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GEOMETRICAL ANALYSIS OF A DESIGN ARTWORK COFFEE TABLE DESIGNED BY THE ARCHITECT AUGUSTO MAGNAGHI-DELFINO

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Abstract. For his home (Caboto Street, in Milan) the architect Augusto Magnaghi-Delfino designed a prototype of coffee table in marble and iron and assigned the construction to his trusted artisan in Carrara. The unusual shape of the top table and the single leg and the lack of the original drawing stimulated our interest in trying to recreate the draw of this coffee table. We create two templates, and then we fitted points with approximating curves. For the components of the leg and the table's top we concluded that the shapes are similar respectively to branches Lamé curve and superellipse, a curve used by the Danish writer and inventor Piet Hein in 1959, which started in Architecture and Industrial Design the use of mathematical curves.

Keywords: geometry, design, rational architecture

Mathematics Subject Classification: Primary 51M15; Secondary 51N20

Art is solving problems that cannot be formulated before they have been solved.
The shaping of the question is part of the answer.
Piet Hein

1 Introduction

Italian design refers to all forms of design in Italy, including interior design, urban design, fashion design and architectural design. Italy is recognized as being a worldwide trendsetter and leader in design: Italian architect Achille Castiglioni, Pier Giacomo Castiglioni and Luigi Caccia claims that *"Quite simply, we are the best"* and that *"We have more imagination, more culture, and are better mediators between the past and the future"*. After World War II, however, there was a period in which Italy had a true *avant-garde* in interior design. With the fall of Fascism, rise of Socialism and the 1946 RIMA exhibition, Italian talents in interior decorating were made evident, and with the Italian economic miracle, Italy saw a growth in industrial production and mass-made furniture. Yet, the 1960s and 1970s saw Italian interior design reach the top of stylishness and, from that time, the phrases *"Bel Design"* and *"Linea Italiana"* entered into the vocabulary of furniture design.

Born from an idea of Giò Ponti and Alberto Rosselli, in 1954 *La Rinascente* company established a prize, named *Compasso d'Oro* that now is the highest honour in the industrial designing circle in Italy, as famous as the first grade international awards like the Red Dot Award. It was the first award of its kind in Europe and soon took on an international dimension and relevance. Augusto Magnaghi-Delfino won the first edition with SAFFA Modular Kitchen designed for the SAFFA firm (Società Anonima Fabbriche Fiammiferi e Affini). Then he studied for FGB, the company Busnelli of Meda, numerous furniture models: the sofa bed Relax, the Maxia armchair, lounge chairs and the living room Talia, exposed to the *Triennale* exhibits.

2 Cultural Environment and career of the architect Augusto Magnaghi-Delfino

The Italian Rationalism (Architettura Razionale) had its maximum exponents in the Gruppo 7, founded in 1929 by a group of young architects: Sebastiano Larco, Guido Frette, Carlo Enrico Rava, Adalberto Libera, Luigi Figini, Gino Pollini and Giuseppe Terragni. Gruppo 7 declared that their intent was to strike a middle ground between the classicism and the industrially inspired architecture.
Giuseppe Terragni, born on 1904 in a family of builders, in 1921 enrolled at the Superior School of Architecture of the Polytechnic of Milan, where he studied with particular passion the architectures of the past and appreciated the new rationalist architecture encoded by Le Corbusier. Obtained the degree in 1926, he engaged himself in the study of the theoretical premises of the new architecture, still in elaboration. Elaborating a personal rationalist language, marked by the search of classical proportions, the architecture of Giuseppe Terragni starts from a pure geometric shape, that corresponds to the square or to the rectangle and for the plan to a solid form that coincides with the parallelepiped.
Le Corbusier, master of the modern movement, in relation to the new way of conceiving the furnishings wrote "*Equipping is to classify, through the analysis of the problem, the various elements necessary for domestic use. Substituting the countless furniture dressed by different forms and names, the "cases-standard" are incorporated in the walls or supported by them [...] equipped inside according to their exact destination, they are only the furniture of a house. Only the chairs and tables remain."*
Chairs and armchairs, therefore, are the master. The famous Sant'Elia armchair by Giuseppe Terragni of the 1936, the Modern Armchair by Le Corbusier of 1928 and the Chaise Lounge; the "Barcelona Armchair" by the German architect Mies Van Der Rohe are widely known. All design works characterized by the combination of chrome-plated steel-leather and simple and essential lines. In the modern armchair of Le Corbusier, for example, the separation between the cage of the structure and the cushions expresses in full the logic of rationalism.
One of the famous phrases of Mies Van Der Rohe "*Less is more*" is a "brand" rationalist. That is, all that does not serve the function ("the most", see for example the decoration) becomes "less". Another famous phrase of the famous architect is "*God is in the details*" so we understand that the true point of architecture and rationalist design is not the excess of decoration but the care of the small detail.
Augusto Magnaghi-Delfino, born in Milan in 1914, graduated in 1939 at the Faculty of Architecture of the Politecnico. During his studies, he knew Mario Terzaghi, with whom he started a lasting professional partnership and a solid friendship, and Luigi Comencini and Alberto Lattuada who later will became famous filmmakers. Still a student, he began to attend the study of Terragni and Lingeri, where he also brought his friend Terzaghi as a further practitioner to help the drafting of the contest project for the Palazzo del Littorio in Rome. In

