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APPLICATION OF SHAPE DISTRIBUTIONS TO COMPARE TRIANGULAR MESHES

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Abstract. In computer graphics and its applications where shape-based recognition problem is solved, a shape distributions are used to calculate a similarity measure and discriminate among similar and dissimilar shapes. In this paper, we apply the shape distributions method to five triangular meshes obtained by optical scanning of the same object. The aim is to find relation between form deviations of individual meshes and similarity measures obtained by shape distributions approach.

Keywords: shape recognition, shape function, shape distribution, triangular mesh

Mathematics Subject Classification: Primary 68U05, 68U10; Secondary 62-07.

1 Introduction

Shape-matching or shape-based recognition problem is widely solved in computer graphics, computer vision, mechanical engineering, molecular biology, etc. Application of shape distributions represents an efficient approach to compute similarity measures of 3D shapes. The main idea is to represent a digital shape signature for a 3D model as a probability distribution sampled from a suitable shape function measuring geometric properties of the 3D model. This generalization of geometric histograms is so called shape distribution [1], [2]. This approach reduces the shape matching problem to the relatively simpler problem, i.e. comparison of two probability distributions instead of traditional shape matching methods, e.g. parametrization, feature correspondence and model fitting. The method for construction of shape distributions from 3D polygonal models to compute a measure of their dissimilarities consists in the following steps: shape function selection, random points sampling, shape distributions calculation and shape distributions comparison. Shape function is usually based on simple measurements of geometrical features, e.g. angles, distances, areas, volumes, [1] to [4]. Shape distributions calculated for a sufficiently large number of random sample points of the surface are compared using well-known curve matching techniques, e.g. Minkowski L_N norm.

In this paper, the shape distributions method is used to determine similarity of five triangular meshes in stl format obtained by optical scanning of ball-bar standard with two precise spheres connected by a cylinder (Fig. 1). This standard is used for calibration of optical scanners when the diameters of the two spheres (obtained by least squares fitting to the corresponding parts of the triangular mesh) and the distance of their centres are measured. The goal of this paper is to find the correspondence between the form deviations of individual meshes and similarity measures obtained by the shape distributions approach.

Fig. 1. Ball-bar standard.

2 Triangular meshes pre-processing

The standard was scanned by portable handheld optical scanner separately five times. Thus, five different triangular meshes, each of them in general position with respect to the coordinate system, were obtained. The meshes contain different parts of scanned handles which have to be removed before evaluation because they can distort the results, see example in Fig. 2.

