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WHEN TURIN CAME TO ROME: A WALK IN THE PRATI DISTRICT AND ITS SYMMETRIES

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Abstract. Symmetry, in mathematics today, is the property of invariance with respect to any "isometry", i.e. with respect to any transformation leaving distances unchanged. Thanks to isometries it is possible reduce complex shapes into primary forms for the purpose of classification, for example. The buildings of the Prati district of Rome, show symmetries at various spatial scales (urban scale, front, decorations). We analyze them and propose some significance to their urban scale symmetry.

Keywords: symmetry, urban scale, architecture, friezes, Rome 20th century

Mathematics Subject Classification 00A67, 58D19, 05B45

1 Introduction

Symmetry, in mathematics today, is the property of invariance with respect to any transformation that transforms an object while leaving distances unchanged. Such a transformation of a space is called "isometry". Thanks to the isometries it is possible reduce complex shapes into primary forms for the purpose of classification, for example. In particular, all information about a symmetrical object can be reduced to the description of a portion of the object, together with a transformation of the space; for instance, a motive with rotational symmetry of angle $2\pi/n$ can be memorized by the graphic information contained in a circular sector of angle $2\pi/n$, and the instructions to rotate it n times of that angle, thus filling the entire circle. It is obvious that this description is shorter, at least from a computational point of view, as it occupies less memory, and therefore it can be legitimately called "less complex". A very symmetrical object is also a simpler one, because it is invariant with respect to many isometries so that the graphic information needed to memorize it is less and less. Under another viewpoint, if you look at an object, close your eyes, and while your eyes are shut somebody performs an isometry characteristic of the object, the fact that the object is "invariant" means exactly that you will not be able to see any difference. The architecture has exploited the symmetries also to speed the design and realization processes of artifacts since antiquity. In this study we show how in the early twentieth century buildings of

the Prati district of Rome, isometries have been used systematically on the large urban scale and on smaller on the scale of individual buildings decorations. In [1,2] we performed studies using planar symmetry in architecture and Cultural heritage in Rome, yielding to hypothesis of reconstruction of patchy floors from existing fragments: the same was later done also in Villa Adriana [3].
Moreover symmetries on urban scale usually suggest some privileged directions, for instance as vanishing point of a large-scale perspective. Prati district was built, as Rome became the Capital of the new Kingdom of Italy, to host its functionaries and employees. We show its urban symmetries are never aligned on the view of the adjacent Vatican City, itself the capital, of another State. We think the choice of orientation of the symmetry is significant.
On the scale of the individual buildings, all buildings of Prati show a highly symmetric front. Many of the buildings also carry decorative friezes. We will look in particular at the front of the Courthouse.

1.1 Mathematical friezes

Given a pattern, the set of isometries leaving it unchanged, forms a group under the operation of composition of transformations. Any such group contains finite and infinite Subgroups (possibly void). The finite subgroup is generated by transformations, which under a finite number of compositions with themselves end up at the identity (mirrors and rotations by a divisor of 2π). If the infinite part of the group is generated by the translations along two independent directions, the group is considered a "tessellation group", or a "wallpaper group", It is called a "frieze group" if its infinite part is generated by one translation. There exist only 7 frieze groups [7]. The repetition patterns of the seven frieze groups can be summarized graphically in the scheme of fig.1