Terragni's architectural firm, he met Filippo Tommaso Marinetti, Lucio Fontana and Adone Asinari.
In 1939, the two young architects built the "Casa dei nidi" (house of the nests) in Fino Mornasco, a residential building with rationalist lines and they won, with Lingeri and Cattaneo, a competition for the Palace of Trade Unions in Como and in the 1941 participate in the primordial Futurist group Sant'Elia.
From 1948 to 1952, Augusto worked for the firm SAFFA. The firm produced matches and derived from cellulose and asked to Magnaghi to draw the first modular kitchen. On 1954, he won the prize "Compasso d'Oro "for this kitchen.
Magnaghi-Delfino, very attentive to public relations, wrote some articles for the magazine Domus, on which, in the fifties, some of their projects of furniture and design objects appeared. They worked fruitfully together in all fields, separating themselves seldom to follow their own personal interests. Magnaghi-Delfino designed for the company Olivari the handles BICA that were then placed on the doors of the transatlantic *Raffaello* and *Michelangelo* and later exhibited at the Museum of Modern Art in New York. After almost 50 years, they are still in production.
The two architects dedicated themselves with particular commitment to public housing, school and hospital. In 1947, they studied the application of prefabricated construction systems to the using popular building in some buildings in the QT8 district in Milan. In 1950, they signed together the *Village for workers* of the paper mill Vita & Mayer in Cairate and the small quarter in Lonate Pozzolo. They designed numerous complexes for the INA-CASA, in Gemonio and Orago in the 1951, in Magenta in 1953 and in Monza in 1955. After the happy realization of the elementary school in Jerago with Orago, followed interventions in the Feltre district and Comasina (Milan) and the INAM clinics of Laveno Mombello (1957-58), Cremona (1960) and Milan, among which the beautiful structure in Piazzale Accursio (1957-60). They collaborated with Ignazio Gardella at the design of the Ivrea Hospital and with Vittorio Gandolfi for the project of the new airport in Lagos (Nigeria). In 1959 they built the house in Caboto street where Augusto Magnaghi-Delfino lived till 1963 when, undermined by a serious illness, get off. This closed the brilliant activity of a knit professional team, while the projects remained entrusted to Mario Terzaghi, which was the only one owner of the architectural firm.

3 Furnishing works: right lines or curved lines?

Civilized man is surrounded on all sides, indoors and out, by a subtle, seldom-noticed conflict between two ancient ways of shaping things: the orthogonal and the round. Cars on circular wheels, guided by hands on circular steering wheels, move along streets that intersect like the lines of a rectangular lattice. Buildings and houses are made up mostly of right angles, relieved occasionally by circular domes and windows. At rectangular or circular tables, with rectangular napkins on our laps, we eat from circular plates and drink from glasses with circular cross sections... [**7**].

