

MODELING AND SYNTHESIS OF CONTROL THE PROCESS OF CASTING

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Abstract. This article is focused on modeling and design of thermal control of distributed parameter systems. Partial differential equations describe the dynamics of these systems. Currently, to solve these equations using numerical methods, concrete the finite elements method. The numerical model was created based on the finite elements method using software COMSOL Multiphysics, which is adequate for modeling the process of casting. This software was linked with MATLAB & Simulink environment, where control of temperature fields in casting mold was simulated. Linked software provides for foundry practice to include the possibility of controlling the process at the very beginning of the design and by the can reduce the cost of casting production.

Keywords: distributed parameter systems, finite element method, simulation, control, COMSOL Multiphysics, MATLAB & Simulink

Mathematics Subject Classification: Primary 60A05, 08A72; Secondary 28E10

1 Introduction

Most of the dynamical systems analyzed in engineering practice, including the casting process, are distributed parameter systems (DPS) defined on complex-shape 3D definition domains. In order to obtain the desired structure during solidification in casting, the process requires a specific temperature field of the mold. Modeling, simulation and evaluation of real-time experiments in this area is now widely accepted as an important tool in product design and process development to improve productivity and casting quality.

Techniques of finite elements method (FEM) based modelling and design of control synthesis methods of DPS is demonstrated on modelling and simulation of control of temperature fields in casting mold. For control synthesis purpose, numerical models in the form of a lumped-input/distributed-output system (LDS) were created by means of numerical dynamical – FEM solution. Analysis of formulated models, simulation of temperature fields, but also design of control synthesis was carried out using the Distributed Parameter Systems Blockset for MATLAB & Simulink - Third-party software product of The MathWorks, developed at the

2 Modeling systems with distributed parameters

In general, DPS are systems whose state or output variables, $X(x,y,z,t)/Y(x,y,z,t)$ are distributed variables or fields of variables, where (x,y,z) are spatial coordinates in 3D. These systems are often considered as systems whose dynamics is described by partial differential equations (PDE) as infinite-dimensional systems, [1]. [2]. However, from point of view of implementation of DPS control in engineering practice, where a finite number of sensors and actuators for practical sensing and control is at disposal, such infinite-dimensional systems need to be approximated by finite-dimensional systems. There are many dimension reduction methods, which can be used to solve this problem, e.g. continuous and approximation theories aimed to control of parabolic systems presents monograph [3].

2.1 LDS representation of distributed parameter systems

In the input-output relation, PDE define distributed-input/distributed-output systems (DDS) between distributed input, $U(x,y,z,t)$ and distributed output variables, $Y(x,y,z,t)$, at initial and boundary conditions given. Distributed parameter systems very frequently are found in various technical and non-technical branches with limited number of manipulated input variables, or actuators and may be interpreted as LDS, Fig. 1, [4]. These lumped input variables by means of interaction of fields and variables generate distributed output of real DPS.

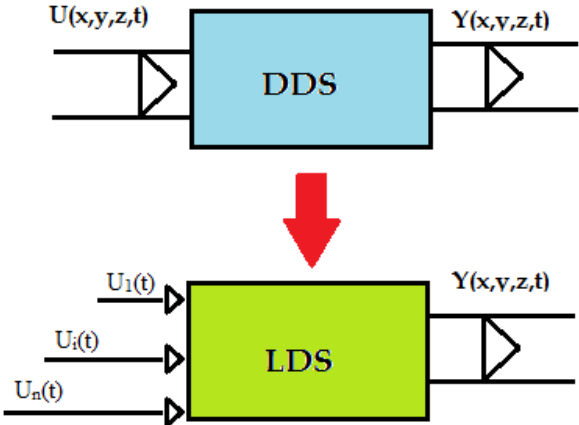


Fig. 1. Lumped-input/distributed-output system representation of distributed parameter systems.

