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TOWARDS A DIFFUSED (OPEN AIR) MUSEUM IN THE HISTORICAL CENTER OF ROME. UNIVERSITY THIRD MISSION INVOLVING ART, ARCHITECTURE, MATHEMATICS

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Abstract. Among third mission of universities is to nurture scientific citizenship. Our *scientific trips* initiated (see [1]) on occasion of the 2014 European Science Week. Here we describe two such trips: *Modularity and symmetries in the city of Rome*, observing the "cosmatesque" geometric compositions in the medieval Roman churches. The second, *Under the bridges of Rome more than one story*, is a walk along the banks of the Tiber, providing a picture about historical and structural aspects affected by the presence of the River Tiber and its bridges.

Keywords: mathematics, architecture, early structural engineering, bridges, cultural heritage, Rome.

Mathematics Subject Classification: Primary 97A30**,** Secondary 00A67

1 Introduction

Diffusion of scientific culture aims at fostering and updating scientific citizenship, and is encompassed in the “third mission” of Universities, as defined in the Horizon2020 European program. We experienced a new way of spreading abstract scientific culture, by exploiting artefacts of Cultural Heritage widely available in Rome, and promoting *scientific trips*. Such field-trips, open to a large public, draw on the beauty of these artefacts and the sense of belonging they instill, to motivate a large public towards a scientific aspect that can be ascertained by direct observation. We initiated Scientific trips [1, 4] on the occasion of 2014 European Science Week, and then maintained them regularly under different institutional settings.

Focussing on the experiential component is crucial to a scientific approach, together with its assessment and possible critique, shared by other scientific disciplines. Given the great amount of available digital resources, the experiential approach risks to be overshadowed. Our experience then merged into the larger multidisciplinary project entitled "From the cosmos to the earth, to life, to culture. A resilient strategy for the dissemination of natural, physical and mathematical sciences: a proposal for a spread museum for the city of Rome", funded by the Italian Ministry of Education (see [2] for details about planners participants, developed material etc.) for 2016. The goal of the project this time went further, as it was to be a pilot experience, so as to develop material useful in the future for other teachers, classes, cultural associations or public libraries, to perform the activities by themselves. The experiences in Astronomy, Botany, Geology and Mathematics have therefore been collected on the website [2], together with scholarly material especially developed, and material realized by the participating classes. In this paper we report on the mathematical activities, delving into the mathematics implicitly weaved into the artefacts.

We describe two scientific trips: *Modularity and symmetries in the city of Rome*, where we observe the geometric compositions of the Roman Marmorari, or "cosmati", in the medieval Roman churches, and *Under the bridges of Rome more than one story*, a walk along the banks of the Tiber, from the bridges around Tiberina isle to Ponte Elio, accompanied by the narration of historical facts, geological evolution, evolution of constructive techniques. These walks offer the opportunity to enjoy monuments and resources such as riverfronts, churches, and bridges, observing them through the eye of a mathematician, an historian of architecture and a civil engineer.

Fig. 1. Some of our scientific trips in previous years, from left to right: Surveys and mathematical models of Titus's arch at Circo Massimo; The Aurelian Wall at Porta Latina, the science of restoration; Geometric flash mob in St. Peter's Square.

The entire realm of activities follows in the path of "diffused museums", much advocated in the 1970's to treasure the available Cultural Heritage on a widespread scale [7].
In Italy there are many examples of widespread museums. The idea of a diffused open air museum is often associated to the idea of virtual and augmented reality. Our work adds the direction of direct experience, of embodied learning and knowledge: direct experience can be assisted by the help of technology, but not replaced. Younger generations, who are often too immersed in technology, can benefit from the direct on-field experience, not mediated by technology, involving not only sight but also the senses of touch and hearing.

These particular trips were aimed at high schools. The structure of the project was planned to provide a project presentation meeting in January, a scientific introduction to the visits in March, the guided visits in April. Then some time was left to the teachers in order to organize some original material to be developed with their classes. Some of them left this as a task to the students for the summer holidays. In october the products were ready, they all can be seen on the website of Museodiffuso [2]. In November we gathered all the teachers and part of students for a final general meeting at our University together with the botanical, geological and astronomical trips, where selected products developed by students togheter with their teachers where shown.