In 1953, architect Saarinen had the idea for the design of his furniture with a single leg. Because the irrational overlap of legs disturbed him, he made of the chair and the table a clear and distinct unity. Dozens of designs will follow the stories of different models: the first in the 1:4 scale, then in proportions corresponding to those of a room of the size of a dollhouse, until it reaches a good effect in space. Knoll International realized the Saarinen collection, which is still in production today and is part of the permanent collection at the Museum of Modern Art in New York. The work on the tables began after 1956 and was the last series of furniture

designed by Saarinen before his premature death occurred in 1961. Saarinen continued to work on the problem of "organic mobile", that is to say the furniture in a single form and in a single material. Visually, the pedestal furniture group has an organic unit. However, given the technical limits of the period, Saarinen failed to make this series with a single mould and a single printed material. The base was made of aluminium casting because the plastic printed in such a form was not sufficiently resistant to hold a tabletop or the weight of a seated person. Saarinen's coffee tables of the Pedestrial Collection have tabletops in a wide range of woods, marbles and granite, features with bevelled edges; the top attaches to base with threaded rod and base is heavy moulded cast aluminium with white or black paint.
The architect Arne Jacobsen, born in 1902 in Copenhagen, is one of the grandfathers of modern Danish furniture and the minimalist Danish style. He is remembered for his elegant and functional chair design. From 1934, he cooperated with Fritz Hansen and in 1952, they made the Jacobsen Ant Chair and the Jacobsen Series 7 Chair. At the end of the 50s, Jacobsen designed the famous Egg chair, Swan sofa and Series 3300 chairs, in which the curved lines are an admired and outstanding design.

3.1 Particular furnishing works: coffee tables

The coffee table can have many uses: a spot for a cocktail or the centre of a conversation area. We have many styles of coffee tables of different forms and materials. Traditional coffee tables tend to have ornate detailing, such as curved legs, carvings or inlaid tops and be made of rich woods. Transitional furniture falls between traditional and contemporary/modern furniture. It has a more casual, functional feel. Transitional coffee tables will suit most decor styles. Though contemporary and modern may seem interchangeable, contemporary style refers to what is currently popular. Contemporary designs often feature stark interiors with bold, bright accents. Glass tops and metal bases are common features of contemporary coffee tables. Mid-century modern refers to the interior design style that was popular between the 1950s and 1960s. Mid-century modern coffee tables are characterized by clean, simple lines and bentwood or moulded plastic construction. Most of the time, wood furniture has a simple finish to show off its natural beauty.

4 The Magnaghi-Terzaghi coffee table

The Magnaghi-Terzaghi architectural firm from 1950 designed sophisticated lounge set and coffee tables with oval top for industrial production For example, we consider the coffee table with top painted etched glass top and brass legs (38.7cm high, 120cm principal diameter).

Fig. 1. Coffee Table (1955).

For his home (Caboto Street, in Milan) Augusto Magnaghi-Delfino preferred to assign to his trusted artisan in Carrara the construction of a prototype in marble and iron. The unusual shape of the top table and the single leg and the lack of the original drawing stimulated our interest in trying to recreate the draw of the top and the legs.

(a) (b)

Fig. 2. Caboto home's coffee table (a), (b).