2.2 Dynamics of lumped-input/distributed output systems

Output of the linear LDS in time domain, or in s-domain is in the form:

$$Y(\bar{x}, t) = \sum_{i=1}^n Y_i(\bar{x}, t) = \sum_{i=1}^n G_i(\bar{x}, t) \otimes U_i(t) \quad (1)$$

$$Y(\bar{x}, s) = \sum_{i=1}^n Y_i(\bar{x}, s) = \sum_{i=1}^n S_i(\bar{x}, s) \otimes U_i(s) \quad (2)$$

where $\bar{x} = (x, y, z)$ is position vector in 3D, $U_i(t)$ - lumped input quantity, $G_i(\bar{x}, t)$ - i-th distributed impulse response, \otimes - denotes convolution product, $Y_i(\bar{x}, t)$ - system response to the i-th input, $S_i(\bar{x}, s)$ - i-th transfer function. When $U_i(t)$ is a unit-step (Heaviside) function, s in the form of distributed transient response function $\mathcal{H}_i(\bar{x}, t)$. For a discrete-time system considering zero-order hold (ZOH) units, the overall distributed output variable of LDS and ZOH can be expressed in the form:

$$Y(\bar{x}, k) = \sum_{i=1}^n Y_i(\bar{x}, k) = \sum_{i=1}^n \mathcal{G}\mathcal{H}_i(\bar{x}, k) \oplus U_i(k) \quad (3)$$

where \oplus denotes convolution sum. For points $\bar{x}_i = (x_i, y_i, z_i)$ located in surroundings of lumped input variables, $U_i(t)$, where partial distributed transient responses $\mathcal{H}_i(\bar{x}_i, t)$ attain maximal amplitudes, partial distributed output variables are obtained in time-domain, or in z-domain respectively:

$$\{Y_i(\bar{x}_i, k) = \mathcal{G}\mathcal{H}_i(\bar{x}_i, k) \oplus U_i(k)\}_{i=1,n} \quad (4)$$

$$\{Y_i(\bar{x}, z) = \mathcal{S}\mathcal{H}_i(\bar{x}_i, z)U_i(z)\}_{i=1,n} \quad (5)$$

2.3 DPS feedback control loop based on LDS structure

Decomposition of dynamics enables also to decompose the control synthesis to time and space control tasks in the DPS feedback control loop, Fig. 2.

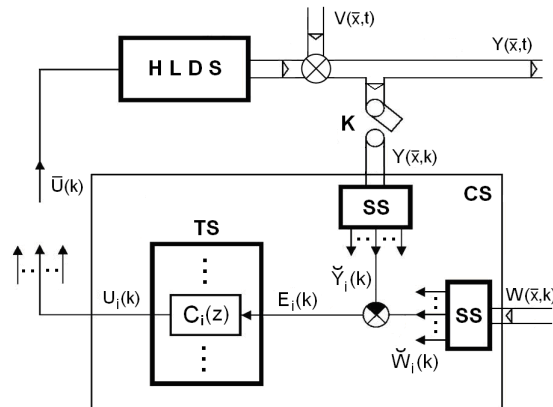


Fig. 2. Distributed parameter feedback control loop: HLDS - LDS with zero-order holds $\{\mathcal{H}_i\}_i$ on the input, CS - control synthesis, TS - control synthesis in time domain, SS - control synthesis in space domain, K - time/space sampling, $V(\bar{x}, t)$ - disturbance variable, $Y(\bar{x}, t)$ - distributed controlled variable, $Y(\bar{x}, k)$ - sampled distributed controlled variable, $\check{Y}_i(k)$ - approximation parameters of controlled variable, $W(\bar{x}, k)$ - reference variable, $\check{W}_i(k)$ -

approximation parameters of reference variable, $E_i(k)$ - control errors, $C_i(z)$ - controllers, $U_i(k)$ - lumped control variables

Let us consider a step change of distributed parameter control variable $W(\bar{x}, k) = W(\bar{x}, \infty)$ and $V(\bar{x}, t) = 0$. The goal of the control synthesis is to generate a sequence of control inputs $\bar{U}(k)$ in such manner, that in the steady-state, for $k \rightarrow \infty$, the control error $E(\bar{x}, k)$ will approach its minimal value $\|\check{E}(\bar{x}, \infty)\|$ in the quadratic norm:

$$\min \|E(\bar{x}, \infty)\| = \min \|W(\bar{x}, \infty) - Y(\bar{x}, \infty)\| = \|\check{E}(\bar{x}, \infty)\| \quad (6)$$

