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SQUARE AND TRIANGULAR MATRICES IN MAGNAGHI-TERZAGHI'S ARCHITECTURES

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Abstract. The encounter, in the past editions of Aplimat Conference, with Paola Magnaghi Delfino, daughter of Augusto Magnaghi who, together with his friend and fellow student Mario, founded the Studio Magnaghi-Terzaghi, was an opportunity to start a systematic research on the drawings and projects carried out by these two architects, collaborators of Pietro Lingeri and Giuseppe Terragni, that were two of the most important exponents of Modern Architecture in Milan. The modern architecture created by the two architects is strongly mathematical in its conception.

Keywords: modern architecture, geometry, geometrical layout, square, equilateral triangle, architectural shape

Mathematics Subject Classification: Primary 00A05, 51F05, 97G99; Secondary 97F90

1 Introduction

The encounter, in the past editions of Aplimat Conference, with Paola Magnaghi Delfino, daughter of Augusto Magnaghi who, together with his friend and fellow student Mario, founded the Studio Magnaghi-Terzaghi, was an opportunity to start a systematic research on the drawings and projects carried out by these two architects, collaborators of Pietro Lingeri and Giuseppe Terragni, that were two of the most important exponents of Modern Architecture in Milan.
What at first came to light, in addition to the quality in itself of their architectural works, is a systematic use of the geometry as a basic layout on which they structured their projects. As a matter of facts, architectural structures based on a geometric form have ancient roots, adopted with continuity throughout history up to modern times.
The rational composition of the geometric elements comes from the evolution of the Vitruvian precepts in which we read the thesis endorsed also by Le Corbusier for which the beauty of a building consists in the rational integration of all the parts, such that nothing can be added or eliminated without destroying the harmony of the whole.
If, however, in the past, the composition of codified orders and ornaments was defined by interpreting geometrical objects as linguistic elements, Le Corbusier’s system from which a

large part of the so-called rational architecture is born, sets the limits of a compositional process regulating the possibility to order and precisely proportion the project as indoor and outdoor spaces as a whole. Consequently, in the architectures we are analyzing geometry more than a language, is a method.

2 Square Matrices

In the year of their graduation (1939), the architects Magnaghi and Terzaghi received the assignment from Mr Cicogna to design a residential building from then on known as the Casa dei Nidi [2] (House of the nests).

Various History of Architecture books quote this building as one of the first examples of Italian Modern Architecture, yet the writings have always returned to a single article written by Giuseppe Pagano in 1940 and published in Casabella n. 150 [4] (Fig. 1).

The drawings published in this article have been the starting point for an accurate redrawing, followed by a comparison with the actual survey. The obtained model was thereafter analyzed with the aim to identify the geometric matrices on which the architects based the project of the building.

The 80 cm-sided square is the module that the architects used to design this architecture. The plan is incorporated into a double square made by 11,5 x 22 modules and the elevation is 22 x 16 modules. The 0.80 m module has an obvious convenience for the control of the façade because the distance from the storey of the floor below to the floor above is 3.20 m, equal to 4 modules.

The control of the project is given both by the tessellation of the plan with squares and of the space with cubes of side equal to 0.80 m. The general measures of the plan are 9.20 x 17.60 = 161.92 square meters (11.5 x 22 = 253 modules). The total volume of the building, all inclusive (calculated empty to full) is 161.92 x 12.80 = 2072.576 cubic meters (253 x 16 = 4048 modules). The gross surface area of 6 apartments from the first to the third floor is 66,24 m (9 x 11.5 = 103.5 modules). A semi-open staircase connecting the different levels of the building is placed in a space of 6.5 x 4 = 26 modules equal to 3.20 x 5.20 = 16.64 square meters. The apartments consist of two rooms and a kitchen. The rationality of the scheme is the essential tool to control the project in all its parts.

The “ribbons” of load-bearing masonry on which the slabs lie are ½ module wide (40 cm) and allow to have apartments free from load-bearing structures visible in the interiors, as well as the possibility of having proportioned openings measured according to a coherent scheme that does not betray the quality of the design philosophy.

The openings on the main façade are equal to 3 modules (2 + 1) with sliding shutters fitting exactly in the filled part of the wall. The ground floor originally had two single-span porches on the front side of building, in plan composed of 9 x 6.5 modules (7.20 x 5.20 = 37.44 square meters), which almost immediately have been closed to obtain two additional housing units. In the lower area of the back side of the building, 6 garages were placed, one for each original housing unit.

As Pagano wrote «from this healthy realism, this very first construction by Magnaghi and Terzaghi was born, and it is thanks to their faith and their intelligence if this house is not an "ordinary house"» [4]. The simple volumetric value of this composition guarantees quality and cost effectiveness for the intervention. Starting from the square, vertical and horizontal rectangles are obtained. In this way the architects managed a sequence of light-dark, full and empty relationships that make this building one of the examples of Italian rationalism. The square grid is the tool used by the two Milanese architects to generate an innovative architecture that is perfectly in line with their times or even, for the period in which it was built, a cutting-edge project. In fact, the aesthetic rhythm of this architecture is based on a geometric grid that generates pure forms. This grid is referred to the classic *ad quadratum* background, so deep-rooted in the Italian tradition to constitute the basis of the new Italian Rationalist Architecture as theorized by the "Gruppo 7".