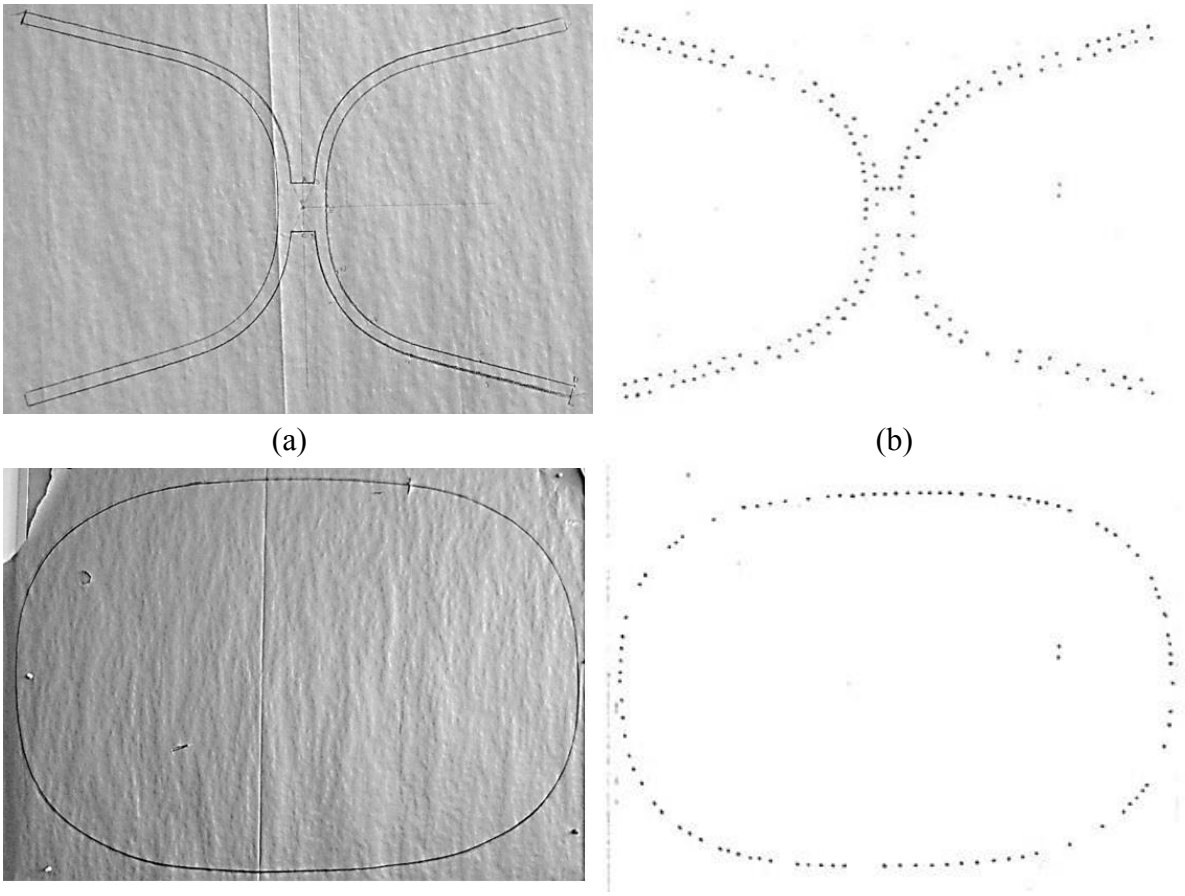
Description of the coffee table:
The top is made in Carrara marble, its measures are:
Length: 116.5cm. Width: 79.5cm. Thickness: 2cm.
Specific weight of the marble: 2.5 Kg/dm^3 Total weight: about 46Kg.
The top is supported by a single iron leg and is removable.
The single leg is composed by two iron pieces jointed, perfectly symmetrical.

Fig. 3

The measures of the legs are:
Length: 48.5cm.
Width: 69cm.
Thickness: 1.5cm.
Height: 21cm.

4.1 Coffee table study

We made templates for both elements, top and leg, and then we tried to recreate the shapes, using GeoGebra. In Fig. 4 we see the templates and the paper with plotted points.



(a) (b)

(c) (d)

Fig. 4. Figures (a), (b), (c), (d).

4.1.1 Leg's study

For the components of the leg, we try with catenaries, Musmeci's curves, generalized parabolas and hyperbolas (not Apollonius' conic sections), superellipses.
The catenary is the curve that an idealized hanging chain or cable assumes under its own weight when supported only at its ends. The catenary curve has a U-like shape, superficially similar in appearance to a parabolic arch. Mathematically, the catenary curve is the graph of the hyperbolic cosine function.
The equation of a catenary in Cartesian coordinates has the form

$$y = cosh\left(\frac{x}{a}\right) \tag{1}$$

Catenaries and related curves are used in architecture and engineering, in the design of bridges and arches, so that forces do not result in bending moments.
Over any horizontal interval, the ratio of the area under the catenary to its length equals a, independent of the interval selected. The catenary is the only plane curve other than a horizontal line with this property. In addition, the geometric centroid of the area under a stretch of catenary is the midpoint of the perpendicular segment connecting the centroid of the curve itself and the

x-axis [5]. In our opinion, the curves designed by Augusto Magnaghi-Delfino are not catenaries, because the curvature decreases too quickly.