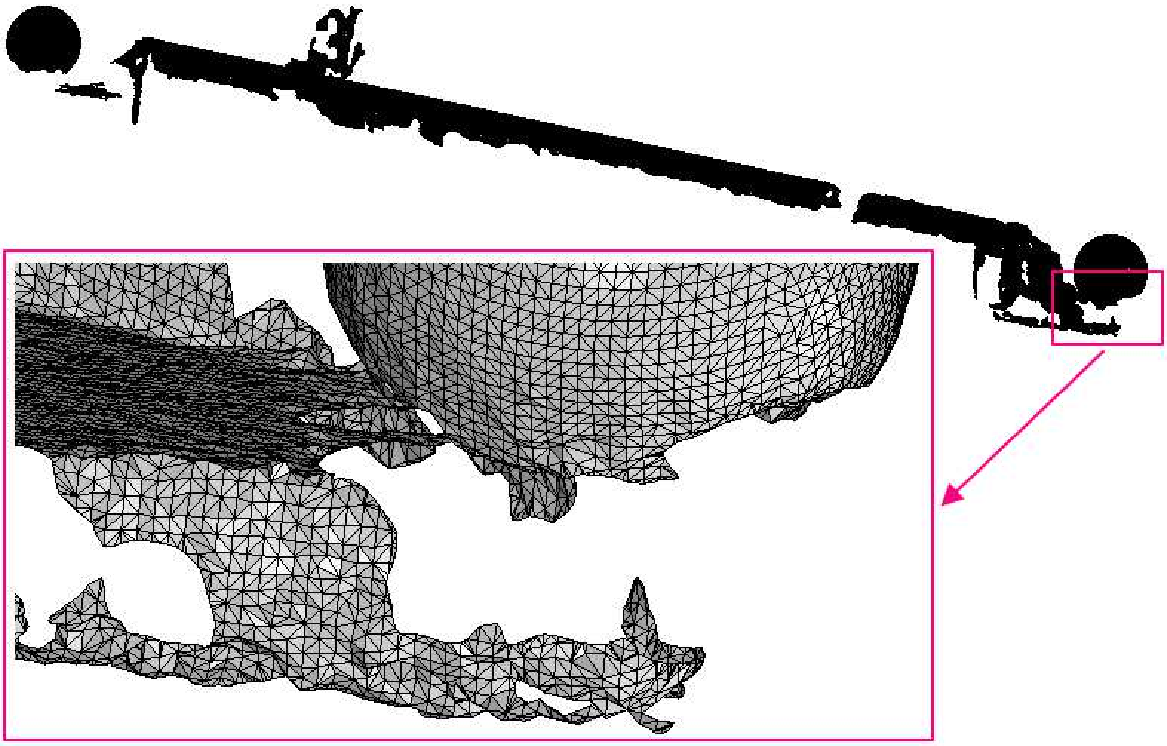

Fig. 2. Ball-bar standard and a detail of triangular mesh.

To remove the same part corresponding to the handles, the meshes were aligned with respect to the coordinate system in the following way. The least square method was used to associate two spheres to the corresponding parts of the triangular mesh. Denoting S_1 and S_2 the centres of the spheres, the centre of straight line segment S_1S_2 lies at origin O and $S_1S_2 \subset x$. Now, it is possible to choose suitable position of section planes and trim inappropriate parts of meshes, see Fig. 3. Vertices of meshes depicted in Fig. 3 represent the input data for further processing.

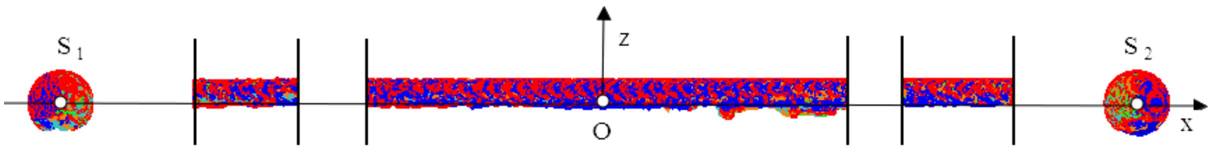

Fig. 3. Five triangular meshes aligned with respect to the coordinate system.

In addition to the five scanned triangular meshes, a triangular mesh of the nominal CAD model, see Fig. 4, of the standard was generated (nominal mesh). The position of the nominal CAD model with respect to the coordinate system is identical to the position of meshes in Fig. 3.

3 Shape distributions

In this section, a detailed description of the method used for construction of shape distribution based on triangular mesh is given.

3.1 Shape function

The first step of shape distribution construction is to choose a shape function. Here, the modification of $D1$ shape function described in [1] is used. Originally, the shape function $D1$ measures the distance between a fixed point (centroid of the model boundary) and one random point on the surface. In this paper, the random sampling is represented by mesh vertices. Then, the shape function is a function measuring the oriented distance of mesh vertex from origin O. As the sampling density strongly influences the precision of the shape distribution, the distances of all mesh vertices $V_i = (x_i, y_i, z_i)$ from origin, i.e.

$$f_i = \text{sign}(x_i)\sqrt{x_i^2 + y_i^2 + z_i^2}, i = 0,1,\cdots,n, \quad (1)$$

are taken into consideration.

3.2 Shape distribution construction

The second step is a frequency histogram calculation, i.e. determination how many values f_i fall into each of k fixed sized classes. The frequency is normalized by number of mesh vertices n to eliminate the influence of different number of meshes vertices. An example of histogram for nominal mesh is shown in Fig. 4. Due to the mutual position of the nominal CAD model and the histogram for nominal mesh depicted in Fig. 4, it is obvious that the oriented distance reflects the shape characteristic of the mesh very well.