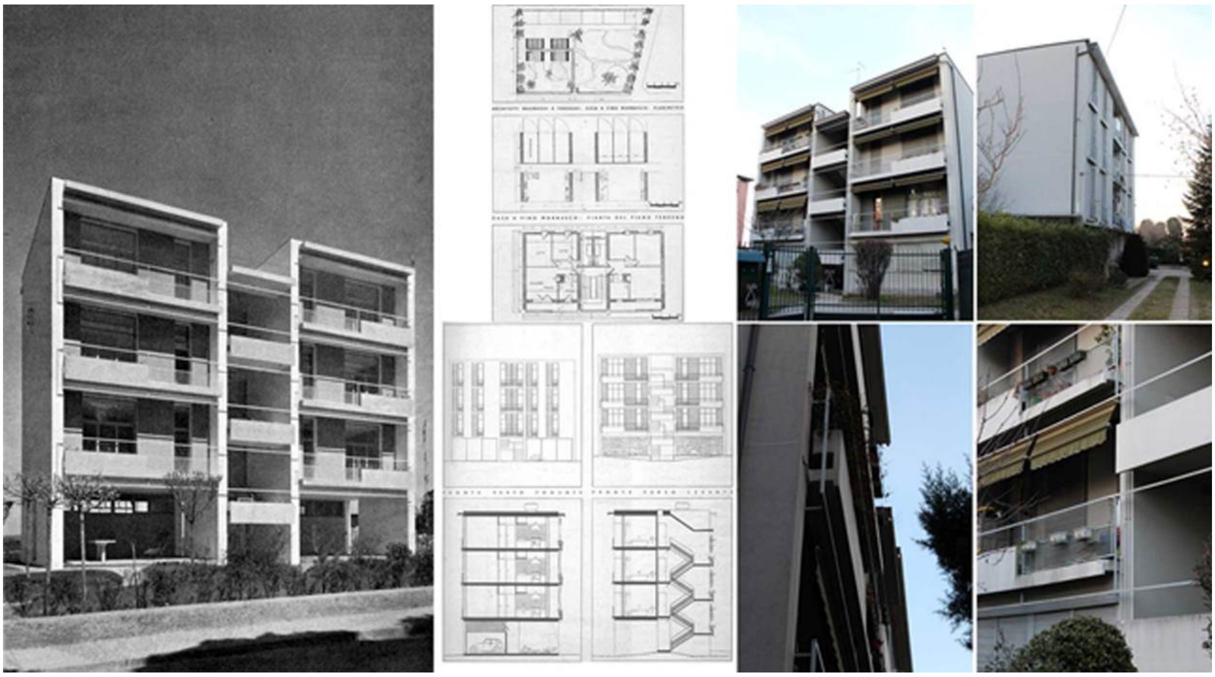

Fig. 1. Plans, elevations, sections [4], and photos of Casa dei Nidi at Fino Mornasco (CO) Italy.

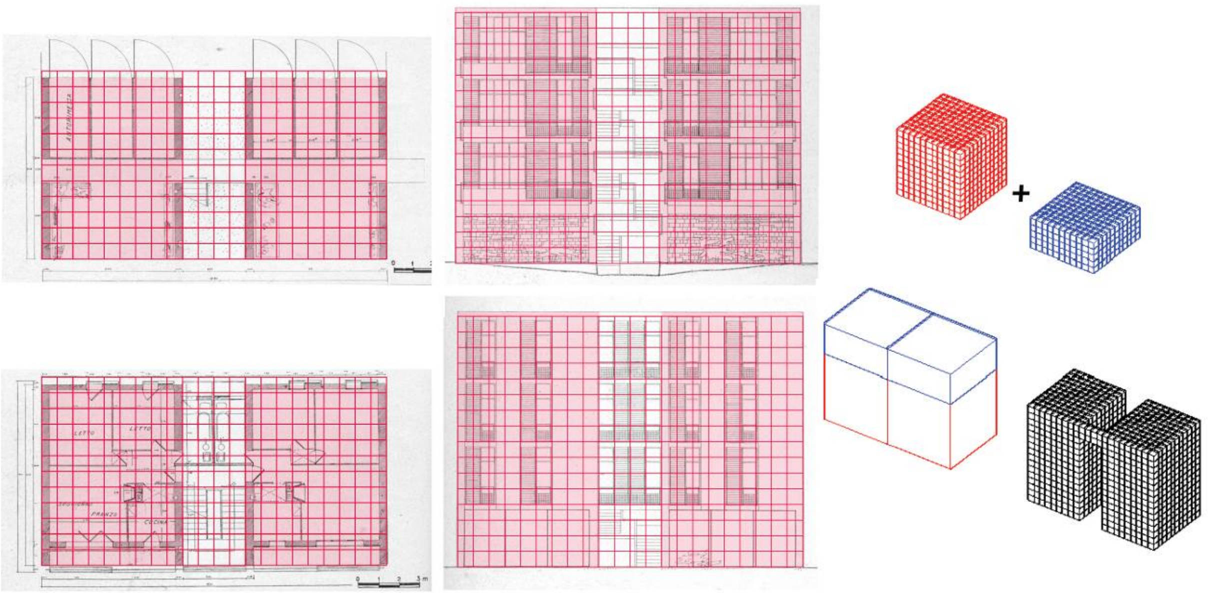

Fig. 2. Grid and geometric schemes for the definition of Casa dei Nidi.