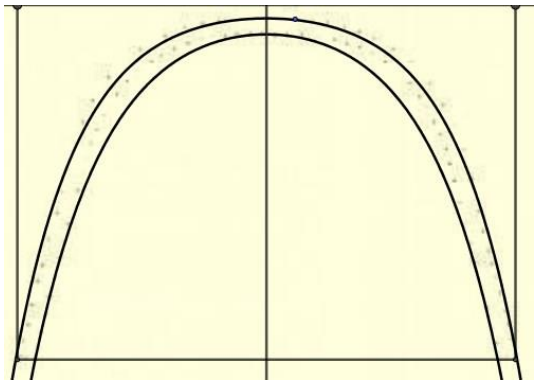


Fig. 5

The curves designed by Augusto Magnaghi-Delfino are also similar to Sergio Musmeci's logarithmic curve. Sergio Musmeci (1926-1981) was a Roman engineer, who worked in the Riccardo Morandi e Pier Luigi Nervi firm and designed many famous bridges, in collaboration with his wife, the architect Zenaide Zanini. The Musmeci's curve has the form

$$y = ln\big(cos(ax)\big) \tag{2}$$

There is a particular geometric property of this curve: the angle that the axis form with the *x*-axis is proportional to *x*-coordinate. Musmeci wrote "*A continuous form is an organic form; it is modern because it represents the solution, in a single structure, of forces and balances that descend from a complex much wider of events"*.
But, in our opinion, the curves designed by Augusto Magnaghi-Delfino do not agree with Musmeci curves.

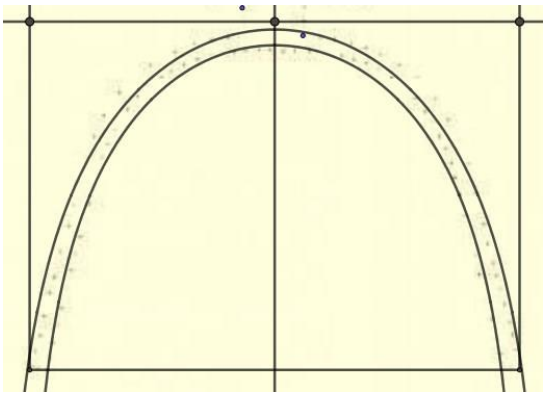


Fig. 6

Generalized parabolas and hyperbolas have respectively the form

$\left(\frac{x}{a}\right)^p - \left(\frac{y}{b}\right)^q = 1$ with p =1 or q= 1 (3)

$\left(\frac{x}{a}\right)^p - \left(\frac{y}{b}\right)^q = 1$ with p and q, positive real numbers (4)

None of these curves fitted well the points of our template. In the parabolic case we have an error of 5.1% that correspond with 3.5cm in the hyperbolic case we have an error of 4.7% that correspond with 3.24cm.

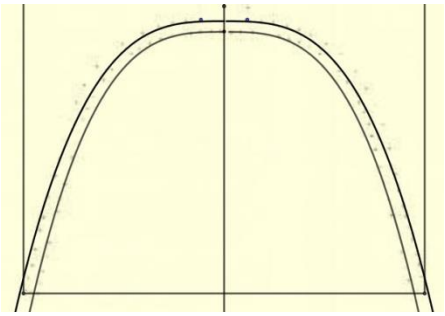


Fig. 7. Generalized parabolas.

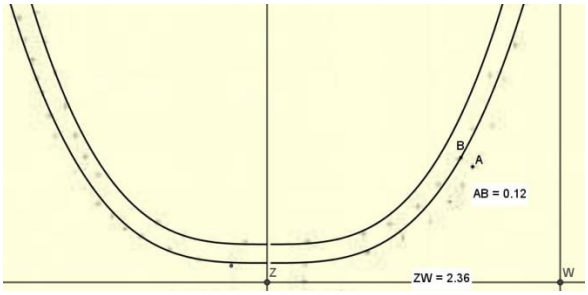


Fig. 8. Error evaluation.

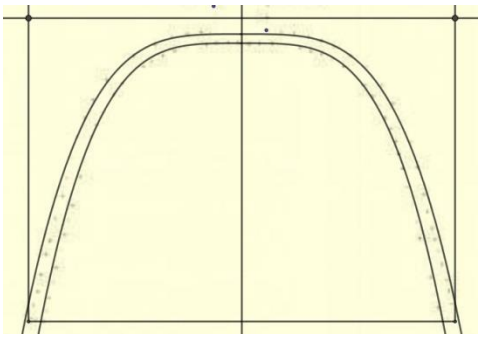


Fig. 9. Generalized hyperbolas.