First, in the SS blocks, the approximation both of sampled distributed controlled variable $Y(\bar{x}, k)$ and reference variable $W(\bar{x}, \infty)$, on the set of reduced steady-state distributed step responses $\{\mathcal{A}HR_i(\bar{x}, \infty)\}_i$, are solved in following form:

$$\min_{\check{Y}_i} \left\| Y(\bar{x}, k) - \sum_{i=1}^n Y_i(\bar{x}_i, k) \mathcal{A}HR_i(\bar{x}, \infty) \right\| = \left\| Y(\bar{x}, k) - \sum_{i=1}^n \check{Y}_i(k) \mathcal{A}HR_i(\bar{x}, \infty) \right\| \quad (7)$$

$$\min_{\check{W}_i} \left\| W(\bar{x}, \infty) - \sum_{i=1}^n W_i(\bar{x}_i, \infty) \mathcal{A}HR_i(\bar{x}, \infty) \right\| = \left\| W(\bar{x}, \infty) - \sum_{i=1}^n \check{W}_i \mathcal{A}HR_i(\bar{x}, \infty) \right\|. \quad (8)$$

3 Numerical modeling of temperature fields of casting mold in COMSOL Multiphysics

For study of the physical phenomena occurring during casting solidification of aluminium-alloy in the permanent mould, from a DPS control point of view, control system development as well as mathematical model validation, a benchmark of the casting processes with a two-part steel mould of a complex-shape has been made, Fig 3. Inside of the mould are built-in 26 electric heating elements in the cylinder shape, each with maximal heating power 400 W. Heating elements are grouped to 5 zones and their heating power is actuated by the input voltage range of (0–10) V. In the body of the mould is also placed 7 water-cooled copper chills and 11 thermocouples, Fig. 4.



Fig. 3. Construction of casting die.

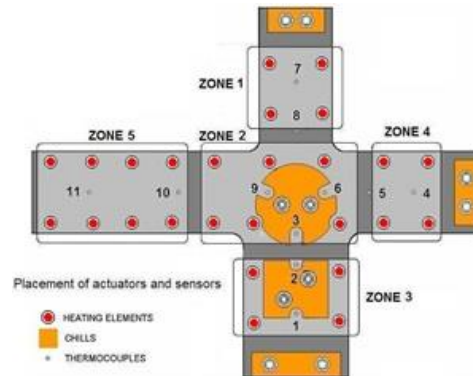


Fig. 4. Casting die with integrated action and measuring members.

The casting mould is one of the key components of a casting process. It is well known, that the quality of the castings is affected strongly by the surface quality and the distribution of temperature in the mould, which has both time and space dependence. Influence of fundamental input factors to the results properties of casting products is depicted in Fig. 5. For various control methods during of casting process is necessary also modelling of temperature fields in casting processes. There exist many methods of control according given requirements, as predictive control, robust control and others, [5], [6], [7], [8]. Numerical modeling of technological processes is also in the frame of concept Industry 4.0, [9].



Fig. 5. Schematic entry-exit models casting system.

CAD model of casting die was imported into COMSOL Multiphysics graphical environment to create finite element network that is avoidable for further numerical calculations. Were defined boundary conditions with environment temperature of 20 °C. The heating zones were defined to match the physical connection of the experimental facility. To solve the model contains several types of COMSOL Multiphysics solver, dealing with linear and nonlinear problems and a task in the frequency and time domain or with the tasks of the selected parameter. When solving systems of linear equations solver can be used directly as a UMPACK spools, GMRES iterative solver, solver with complex gradients, or with geometric multigrid. After finding performance ratio simulations were performed stepwise increase in performance in individual areas. Return to particular zones was made of the performance increase of 30%. The results of these simulations were nuts transient responses measuring 11x61. The number of lines is 11, because the measurement points have eleven. Number of pillars 61 tells us that the length of the simulation was 18,000 seconds with sampling plus 300 seconds is added to the first sample at the beginning of the simulation. For modelling of temperature fields as DPS based on FEM, wide possibilities are offered by COMSOL Multiphysics with heat transfer module, [10], [11]. Distribution of temperatures $T(\bar{x}, t)$ in the casting die over the definition domain $\Omega \in E_3$ for lumped input variables $\{U_i(\bar{x}, t)\}_{i=1,5}$ in the form of heat sources (W/m³) is modelled by PDE of parabolic type with boundary conditions (BC) and initial condition T_{init} for heat flux:

$$\begin{aligned} \frac{\partial T(\bar{x}, t)}{\partial t} - a \nabla^2 T(\bar{x}, t) &= \sum_{i=1}^5 U_i(\bar{x}, t) \\ -n(-\lambda \nabla T) &= h(T_{ext} - T) \\ T(\bar{x}, 0) &= T_{init} \end{aligned} \quad (9)$$

Parameters of PDE and BC are in Tab. 1

| Quantity | Value | Unit | Description |
|-----------|-------|-----------------------|---------------------------|
| λ | 76.2 | W/(m.K) | Thermal conductivity |
| ρ | 7870 | kg/m ³ | Density |
| C_p | 440 | J/(kg.K) | Heat capacity |
| h | 10 | W/(m ² .K) | Heat transfer coefficient |

Tab. 1 Parameters of PDE for Iron

In Fig. 6 we can see distributed transient responses in steady-state and steady temperature at 11 measuring points for actuating of the 5. zone, transient responses of systems after the power increase of the 5. zone are depicted in Fig.7.

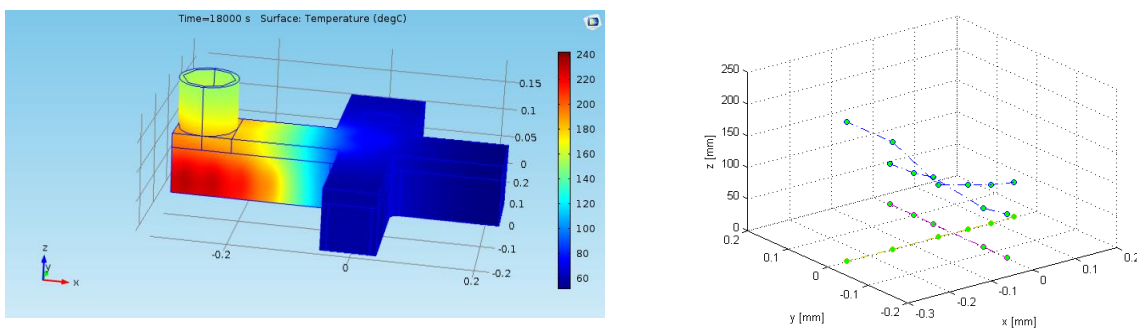


Fig. 6. Distributed transient responses in steady-state and steady temperature at 11 measuring points for actuating of the 5. zone.

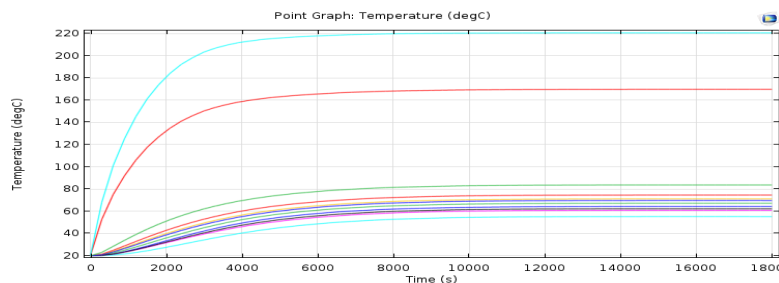


Fig. 7. Transient responses of system after the power increase of the 5. zone.

3.1 Identification of transient responses

After performing simulations in COMSOL Multiphysics data were downloaded using COMSOL LiveLink with MATLAB to the MATLAB, see Fig. 8. Subsequently, the step responses were identified using Identification Toolbox in MATLAB [12]. The role of the Identification Toolbox was to find the coefficients of the transfer system. System represents a step response obtained from simulations in COMSOL Multiphysics. Mathematical models in the form of PDE with given BC will be associated with distributed-input/distributed output system.