Also in the Bica-Montecatini office building in Via San Giovanni sul Muro – Milano [2], designed in 1955, the two architects chose the square as a basic module by adopting a 3,34 x 3,34 meter grid that generates the shape in plan and scans the rhythm of the façade. The plan of the office building is set on a 5:14 ratio grid and the main elevation (front view from Via San Giovanni sul Muro) is a rectangle of 8:14 ratio. As already mentioned, the starting module for a first sizing of the building is the square with side equal to 3.34 meters which is reduced by a few centimeters following the adjustments due to the presence of the via Giulini passage that crosses the building and in particular interrupts the rhythm of the ground floor and of the first floor. In this two levels are located a number of commercial activities.
The structure of the main façade towards Via San Giovanni sul Muro presents a protrusion of 0.3 meters wide that emphasizes the verticality of the building and of the grid. The measurement of the span rating of the pillars is the same as the side of the module chosen to draw the building (3.34 meters). The number of pillars is not equal to the number of modules: they are 13 instead of 15 because of the presence of the span without pillars under which the road passes through. This reduction has been wisely incorporated by the measured drawing of the façade. Indeed, 3.34 x 14 would give a total length of 46.76 meters, but due to the lack of the two pillars the real length of the façade is 46.16 meters (2 x 0.3 = 0.60; 46.76 - 0, 60 = 46.16 meters). The size of the aluminum window-frames that characterize the façade is a consequence of the choice of the module and the dimension of the pillars. The size of the window frames, equal to 1,033 meters, derives from the subdivision of the measure of the module minus that of the pillar (3.34 - 0.30 = 3.04 meters, 3.04 / 3 = 1.033). Therefore the window frame has a ratio of 1: 3. In height the square shape of the module is emphasized by the non-transparent window part that covers the sides of the slabs on each different floor. As previously mentioned, the size of the window and the lack of two pillars cause a reduction in the overall length of the project. This length has been cleverly rationalized, in fact, its calculation is simple when the structure of the matrix that generated the project is known. If we consider, in fact, the number of spaces between the pillars (11) and the measure of the span (3.34 mt), the total length is 3.34 x 11 = 36.74. To this measure we should add the span of 9.42 meters and the width of a pillar 0.30 meters (0.15 x 2, i.e. ½ pillar per side). The result is equal to 46.46 meters. It could be precisely 46.76 if the width of a pillar was that theorized in the drawing.

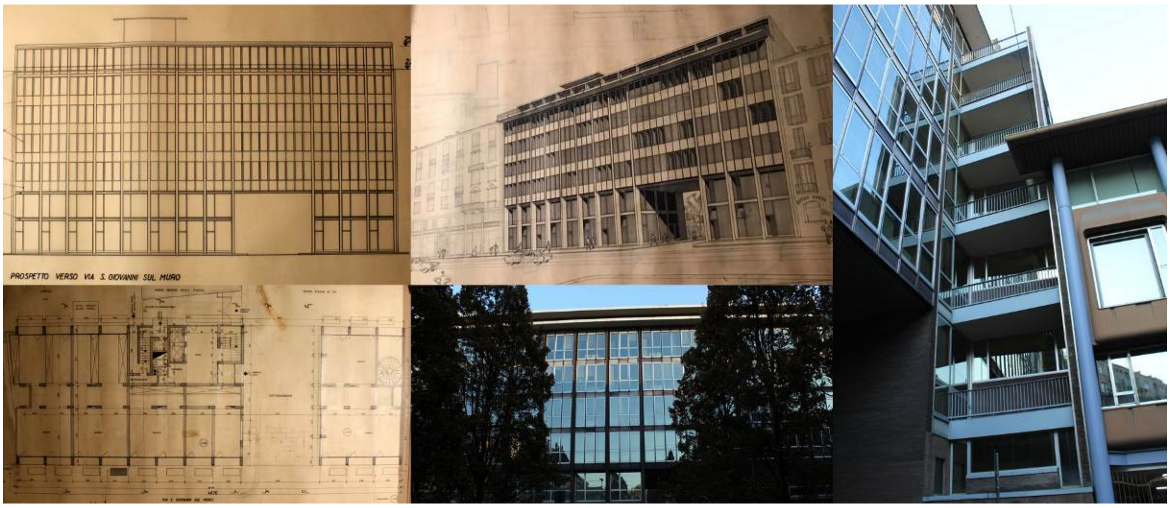

Fig. 3. Plan, elevation, perspective view [2], and photos of BICA Montecatini office building, via San Giovanni sul Muro – Milano.