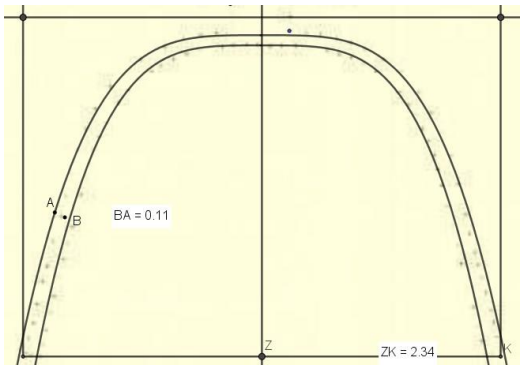


Fig. 10. Error evaluation.

So we considered to approximate with branches of quartic curves. The quartic we considered is a particular case of superellipses, also known as Lamé curves, after the French mathematician Gabriel Lamé, who generalized the equation for the ellipse.
The superellipse was named by the Danish writer and inventor Piet Hein (1905-1996), that considered this curve in order to answer to the question to fit harmoniously an oval shape into a rectangular space.
The question had been suggested by a complicated city-planning problem that first arose in 1959 in Sweden. Many years earlier Stockholm had decided to raze and re-build a congested section of old houses and narrow streets in the heart of the city. Two broad new traffic arteries running north south and east west have been cut through the centre of the city. At the intersection of these avenues, a large rectangular space about 183m long is being laid out. At its centre, would be construct an oval basin with a fountain surrounded by a large oval pool containing several hundred smaller fountains.
Daylight would filter through the pool's translucent bottom into an oval self-service restaurant, below street level, surrounded by oval rings of pillars and shops.
In planning the exact shape of this centre, the Swedish architects ran into unexpected problems. The city planners next tried a curve made up of eight circular arcs, but it had a patched-together look with ugly "jumps" of curvature in eight places. In addition, plans called for nesting different sizes of the oval shape, and the eight-arc curve refused to nest in a pleasing way.
The architectural team in charge of the project consulted Piet Hein. To understand Piet Hein's novel answer, we must first consider the ellipse, as he did, as a special case of a more general family of curves with the following formula in Cartesian coordinates

$\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^p = 1$ with p is a positive real number (5)

where a and b are unequal parameters (arbitrary constants) that represent the two semi axes of the curve. When $p>2$, the oval develops flatter and flatter sides, becoming like a rectangle; indeed, the rectangle is its limit as p approaches infinity.
Moreover, such curves could be nested, to give a strong feeling of harmony and parallelism between the concentric ovals. Piet Hein called all such curves with exponents greater than 2 "super-ellipses."
Bruno Mathsson, a well-known Swedish furniture designer, has enthusiastically adopted the Piet Hein's superellipse. He first produced a variety of superelliptical desks, followed by superelliptical tables, chairs and beds. Industries in Denmark, Sweden, Norway and Finland have turned to Piet Hein for solutions to various orthogonal-v. -circular problems and he has been working on superelliptical furniture, dishes, coasters, lamps, silverware, textile patterns and so on.
We thought to generalize the Hein's curves, creating a non-Hein superellipse

$\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q = 1$ with p =1 and q = 4 (6)

We obtained a pleasant shape, approximately near to Magnaghi's.