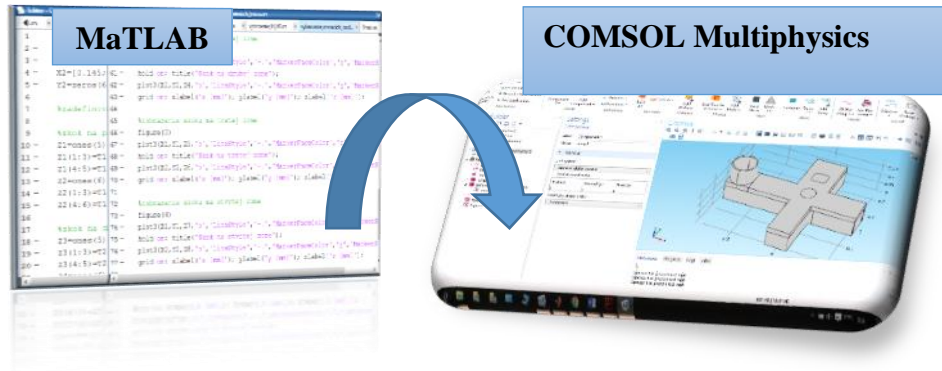


Fig. 8. Communication between software MATLAB and COMSOL Multiphysics.

Next, transient responses with maximal amplitude (e.g. for actuating in 5th zone it is position 11) were used for identification of transfer functions $\{S_i(\bar{x}_p, s)\}_{i=1,5}$. In Tab. 2 we see the setting for the estimates of our transmission. The following Fig. 9 it can be seen that the estimated parameters of the model is almost identical to our model. The conformity of those two models is 97.6%.

| Zone | 1 | 2 | 3 | 4 | 5 |
|---------------------------------|------------------------------|------------------------------|----------------------------|-----------------------------|------------------------------|
| Position of thermocouple | 7 | 4 | 1 | 9 | 11 |
| $\{S_i(\bar{x}_p, s)\}_{i=1,5}$ | $\frac{140,61}{1424,2s + 1}$ | $\frac{187,71}{1307,4s + 1}$ | $\frac{114,57}{1828s + 1}$ | $\frac{86,87}{2410,2s + 1}$ | $\frac{220,24}{1230,5s + 1}$ |

Tab. 2. Position of thermocouples and identified transfer functions.

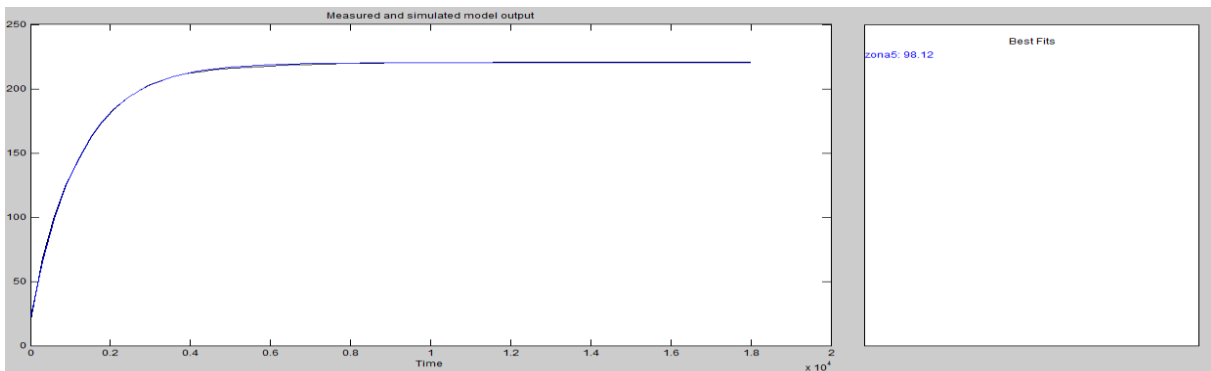


Fig. 9. Comparison of the original and the estimated model.

4 Temperature control of the casting mold

In MATLAB & Simulink we made synthesis of control with help of software packages DPS Blockset, [13], [14]. DPS Blockset is a special software product, which was developed at the Institute of automation, measurement and applied informatics In the Fig. 10 we can see scheme of control with using DPS Blockset in graphical environment MATLAB & Simulink.