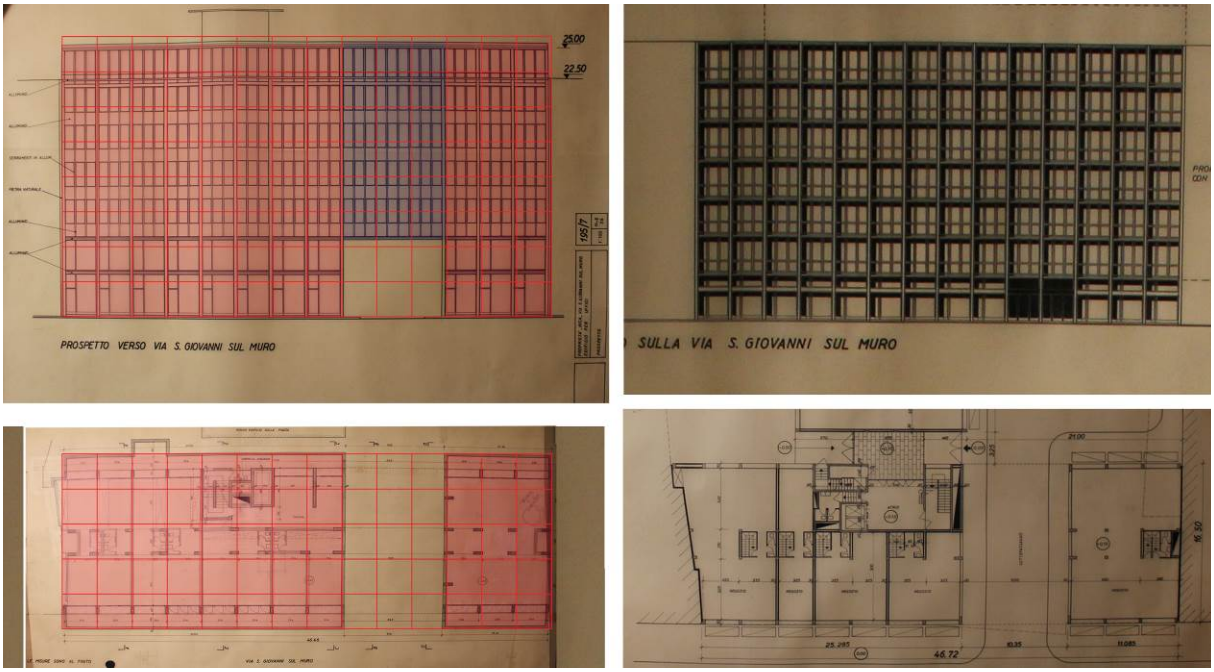

Fig. 4. Square grids generating the general shape of BICA Montecatini office building [2].

The aim of the present description is to explain the method used by Magnaghi and Terzaghi to outline the project of this building, starting from the square grid. It is a process of rationalization of the measure that contributes to the construction of the shape of the building for the Beni Immobili Civili Agricoli Montecatini company whose offices are located from the second to the seventh floor. The top floor is set back on all sides of the building and its height reaches the value identified in the grid. The façade on Via Polezza has the same characteristics of the front on Via San Giovanni sul Muro with two different aspects: the insertion of the glass block in correspondence of the stairwell - a material that the two architects had never used before this period - and the advanced position of the stairway, with respect to the façade, memory of a previous unrealized project that included a building orthogonal to the constructed one. This office building as well can be defined as rational because the method for its design is highly similar to that of the house in Fino Mornasco. The collaboration, when still students, with the architects Lingeri, Cattaneo and Origoni and the awards obtained in 1936 and 1938 [2] are evidence that even at that time Magnaghi and Terzaghi mastered this method, learned from the rationalist masters.

3 Triangular Matrices

On the other hand, in the projects for some religious buildings, (the Cagnola villa church in Gazzada and San Filippo Neri in Bovisasca district in Milan, designed respectively in 1959 and 1961), instead of the geometry of the square, they used the equilateral triangle at the basis of the matrix generating the shape.
This regular geometric figure, closely linked to the number three, represents both the concept of stability and the religious concept of the Trinity,
In fact, the triangle, in the biblical symbolic language, represents the transcendence of God and the Trinity. The latter should be considered the absolute postulate of the New Testament that professes one God in three persons. This is why, as an elementary geometric figure, the

equilateral triangle is often used to generate the form of sacred spaces and, above all in the Baroque period, a composition of triangles becomes a generator of planimetric schemes with a high symbolic value. In the form of the intersection of two equilateral triangles which determines the figure of the Star of David, it generated a whole series of important monuments: among them it is enough to remember the Borromini's church of S. Ivo alla Sapienza, where the concept of wisdom (Sapienza means Wisdom and is the name of the ancient academic institution), inherent in the geometrical figure of the 6-pointed star, becomes the basic form of the plant. The triangle therefore, and in particular the equilateral triangle, is considered the symbol of an absolute perfection, both material and spiritual, and has been found since ancient times in all religions. With the vertex upwards or downwards it indicates the four elements (fire, air, water, earth), which once again, in the fusion of the two symbols in the one quoted above, with the meaning of "wisdom" is in the ancient iconography as a seal in the ring of King Solomon.