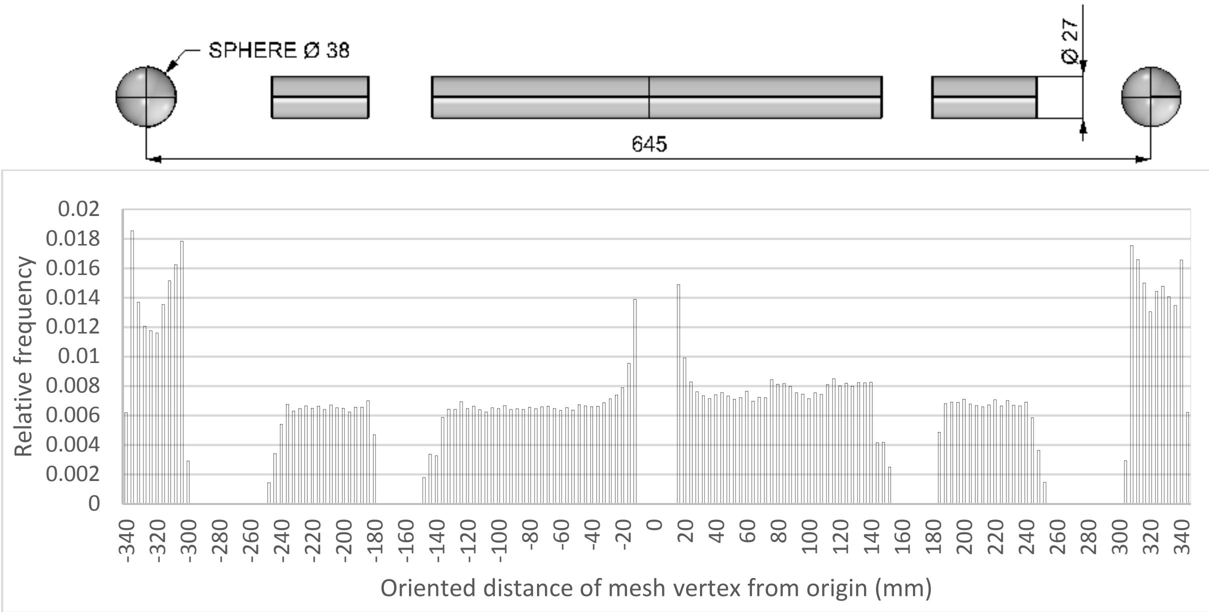

Fig. 4. Nominal CAD model and relative frequency histogram of nominal mesh.

A piecewise linear function constructed from the relative frequency histogram forms the representation for the shape distribution, see charts in Fig. 5.

Additionally, it is possible to express shape distribution by a cumulative frequency distribution, i.e. number of values f_i which are less than or equal to a reference value, see charts in Fig. 6.

3.3 Shape distribution comparison

In case the shape distribution is represented by the relative frequency histogram, the similarity measurement is based on Minkowski L_1 norm

$$D(f,g) = \sum_{i=1}^{k} |f_i - g_i|, \tag{2}$$

where f_i and g_i calculated according to (1) represent the relative frequency of two triangular meshes. In case the shape distribution is represented by the cumulative frequency distribution, the Minkowski L_1 norm is given by

$$D(\hat{f},\hat{g}) = \sum_{i=1}^{k} |\hat{f}_i - \hat{g}_i|, \tag{3}$$

where $\hat{f}_i$ and $\hat{g}_i$ are cumulative frequency of two triangular meshes

4 Experimental results

The shape distributions in form of relative frequency histograms calculated for all triangular meshes $M1, M2, \cdots, M5$ and the Nominal mesh are depicted in Fig. 5. The resemblance of characteristics is obvious. The resulting shape distributions for all triangular meshes are plotted in Fig. 6. The more the curves overlap the greater their similarity.