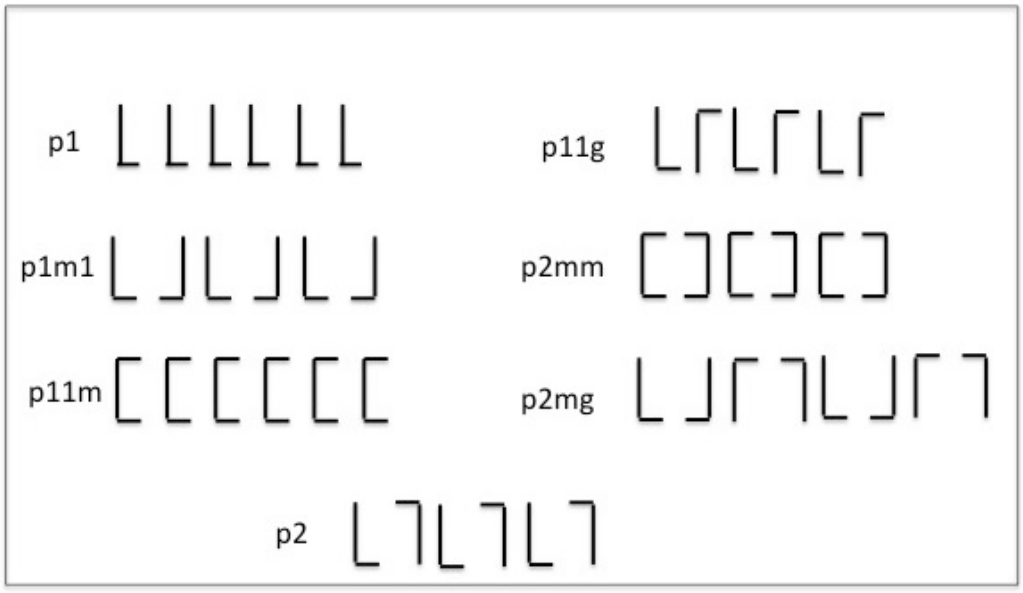

Fig. 1. The seven frieze groups: graphical repetition patterns, labeled by international classification of the groups.

2. Prati District in Rome, some history

The Prati district's territory consisted of fields that during the reign of Nero (54-68 A.D.), were called Prata Neronis and later were part of the properties of Domitia, wife of Domitian (81-96 A.D). During the Middle Ages, the area was called Prata Sancti Petri for its proximity to St Peter's Basilica. [4]
The king Umberto I (1844-1900), first king of a united Italy, decided to place in the Prati neighborhood the housing for the new administrative class that would move to Rome after it was proclaimed the capital of the Kingdom of Italy. [4]
In fact, until the 19th century the area was used for agriculture and only in 1883 it was urbanized in the part that was intended for military exercises, called the parade ground.

2.1 Symmetries at urban scale: orientation of Prati district in the surrounding town

The new district was planned in a very symmetric distribution, much as it happens with the plan of Turin, from where the King and his architects came. When a plan has symmetries and large straight streets, it necessarily also permits long scale perspectives. However at this time the climates between the Kingdom and the adjacent Vatican City was still politically very tense and, probably for this reason, the urban plan was designed in such a way that none of the new streets has as a vanishing point the dome of St. Peter's Basilica. Vatican City was to become the capital of a separate state: the Holy Siege. In addition the names of the streets in the new neighborhood were chosen among all historical characters belonging either to the pagan world of ancient Rome, or else to the lay world of the Italian unification. The main street of the neighborhood was named after Cola di Rienzo, nickname of Nicola Gabrini, a Roman tribune who in the 15th century rebelled against the papal power trying to restore the republic of Rome [8]. This marks even more the separation of the district from the adjacent capital of the Holy Siege.