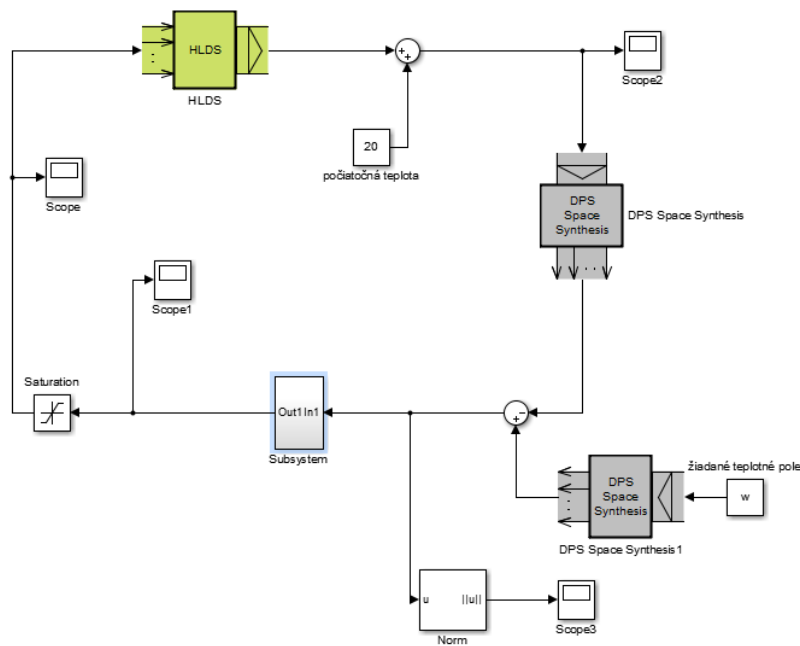


Fig. 10. Scheme of control loop with using DPS Blockset.

This product is used to control synthesis of distributed parameter systems. The next important block is DPS Space Synthesis in this block runs space synthesis, because we have 5 PI controllers in the time synthesis. Block of DPS Space synthesis adjusts the regulation aberrance to the required measure. The transmission parameters derived from the System Identification Toolbox were applied to a transfer function block. This will serve to controller parameter settings. For adjusting the controller parameters will serve us PID Controller block in which the possibility of tuning parameters automatically. This toolbox then gives the user the result of parameter estimation or it is tuned properly remains on the user. Placed to make the final effect of the regulator is correct, basic knowledge of automatic control. The following Fig. 11 see the progress resulting from temperature control to the reference variables required by us.

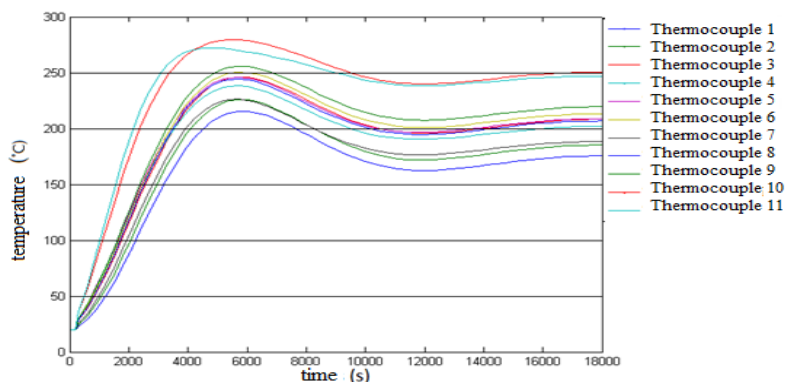


Fig. 11. Controlled variables at given position of the casting mold.

5 Conclusion

MATLAB & Simulink synthesis was carried out with the help of post control software tools DPS Blockset. Control of the scheme was carried out with the help of five PI controllers. Block PID Controllers will serve us to adjust the proportional and integral components of the various controllers. Reached a steady temperature limits and moving around the desired temperature. Linking the graphical environment COMSOL Multiphysics with MATLAB & Simulink package offers the possibility to control the processes already in their proposal. This option, which presents and offers foundry practice to achieve optimal control, which would increase production of castings that would meet our specifications.

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