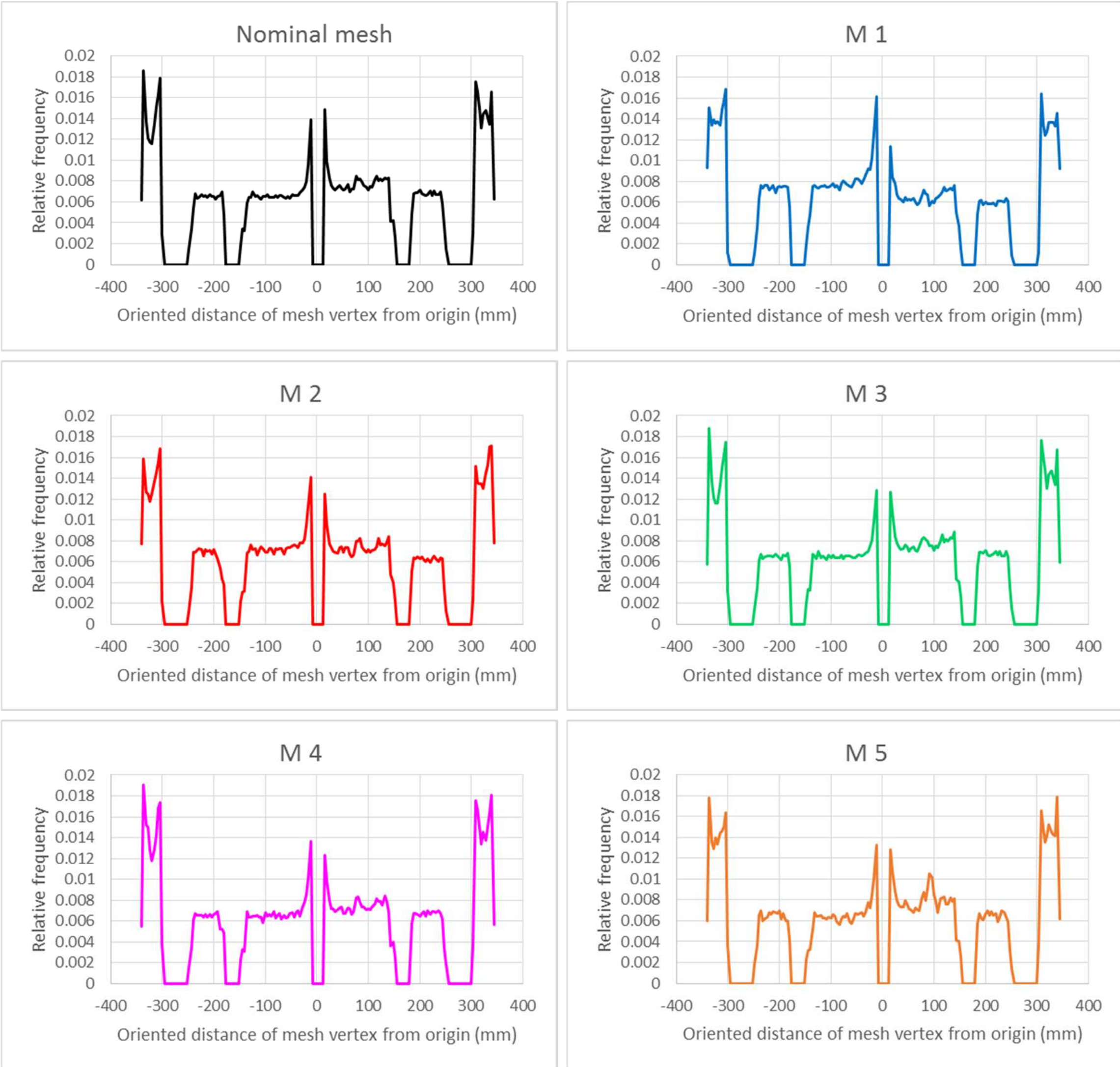

Fig. 5. Shape distributions (relative frequency histogram).