As regards instead the hexagonal plan, it had the precise symbolic function related to the creative force of God: the hexagon is, in fact, composed of six triangles that allude to the Trinity and to the number of days of creation. Hexagonal plans in ancient times were adopted for central-plan systems, typical of mausoleums, baptisteries, imperial chapels or temples and churches of great importance, all declined with a close symbolic relationship with the shape of the plan, which was more often octagonal or circular.

In the modern design of the churches the questions of setting up the plan and related spaces, maintained an underlying continuity with the classical symbology, but subordinating it to functional choices and of architectural language. From the latter, an important component was the renewed interest in the adoption of geometric and arithmetic proportional relationships that derive from rationalist theories, but also from the organic architecture. While the rationalist research developed within the Modern Movement get back to the golden ratio as the basis of Le Corbusier's Modulor or of the research on the measure of the Raumpan of Loos or of the existenzminimum and of the researches born within the Bauhaus, the organic architects, in Europe and America (from Alvaar Aalto to Frank Lloyd Wright) adopted a formative process that instead considered the creative event as an extension of the action of nature, which works according to the same geometric rules. The subtle difference compared to the method previously described concerns the fact that in order to reach the completed work no rules defined a priori are applied, but the discovery of its own is identified from time to time; only in this way creativity and production go hand in hand shaping the composition "according to such a practice that while it does, it invents the way of doing" [7].

Both, therefore, refer to an ordering principle as an indispensable condition of the creative moment, which remains, at times, linked to the a priori search for creative action as the reason for the composition.

The organization of the modern project spaces, abandoned the safe way of the systematic adoption of the proportional rules, that guaranteed a "beauty-just-like-nature architecture", is subjected to the analysis and the "a posteriori" interpretation; a literary, narrative, sometimes post-planning exercise, which is the principle of a self-reproducing architecture that ends up identifying its model in similar architectural objects, in the questionable logic of references and citations. It is therefore necessary to reason on the compositional structure as a place of regulated expression not so much and not more by the use of data elements, linguistic codes, prefixed forms, but by the mathematical concepts that regulate the making and, if we accept the pregnant image of "aesthetics of the formativity", of the constitutive simultaneity of idea and form, the act of thinking [7].

For the purpose of our research it is important to set some dates of the revival of the theory of proportions applied to architecture: in particular the decade of the 1940s was crucial. The discourse emanated from two different sources: one established by art and architectural historians, the other by practicing architects, the former with analytical goals, the latter with creative purposes. Wittkower's theory of harmonic proportions was first published in 1949, in a chapter of his seminal Architectural Principles in the Age of Humanism [6] and in 1951 he participated in the IX Triennale in Milan, entitled 'Divina Proporzione'. The projects we are analyzing are born in this context. For the two churches, which were built between 1959-63 and 1961-64, at the turn of the liturgical reform of the Second Vatican Council that changed the functional and spatial relationship of the altar and the assembly, the adopted geometric system allows us to make further reflections.
We have seen that the hexagonal plan by its nature generates a central symmetry system. Still typical of the Orthodox Christian rite or of different religions such as the Muslim, the Catholic churches over time have adopted a basilica system, apparently in contrast with a tessellation of the plan with the exact triangular shape. It should be noted, however, that many post-conciliar churches designed on the basis of a composition of triangles also have an organization of the basilical liturgical action within a space that is geometrically similar to that generating a central symmetry. After careful analysis, many of these architectures adopt isosceles triangles, derived from the intersection of squares along the diagonal, then with the base angles of 45 degrees and the consequent rotation of the axis joining the altar with the entry of the church that is no longer parallel to the perimeter of the building, but diagonal. This produces a dynamic effect that represents, once again in a symbolic and metaphorical form, the changed relationship of man towards God, which is no longer passive and static, but active.

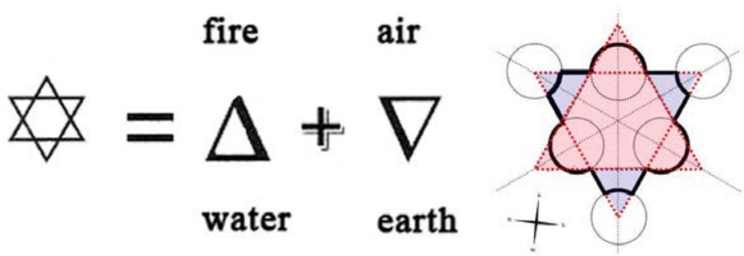

Fig. 5. The symbols related to the equilateral triangles and a compositional scheme generating the plan of Borromini's Sant'Ivo alla Sapienza in Rome.