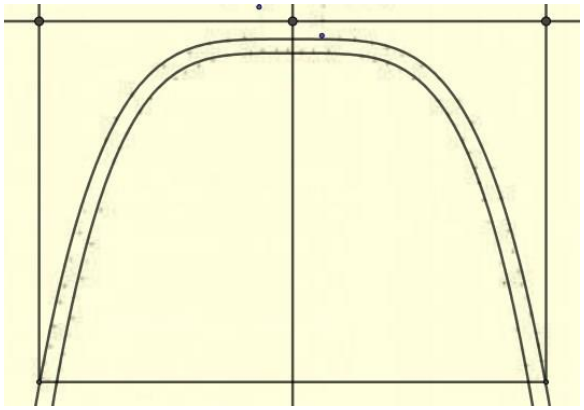


Fig. 11

4.1.2 Top table's study

For the top table, we have considered alternative solutions: ellipse, polycentric curves, quartic curves. We rejected ellipse because its pointed ends would interfere with smooth flow around it; moreover, it did not fit harmoniously into a rectangular space. We rejected polycentric curves although they look like more simple to carve in the marble. Mathematically the polycentric curves are curves made of circle arcs subsequently connected at a point where they share a common tangent. Their use in architecture has been and still is widespread. An application of the polycentric curves is to drawing a class of egg-shaped curves, see [8]. A simple conjecture about ovals, which he also applied to eggs, showed how it is possible to decide, among an infinite number of possibilities, which oval to draw once the measures of the two axes are given. Franco Ghione in the same paper gave a rigorous mathematical proof of this conjecture. Ragazzo's conjecture on how to draw all possible ovals inside a rectangle reduces the problem to *quarter* ovals inside rectangles. In any rectangle, there is only one inscribed ellipse and

infinite number of four centre ovals. The connection's problem in polycentric construction is the curvature's discontinuity.
We drew polycentric curve with eight centres.

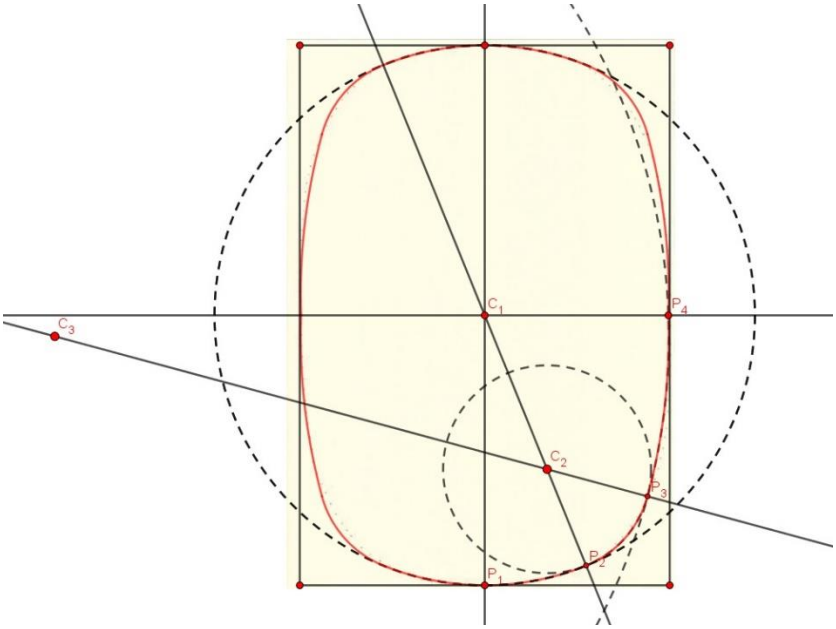


Fig. 12

Then we tried to use quartic curve, but we had more success with Hein superellipse.
For $p = 2.78$ the superellipse fit the points of our template, and we obtained a pleasant shape, approximately very near to Magnaghi's.

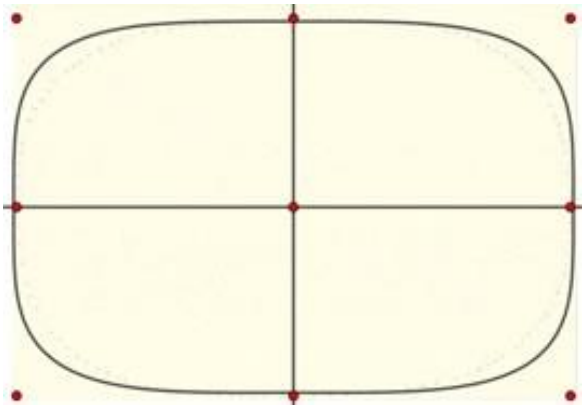


Fig. 13. Quartic curve.