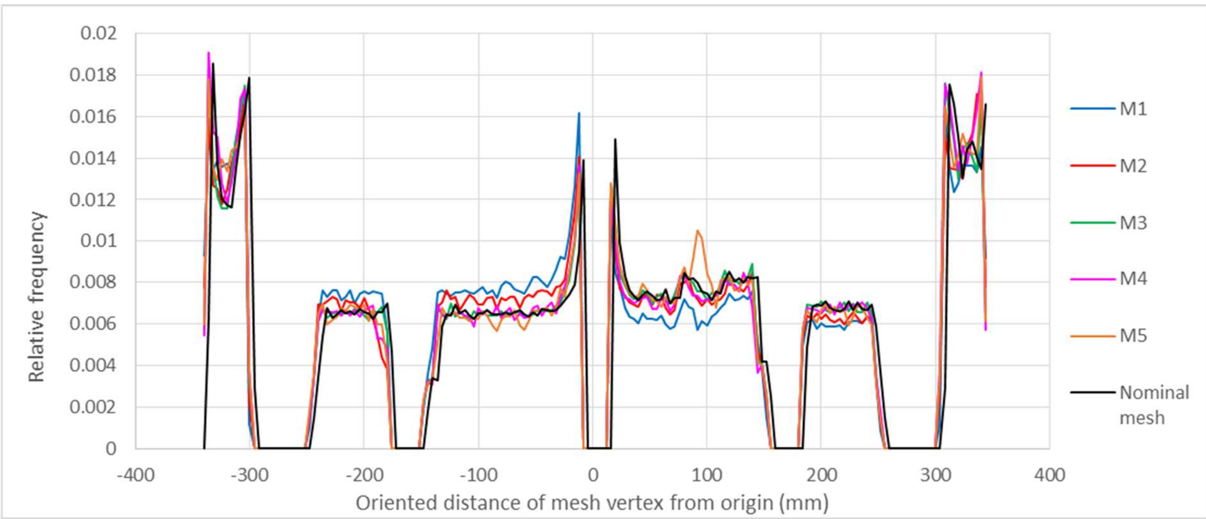

Fig. 6. Shape distributions (relative frequency histogram) of 6 triangular meshes.