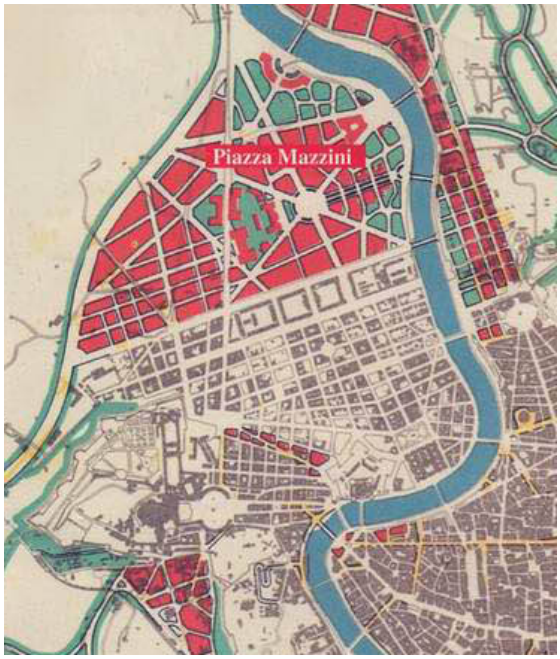

Fig. 2. Plant of Prati 1909. Streets form an ordered lattice.
Rome Town plan 1909, Edmondo Sanjust.

In the 1929 the Lateran Treaty between the Holy Siege and the State of Italy was finally signed, conciliating the two. In fact in 1933 via della Conciliazione was achieved, demolishing the neighborhood "Spina di Borgo" that did impede the view of the basilica from the center of Rome. This road was built as a visible urban sign of reconciliation. The new street, much photographed today, allows a new perspective with a St. Peter's in its vanishing point; as a consequence the Vatican can in fact be seen from several viewpoints in Rome.

Fig. 3. The three adjacent centers of power in Rome: religious, Military, Judiciary. (Aerial photo from Google earth).

2.2 Piazza Cavour

Piazza Cavour is the heart of the Prati district. The square was designed by Savoy planners as a symbol of laicism and for this reason they decided to place in this area the new Courthouse and a theatre: two public places, highly significant for the administration of a new state; the independent action and administration of culture, art, law. The Adriano theatre was inaugurated in 1898.

The planners made impossible the view of Saint Peter's Basilica from any point of the square and they named it to Camillo Benso Conte di Cavour, the Piedmontese statesman who worked to bring the union of the Italian kingdom, to emphasize the separation of the Kingdom of Italy by the Papal States [8]. One of his most famous quotes, for all Italian citizens, is "libera Chiesa in libero Stato", i.e. "a free Church in a free State". He advocated separation of Church and State.

In the middle of the square there is a garden designed by Nicodemus Severi and at the center of which stands a bronze statue of Camillo Benso Conte di Cavour by the sculptor Stefano Galletti between 1885 and 1895.

The square is rectangular. Standing in it, one perceives the sides as theatrical wings.

The wing towards the river Tiber consists of the Justice palace, or Courthouse designed by architect Guglielmo Calderini and built between 1889 and 1910, while one of the short sides hosts the Waldensian Church (Evangelica Valdese) church built in 1911, in Liberty style.

Waldensian is another religion, whose followers had had quite some difficulties with the ruling Catholic one in France and in Italy.

3. The Courthouse

The Courthouse hosts currently the headquarter of the Supreme Court of Cassation, the Lawyers Bar Council of Rome and the central Law library. The people of Rome call it "palazzaccio" or ugly palace, probably due to its role.
Built between 1889 and 1911 by the architect Guglielmo Calderini, it is one of the major works created after the proclamation of Rome as capital of the Kingdom of Italy. The prime minister Zanardelli of that era, an illustrious jurist, wanted the new Courthouse to reunify the various Courts, or Tribunals, of the capital in prestigious headquarters, in the Prati neighborhood that was then being built. [5]
The Courthouse was designed following the principles of “eclectic style”, then in vogue, combining elements of Renaissance to Baroque. The building has an extraordinary size, competing in stateliness with more ancient buildings present in town. Its plant is a rectangle 170x155 m.; its fronts are entirely covered with travertine blocks. [6]
On the sides of the entrances of the building have been placed the travertine statues of the great Latin jurists and on top of the facade towards the Tiber is placed in a large bronze Quadriga by the sculptor Ettore Ximenes [8]. The quadriga can actually be seen from the entire Prati district.