In the churches we are analyzing this character is also evident in the adoption of a tessellation of the plan made with the equilateral triangle. In the case of the church of Villa Cagnola which is of basilica type, the covering of the plan, as we shall see, determines the architectural-landscaping solution described below, with the exception of mediating the rectilinear perimeter of the wall with the need to choose how to finish the composition arriving at the edge of the rectangular hall: that is, to choose to halve the series of triangles along the height intercepted by the long side of the rectangle, or to use volumetrically this exception. For the large hexagonal hall of the church of San Filippo Neri, however, it will be precisely the position of the accessory spaces to entail a sort of hybridization of the distributional-functional system with the geometrical-spatial scheme.
The church of Villa Cagnola has a hall-plan with a fully windowed wall that opens the view from the altar to the pre-existing historical park with a secular Lebanon cedar. The project turns its attention to the old tree and does it by opening one of the long sides of the church. A

series of equilateral triangles, in plan, have one of the sides consisting of a full-height wall and the other consisting of a stained glass window, which allows you to see in transparency the magnificent tree that in some way reminds the biblical tree of the life. On the opposite side of the church, the continuous wall is composed as a succession of full vertical panels, positioned across the reinforced concrete beams that support the roof and overhang from the façade. Under the beams there are long windows that reach the floor. The walls are completely covered with brown tesserae. The plan of the church is rectangular. The ratio of the rectangle is 24:43. The side of the equilateral triangles of the open wall towards the garden is 8 modules. The height of each equilateral triangle of side 8 is 6,9282. The thickness of the walls is equal to 1 module. Also in this case the number 8 is linked to 3 and 6. In the Christian religion these three numbers are meaningful from the symbolic point of view: 6 is the number of creation, 8 is related to the resurrection and 3 to the Trinity.

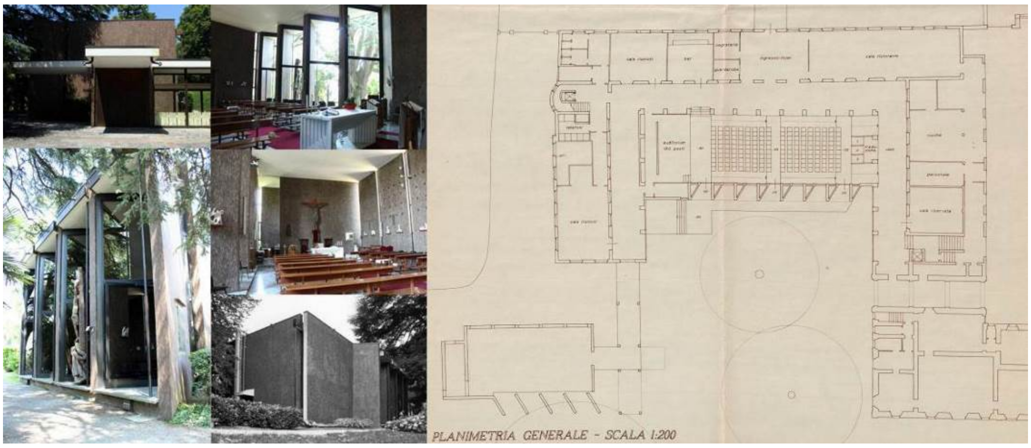

Fig. 6. Photos of the church of villa Cagnogla at Gazzada (VA) and general plan.

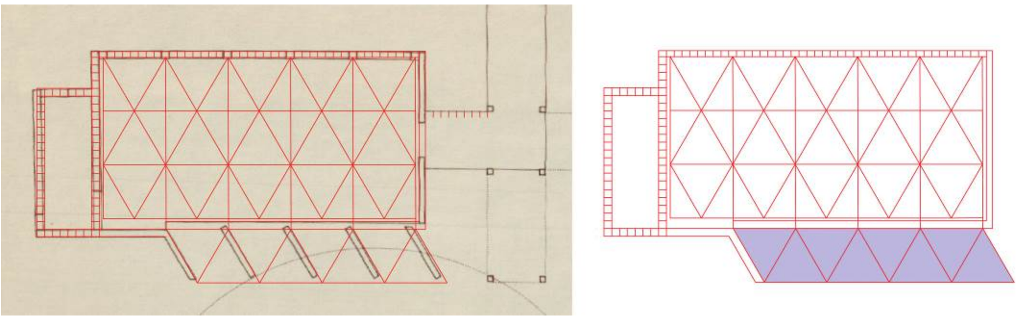

Fig. 7. Geometric studies of the church of villa Cagnogla.

In fact, if we divide the short side of the rectangle of the church plan into three, we get the side of the square generating the plan of the portico connecting the church to the remaining part of the structure of the villa Cagnola. The number of equilateral triangles that generates the open wall is 8. The ratio of the rectangle that composes the main façade is 16:24. The sacred building is a single hall, and is developed like a simple volume at a constant height with a small sacristy on the back of the same height as the church with a rectangular plan of ratio 8:19. Outside, the long façade is punctuated by a series of gargoyles of 1 module that are separated from each other by 7 modules. The church, even in the simplicity of the lines, shows a rational geometry that binds number, form and measure to create that ascensional path that brings the spirit to God. The interior space is rich in symbols while reflecting simplicity and geometric purity.