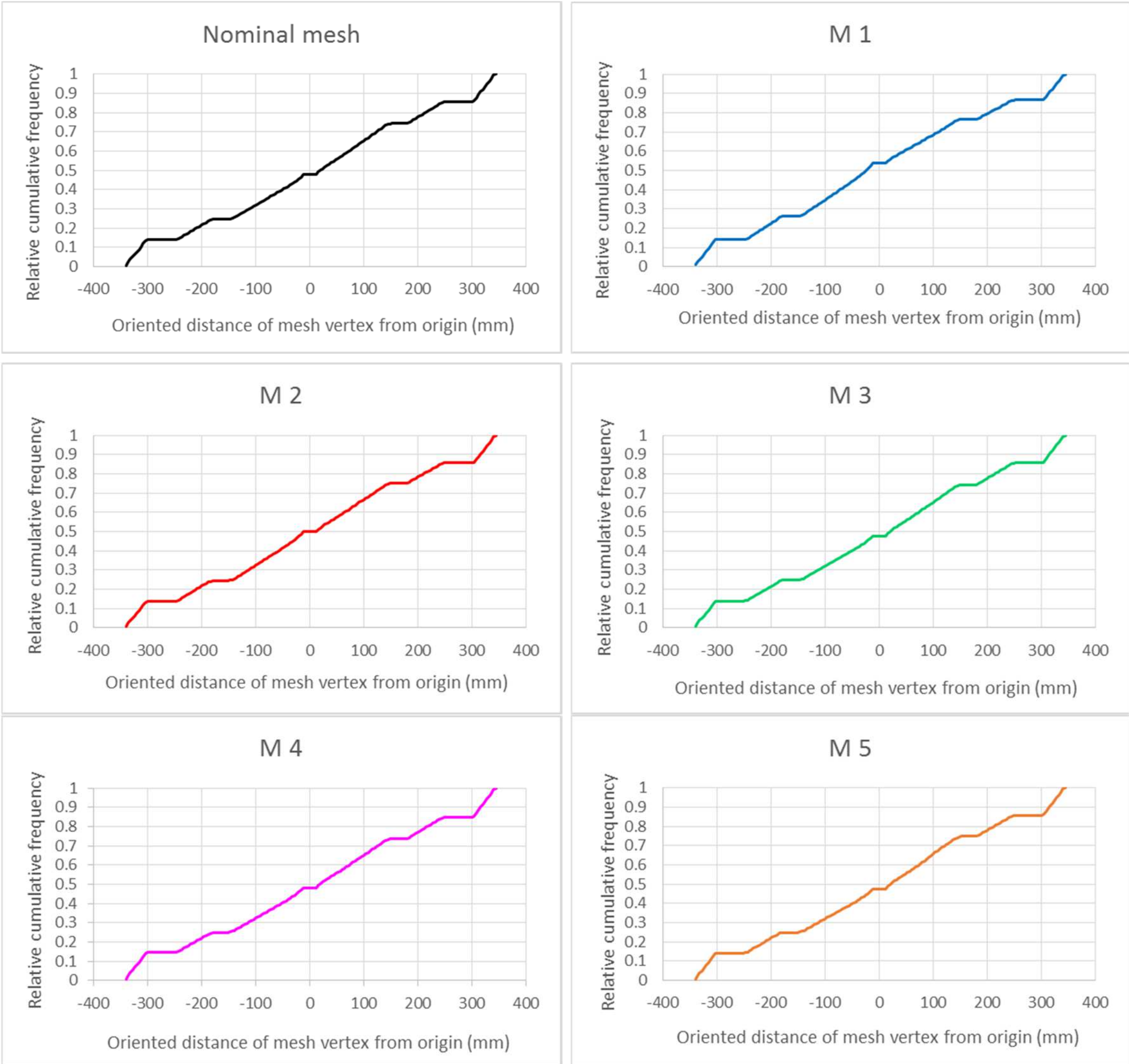

Fig. 7. Shape distributions (relative cumulative frequency).

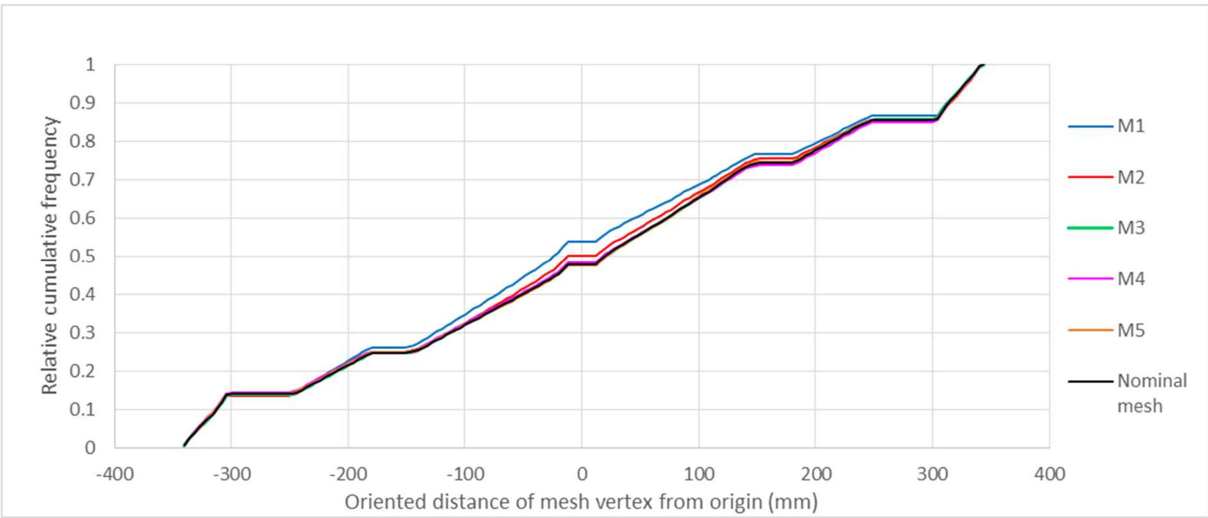

Fig. 8. Shape distributions (relative cumulative frequency) of 6 triangular meshes.