Fig. 4. Front of Courthouse facing Piazza Cavour; notice the bronze quadriga on top. (Photo Tricarico)

3.1 The friezes of the Courthouse

Anybody who has performed this kind of mathematical analysis on an actual architectural or art piece, knows that symmetry groups are “mathematical models” to such end, and therefore applying the models requires some modelling assumptions. We think this is one of the instructive things to do. At times we consider various possibilities.

Fig. 5. The friezes of the Courthouse.

In fig. 5 we marked in red the friezes we are going to classify, on the front of the Courthouse in Rome. They will be shown ordered from bottom to top.

In Figure 6, a classification of three friezes.
Motive A: if we consider half of the strip, the frieze with waves is invariant only under a horizontal translation, (group p1 in the scheme of fig.1). On the other hand, if we take into consideration the mirror axis in the center, the strip pattern cannot be considered invariant under translation anymore, and it can be considered as a single element, with an axis of symmetry. (So it is not really a frieze group).
Motive B in the balustrade, shows two vertical and one horizontal mirror axis (group p2mm).
Motive C shows a vertical axis of symmetry and a rotation of π, corresponding to group p2mg.

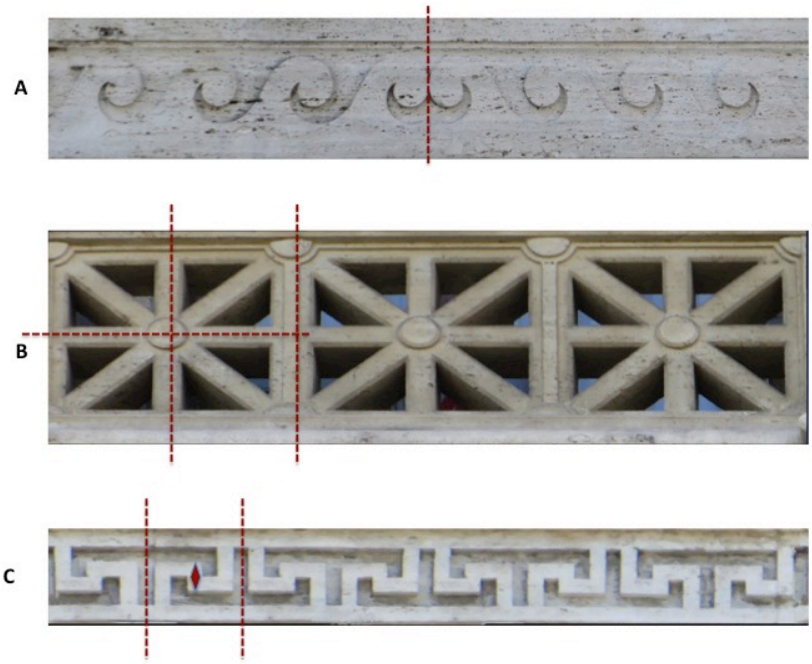

Fig. 6. Three friezes belonging to the front of the Courthouse. (Photo Scirocchi)

Motive D, pictured in fig.7, is architecturally positioned on a drainer. It can be analyzed either as frieze or as planar group, depending on what you presume to allow being infinite.
As a frieze, pattern D displays a vertical mirror axis (group p1m1), and then repeated along horizontal translation. If we take into account the obvious repetition of the pattern, the planar group sees a glide-reflection, with axis parallel to the mirror axis.