2 Modularity and symmetries in the city of Rome

Modularity procedures go through the whole history of art and architecture. The theoretical-scientific systematization of tessellations was born in the 18th century, with the research on the crystallographic groups in the geological field, and finds its definite structure in Polya's Theorem of algebraic geometry of 1926 [9, 15]. On the other hand, discovering repetitions and modules assists in the survey activity, so much that the first Escher's studies about tessellations were born out of his architectural surveys in Alhambra [8]. With this point of view we led our groups of students to observe the geometrical compositions of the Roman Marmorari, or "Cosmati", in some medieval Roman churches and to perform surveys. This scientific trip was planned by Alessandra Carlini and Laura Tedeschini Lalli [3, 5], the trips were guided by the young architect Monica Del Grasso. To stay in the theme of diffused museum, the team developed scientific records such as for museum catalogues, and made them available on the website.

The diffused museum of cosmatesque floors way organized around an itinerary touching five churches: San Clemente, Santi Quattro Coronati, San Lorenzo out of the walls, Santa Maria in Cosmedin, Santa Croce in Gerusalemme.

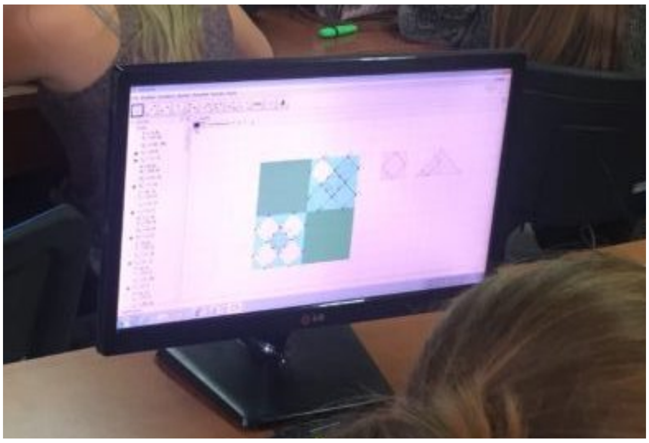
Fig. 2. The map of the itinerary "Modularity and symmetries in the city of Rome"

The guided tours expect the group to perform a real field experience, for this purpose the leader carries mirrors and tissue paper, to survey correctly. Looking for the fundamental domain of the symmetry group of each piece of floor, will save time, redundant information, and will avoid errors in the survey. A symmetry for a mathematician is the invariance of the

graphical motive, under rigid transformations of the plane. So mirrors are used to verify axes of symmetry of the observed motifs, and ascertain when there are no mirror symmetries. i.e. when the composition is in fact not invariant when looked at into the mirror. In medieval churches often the immediate intuition is of greater symmetry, so this is crucial, and medieval floors are good grounds. Tissue paper is used to make a contact survey (fig. 3-4) of the pattern from the ground, and then verify symmetries by actually moving the drawing with one of the rigid movements of the plane. We have proposed to survey mainly the symmetrical motifs in what are called *tappetini*, rectangular areas, much recalling muslim prayer carpets, usually placed in the left and right sides of the churches, where the devoutees would stand. They are decorated with geometric motifs made of stone tiles of various sizes, mostly in red, white, green, yellow.
So the students can use the survey drawn by themselves on tissue paper, to analyze the rigid movements that would leave the pattern unaltered, by posing it on the floor and verify that the superposition of motives is still exact after the movements. To find the fundamental domain, i.e. the minimum region necessary to regenerate the whole pattern under the action of the isometries we similarly invite the students to use their drawing on the tissue paper.
Participating schools and teachers (67 students in total): Prof. Simonetta Martignani (mathematics), Prof. Paolo La Civita (history of art), Liceo Peano in Rome: Prof. Lidia Ansini (mathematics), Istituto Einaudi, Roma; Prof. Letizia Sassolino, Prof. Caterina Palla (mathematics), I.I.S. Federico Caffè in Rome.

Fig. 3. Surveys of the floors, students of Einaudi Institute in Santa Croce in Gerusalemme

Fig. 4. Surveys of the floors, students of Einaudi Institute in Santa Croce in Gerusalemme



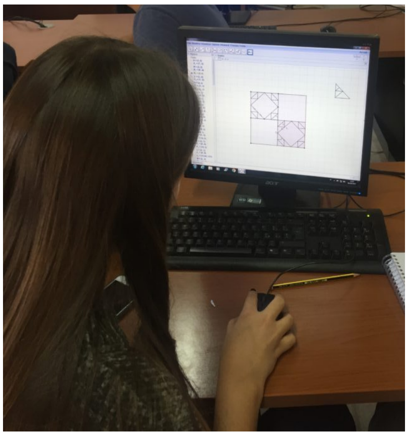


Fig.5. Students of Liceo Peano working at school to reconstruct the symmetries of patterns

3 Under the bridges of Rome more than one story

How was the river of Rome formed, the relationship with the city and the challenge of the bridges, from the Roman antiquity to the nineteenth century. We proposed a walk along the banks of the Tiber river, from the bridges around Tiberina to Ponte Elio, in front of Castel Sant' Angelo, fig.6. The activity was first conceived and activated by Fabio Brancaleoni and Maurizio Gargano (respectively structural engineer and historian of architecture of the Department of Architecture of Roma Tre University) in their University courses. In the diffusion project, the school groups were led by the young engineer Marco Mininni.