The church of San Filippo Neri has a central hexagonal plan with a series of annexes generated by the same geometrical shape based as well on the geometry of the equilateral triangle. The reference to regular polygons as the principle of the form and the related traditional value of geometric "perfection," becomes a model of symbolic rationality, and the courtly instrument of conception of the sacred space.

The church built in 1963 presents a uniform height with annexed hexagonal spaces around the perimeter. On the right of the entrance is the weekday chapel, on the left the baptistery and another little chapel with a secondary altar. Natural light pours from above through 6 semi-transparent plastic domes with a slight underwater twilight effect. A further series of 13 stained glass windows are situated in the upper part of the vertical structure representing the stations of the Via Crucis.

The basic module that allows the description of the project is the equilateral triangle of side equal to 4.66 meters. The relationship between the side of the basic module and that of the large hexagon of the church's hall plan is 1:4. The weekday chapel and the entrance porch are generated by the composition of 18 triangular modules (6x3). The chapel and the baptistery are made out of 6 modules that generate the hexagons of each plan and are connected by a space composed of 3 modules which generates a half hexagon. The church's hexagon consists of 144 triangular modules. The height of the church is two modules. Also in the case of the church of San Filippo Neri, number, form and size contribute to generate a shape with great symbolic value. The 3, 4, 6, and 8 are contained in this scheme; about the symbolic meaning of these numbers we have already spoken.

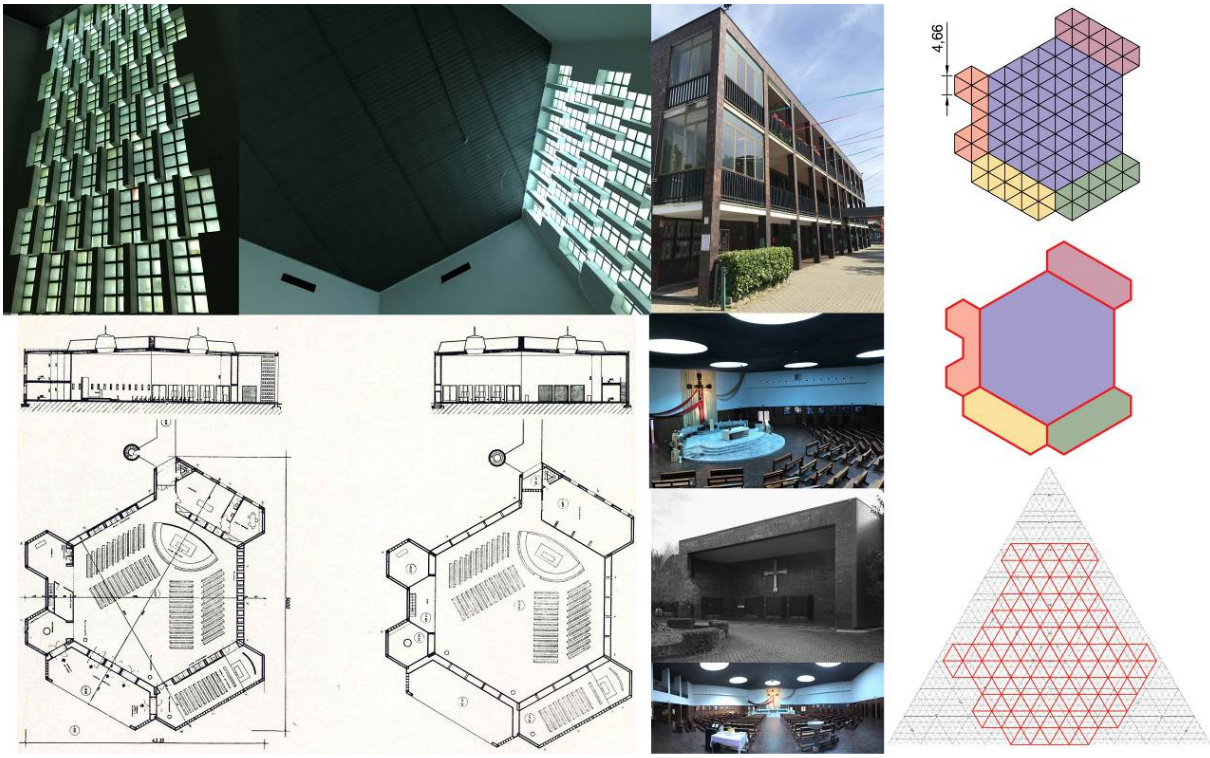

Fig. 8. Plans, sections, photos and schemes of the church dedicated to San Filippo Neri in Milano.