The shape distributions in form of relative cumulative frequency calculated for all triangular meshes $M1, M2, \cdots, M5$ and the Nominal mesh are depicted in Fig. 7. The resulting shape distributions for all triangular meshes are plotted in Fig. 8. Again, the resemblance of the tested meshes is obvious.

The values of Minkowski L_1 norm given by (2) and (3) are listed in Tab. 1 and Tab. 2, respectively. Both tables represent a symmetric similarity matrix with zeros on the main diagonal. The minimal nonzero value ($D_{min}(f,g)$ =0.05 between $M3$ and $M4$ and $D_{min}(\hat{f},\hat{g}) = 0.416$ between $M3$ and $M5$) indicates the best similarity between the two corresponding meshes. The maximal value ($D_{max}(f,g)$ =0.153 between $M1$ and $M5$ and $D_{max}(\hat{f},\hat{g}) = 3.834$ between $M1$ and $M4$) indicates the worst similarity of meshes.

To compare the obtained similarity measure L_1 with real dimensions, the form deviation $d_1, d_2, \cdots, d_5$ (the normal distance between the farthest mesh vertex and nominal CAD model) for each mesh $M1, M2, \cdots, M5$ was evaluated. After that, the mutual deviations between each two meshes were calculated by

$$d_{i,j} = \left|d_i - d_j\right|, \ i,j = 1,2,\cdots,5, \tag{4}$$

see Tab. 2. The minimum nonzero value $d_{3,5} = 0.062$ mm represents the best shape similarity between $M3$ and $M5$, the maximum value $d_{1,4} = 0.582$ mm represents the worst shape similarity not only between $M1$ and $M4$ but also among all triangular meshes $M1, M2, \cdots, M5$. Moreover, when comparing the values of all three similarity measures $d_{i,j}$, $D(f,g)$ and $D(\hat{f},\hat{g})$, it is obvious that $D(\hat{f},\hat{g})$ and $d_{i,j}$ indicate the same results, i.e. the best shape similarity between $M3$ and $M5$ and the worst shape similarity between $M1$ and $M4$.

L_1	M1	M2	M3	M4	M5
M1	0.000	0.093	0.146	0.147	0.153
M2	0.093	0.000	0.087	0.083	0.093
M3	0.146	0.087	0.000	0.050	0.059
M4	0.147	0.083	0.050	0.000	0.069
M5	0.153	0.093	0.059	0.069	0.000

(a) $D(f,g)$

L_1	M1	M2	M3	M4	M5
M1	0.000	2.798	3.823	3.834	3.701
M2	2.798	0.000	1.268	1.587	1.329
M3	3.823	1.268	0.000	0.753	0.416
M4	3.834	1.587	0.753	0.000	0.819
M5	3.701	1.329	0.416	0.819	0.000

(b) $D(\hat{f},\hat{g})$

Tab. 1. L_1 Similarity matrices for triangular meshes $M1, M2, \cdots, M5$.

$d_{i,j}$	M1	M2	M3	M4	M5
M1	0.000	0.112	0.253	0.582	0.315
M2	0.112	0.000	0.141	0.470	0.204
M3	0.253	0.141	0.000	0.329	0.062
M4	0.582	0.470	0.329	0.000	0.267
M5	0.315	0.204	0.062	0.267	0.000

Tab 2. Mutual form deviation similarity matrix for triangular meshes $M1, M2, \cdots, M5$.

5 Conclusion

In this paper, a shape similarity of five triangular meshes obtained by portable handheld optical scanner was investigated. The method was based on application of shape distributions commonly used in computer vision for recognition of shape of 3D models. The shape function measuring the oriented distance of each mesh vertex from suitable chosen reference point was defined, its function values were evaluated. The shape disributions in the form of relative frequency histogram and relative cumulative frequency were constructed. Three different similarity measures were evaluated – the Minkowski L_1 norm for the relative frequency distribution and for the relative cumulative frequency distribution as well as the difference of form deviations between each two tested meshes. The obtained results were compared and the relation between L_1 norm and real form deviation of scanned meshes was determined.

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