The main objective of this activity was to provide a picture, as complete as possible, with outstanding features of interdisciplinarity, about several aspects conditioned by the presence of the River Tiber. In particular, its bridges [13]. To this purpose we told about the evolution of constructive techniques, the floods and overflows of the Tiber River and the great change of the landscape around the river due to the construction of the walls and all the related works. Providing a scientific explanation for the presence of a number of architectural and structural elements the visit aims to give the participants the necessary tools to allow them to understand and interpret by themselves other similar elements in different places.

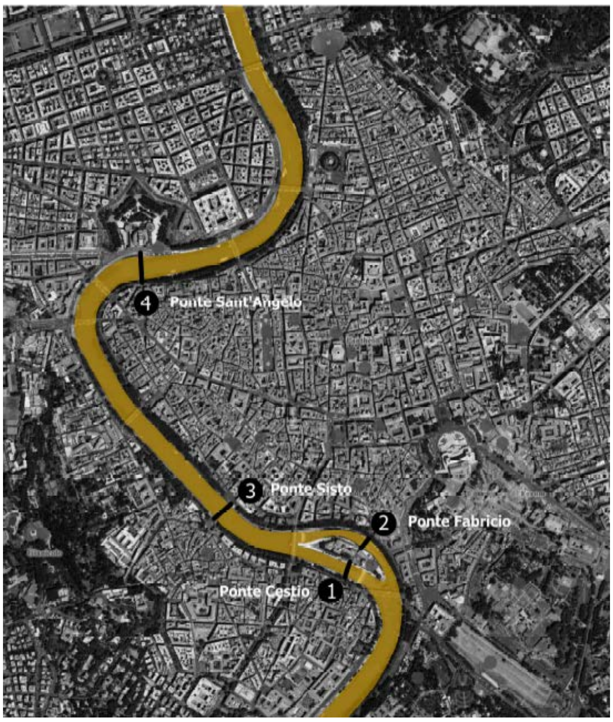


Fig. 6. The map of the itinerary "Under the bridges of Rome more than one story"

During the visit the guide explains how some issues were solved time by time, for example about the construction of ancient bridges in masonry [10], such as Ponte Cestio. Because of the circular shape of the round arch, as the bridge increases in size, also its height grows, therefore there is a difference in height between the top of the arch and the banks of the river. This drawback was overcome with the construction of two inclined access ramps. The base of the roadway was completed with concrete reinforcements necessary to balance the lateral thrust (fig. 7, historical picture taken from [6]). The arches were built with the aid of "centrings", i.e. temporary wooden structures whose goal was to support the arch during its construction. The phase of removing the centrings was called "striking" of the arc. It was a very delicate phase: the ribs had to be removed in a sudden manner so that each and all the segments of the arch took the load at the same time, settling into equilibrium.

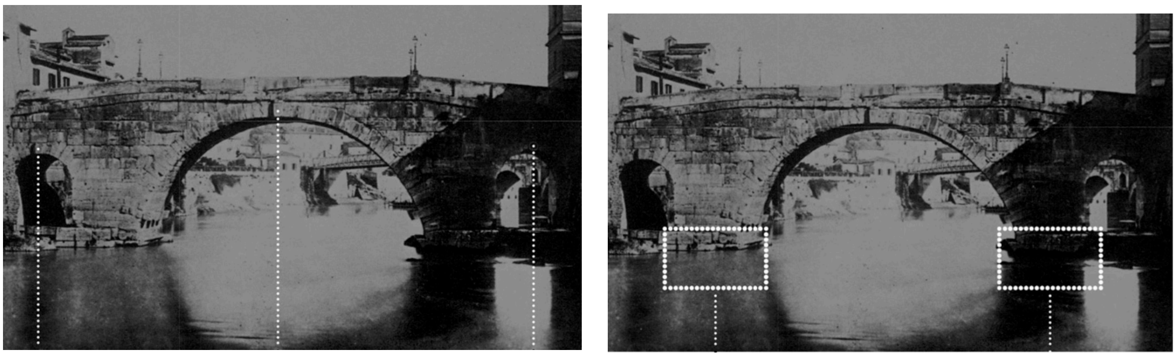


Fig. 7. Ponte Cestio before the construction of the walls, marking structural elements. On the left, the white dots point towards the arcades, on the right the dots mark the piers.