About the 144 it is important to underline that it is a number that belongs to the sphere of the 60 divisors of 5040 (number of the ideal city described in Plato - Book V, Le Leggi) [3] [6]. This number has an important reference also in the description of the Heavenly Jerusalem: is clear in fact 144 cubits is the measure of the side of the cube of the walls of the holy city described in the Apocalypse: "*15. And he who was talking with me had a gold measuring-rod to take the measure of the town, and of its doors, and its wall. 16. And the town is square, as wide as it is long; and he took the measure of the town with the rod, one thousand and five hundred miles: it is equally long and wide and high. 17. And he took the measure of its wall, one hundred and forty-four cubits, after the measure of a man, that is, of an angel.*"[1].
So 144 is a symbolic number and its geometric arithmetic quality is a perfect concept to base the design of a sacred building. The design of the church of San Filippo Neri despite the rationality of the scheme, has a high symbolic value as well, just like the historical churches. Our question is: what is the genesis of this design? As a matter of fact, the purpose for this scheme is not only symbolic but also practical, in fact, the hexagon with the same perimeter covers a larger surface than the square or the equilateral triangle. The hexagon is therefore a convenient form for a large church like this one in the Bovisasca district of Milan, which can house more than 1000 faithful, because it allows to cover, with a defined perimeter, a larger surface.

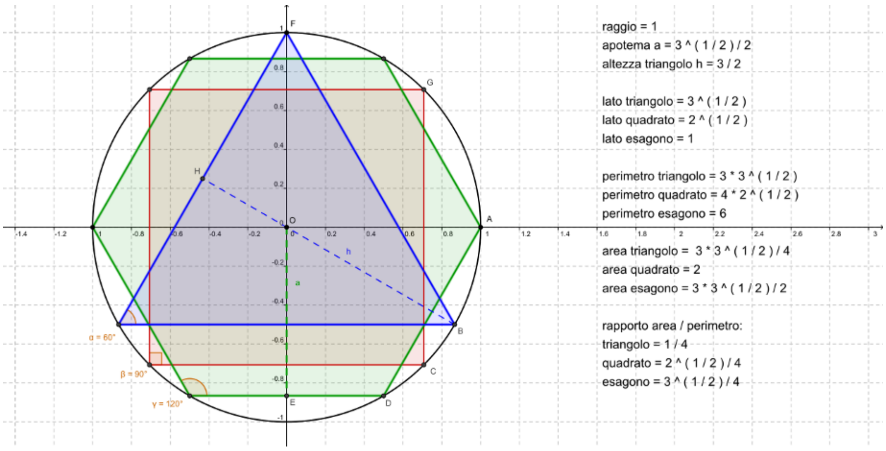

Fig. 9. Comparison of the ratio perimeter-area among equilateral triangle, square and regular hexagon.

4 Conclusions

The modern architecture created by the two architects is strongly mathematical in its conception.
The square and the equilateral triangle at the base of these architectures show a rationality that binds number, shape and size, generating a modern form that bases its rationalist roots in the classical tradition of the History of Architecture of the past.
The reference to the periodic tessellation of the plan is evident and guarantees a strong and rational design method. The geometric figures that generate it are precisely the square, the equilateral triangle and the hexagon. In fact, a tessellation of the plane is a set of limited plane figures and a way of placing them on the **R2** plane, so as to cover it without overlapping and without empty spaces. A tessellation of the plane is periodic, if there are two independent translations **v**, **w** and a finite portion **U** of the tessellation, such that all the drawing is obtained

by copying U and its translations with all the translations generated by **v w**, i.e. all the drawings of the type

$$\mathbf{U + mv + nw}$$

with **m**, **n** part of **Z**. Translated from the mathematical language, it means that when we say that we are tiling a plane we mean that we are covering it completely (without leaving holes) with figures that repeat and do not overlap, and are periodic if these figures are always the same ones next to the others.

Fig. 10. Cartesian graph paper and isometric or triangular graph papers, irreplaceable tools in the Magnaghi-Terzaghi architectural firm.

Moreover, the Cartesian graph paper and the isometric, trilinear or triangular papers were irreplaceable tools never missing in an architectural office, useful for the first layout of the project based on regular grids arising from the periodic tessellations of the plan. Even though nowadays the Cartesian graph papers are still widespread and known, the triangular graph paper or isometric papers that were usually employed for trilinear diagrams or for quickly obtaining isometric axonometric projections and planivolumetric drawings, are less known. Triangular graph paper, in particular is useful to graph the relationships between three variables on an equilateral triangle. This type of two-dimensional graph can be made when the sum of the three variables always adds up to a constant - usually 100% or 1. The charts produced from this method of graphing are generally referred to as "ternary plots" or "ternary diagrams." They are frequently used to plot the compositions of mixtures in physical

chemistry, petrology, mineralogy, metallurgy and other physical sciences. Ternary plots are also used in genetics and game theory.
In conclusion, the tools and rational methodology described are at the basis of the architectural composition and in particular in Augusto Magnaghi Delfino's and Mario Terzaghi's design method. They handled a logical deductive approach with practical purposes that in geometry found an important tool to generate the architectural form.

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NB. In the present research Alessandra Capanna's contribution is in the analysis of the projects and Giampiero Mele's is in the study of the geometric matrices.

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