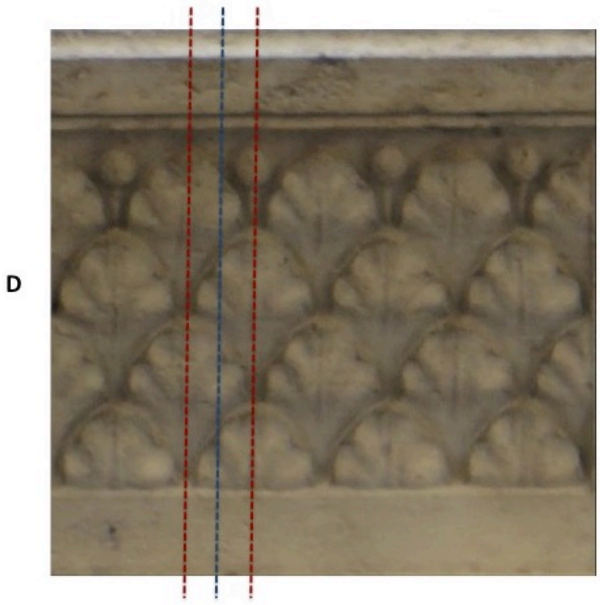

Fig. 7. The drainer. The red dotted lines mark the reflection axis; the blue dotted line marks the glide reflection axis. (Photo Tricarico)

Motive E, the beautiful and complex frieze of the summit cornice in Figure 8 shows a vertical mirror axis, going through the head of the oxen (group p1m1). We could also think to analyze some of its element separately.
From top: the first cornice has two mirror axes. If we extend it we notice that it passes in the middle of the oxen heads, in other words this axis belongs to the group of symmetry of the whole pattern.

Motive F: Flowers below the first cornice. The axis passing through the flowers is not part of the symmetries, which leave the top cornice unvaried. The flower as a single pattern is invariant under rotation of $2\pi/6$, and has six axes of symmetry.
In any case, the entire frieze and the single cornice, belong to group p1m1.

E

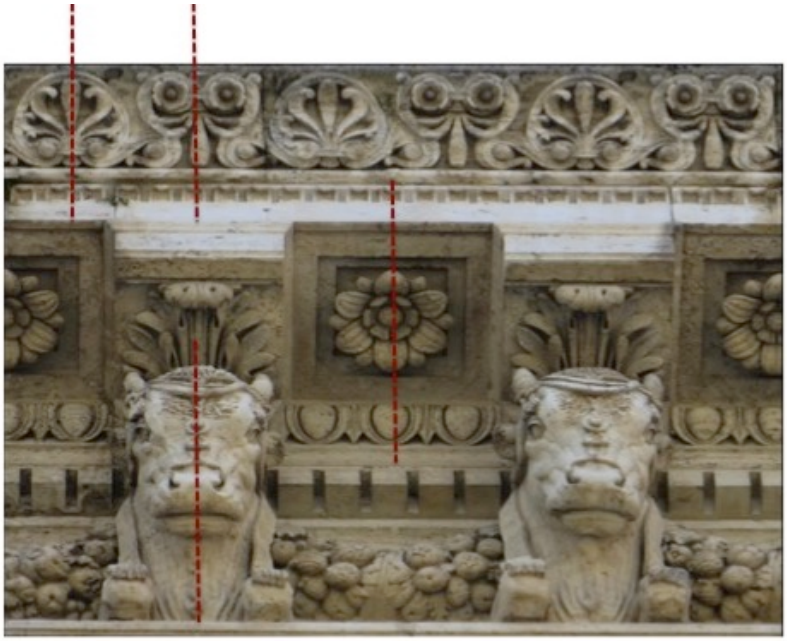

Fig. 8. The frieze of the summit cornice, and the axis of its symmetries. (Photo Scirocchi)

F

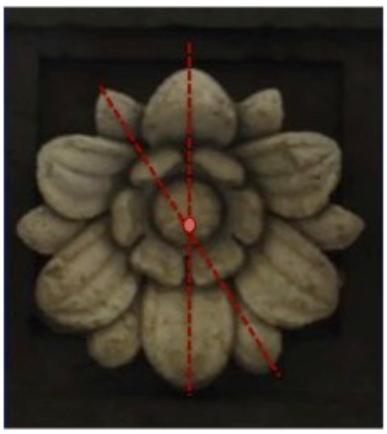

Fig. 9. The flower in the top cornice, the axis of symmetries, the center of rotation. (Photo Scirocchi)

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