The piers ere equipped with cutwaters, or protrusions (figure 8) necessary to facilitate the outflow of the current reducing the vortices and thus limiting the possible impact damage.

Fig. 8. Ponte Sisto, the cutwaters. Photo: Marco Mininni

The foundations, obviously immersed in water, were made by means of a *tura*. The tura is a parallelepiped-shaped structure, with a double wall made from oak poles. The in-gap between the two walls is emptied from the silt and re-filled with matte and quick-setting tufa scales. Then the tura constituted a watertight chest, which was emptied of lime and water by means of Archimedes' screws, and was then filled with concrete. The tura thus allowed to improve the quality of the ground on which to then realize the piers.

Historical aspects are the key to understand differences in material and in position of the bridges [11, 12] therefore much time is spent in front of Ponte Sisto, one of the most important historically.

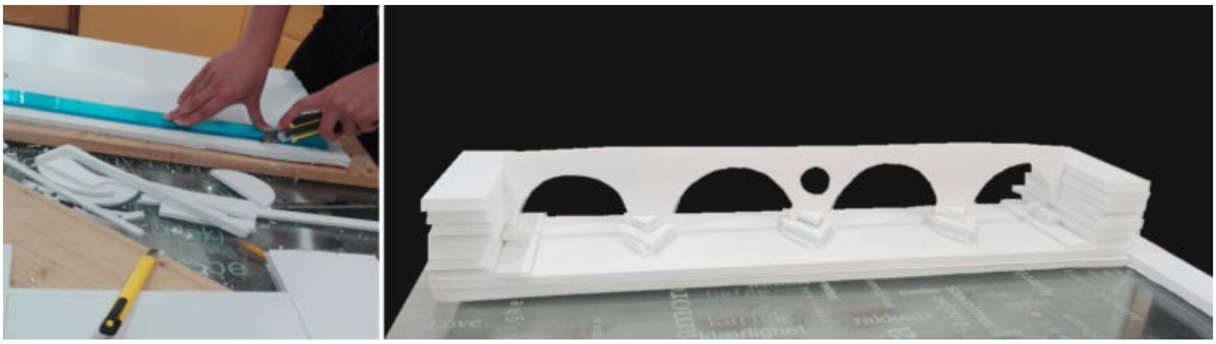


Fig. 9. Three-dimensional maquette of Ponte Sisto, students of Liceo Cavour

We realized a three-dimensional portable maquette, showing the orography of the center of Rome (fig. 10). This model, realized in forex at the Laboratory of Models and Prototypes, Department of Architecture Roma Tre, is a scale model, whose ratio between the horizontal and the vertical scale is approximately equal to 1/4, in order to make the differences in level more evident. The model has proved to be an excellent tool for the concentration of attention, and it allows also to focus the area of the historical center of the city with respect to the different hills that constitute the orography of the city. Moreover it inspires new projects that schools can develop in the neighborhood of their own premises, drawing on the complicated orography of the town: the famous seven hills count only those in the initial district. All around the land is hilly and at time with deep gorges (forre). This activity implies searching the data and then implementing a model to be cut digitally.

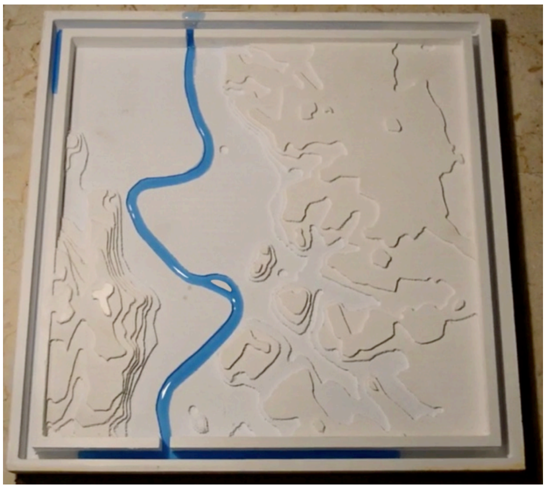


Fig. 10. Three-dimensional portable maquette showing the orography of the center of Rome, Designed by Alessandra Carlini.

Participating schools and teachers (79 students in total): Prof. Alessandra Carlini, Teresita D'agostino, Liceo Scientifico Cavour. Prof. Maria Grazia Nuzzo, Liceo Linguistico Margherita di Savoia. Prof. Maria Teresa Califano I.I.S. Luigi Einaudi.

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