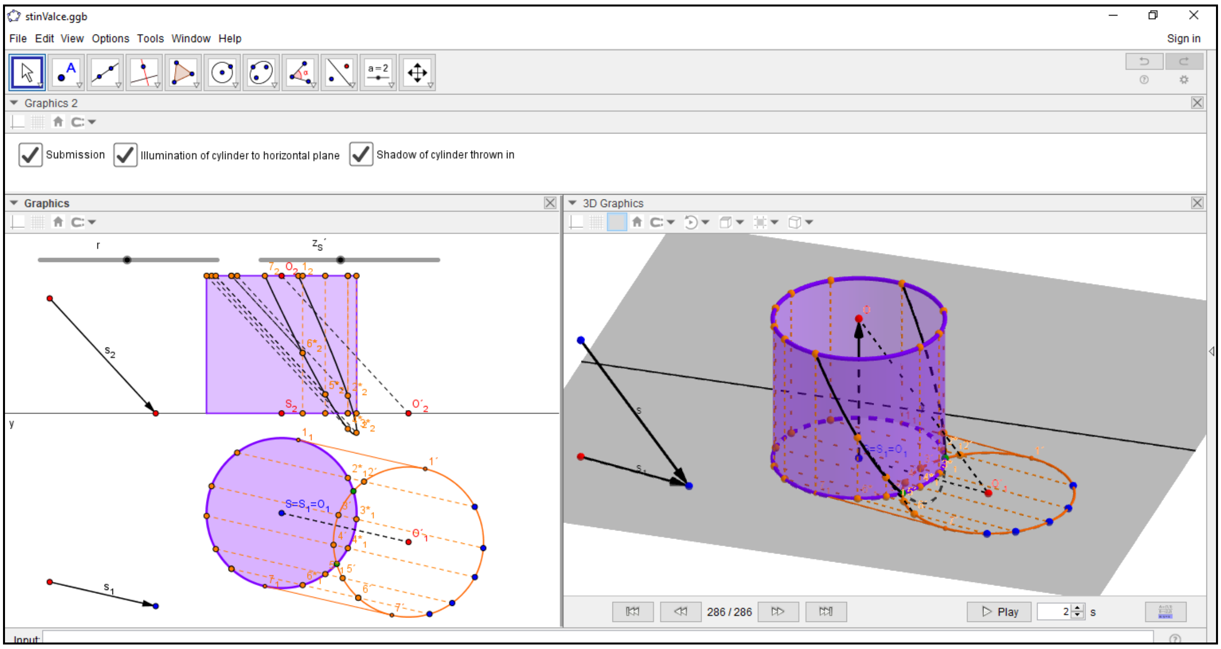

Fig. 14.

3 Conclusion

Teaching of descriptive geometry, not only parallel illumination using software GeoGebra is more visual. Students obtain a better idea of the spatial situation and the solution of the exercises. It is also possible to show different submissions and their solutions in one exercise.

References

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