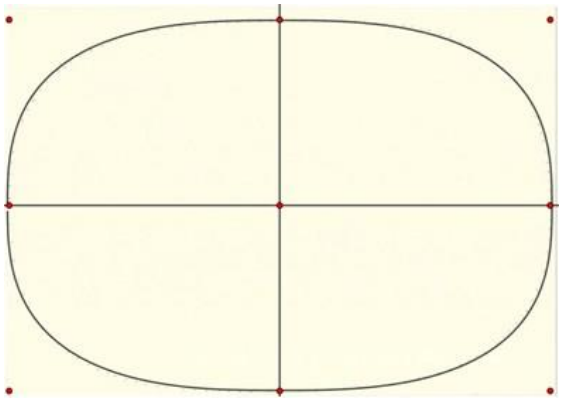


Fig. 14. Superellipse.

5 How Augusto Magnaghi-Delfino did sketch the table?

In 60's the designers did not have computers or electronic devices but only French Curves. French curve is a flat drafting instrument with curved edges and several scroll shaped cut-outs, used as a guide in drawing curves. In the 20th century, among mathematicians and engineers, there was a renewed interest in kinematics, the geometry of pure motion. Study of these mechanisms can be used as a way to show interconnections between mathematics and technology.

Descartes in the 17th Century considered only those curves that could be drawn with mechanical devices. Curves were constructed from geometrical actions, many of which were pictured as mechanical tools. After curves had been drawn, Descartes introduced coordinates and then analysed the curve-drawing actions in order to arrive at an equation that represented the curve. Equations did not create curves; curves gave rise to equations.
A common set of curves is the Burmester set [4] displayed here. The first item is very handy for ellipses, the second very often fits large parts of parabolas and the third item is used most for hyperbolas.

(a) Ellipses (b) Parabolas (c) Hyperbolas

Fig. 16. Figures (a), (b), (c).

French curves, that are flat drafting instruments with curved edges and several scroll shaped cut-outs, were used in manual drafting to draw curves of various shapes. In general you would plot various known points on the paper, and seek to join them up on the edge of a French curve. The one on the top left is meant to be good for hyperbolas, top right for ellipses**,** and the big one for parabolas **(**all the conic sections except the circle, which can be drawn easily anyway).
They can make smooth complicated curves. If you have a bunch of points you want to fit a curve to you sort of pick a piece of French curve that fits through the three points. You then pick another slice of curve that fits enough the adjacent points and so on.
There's no standard for French curve shapes. The complicated shapes are just a means to get all kinds of curves on one piece of plastic.
We considered the templates and we tried to join the plot points with French curves, obtaining

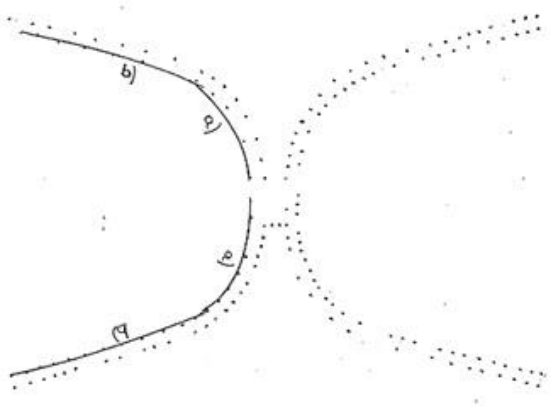


Fig. 17

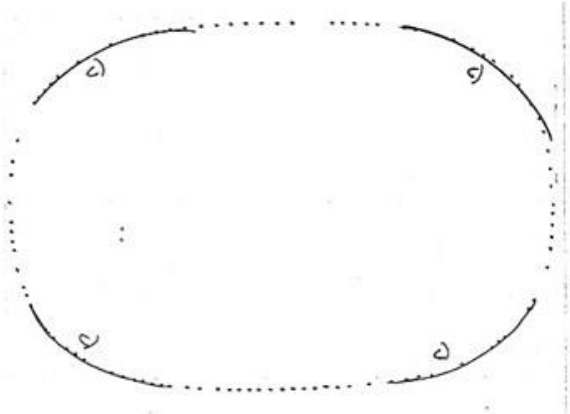


Fig. 18

In this approach we used 16 (a) and 16 (b) for the leg and 16 (c) for the top.

6 Conclusions

In all the works of the architect Magnaghi-Delfino, both of architecture and design, we find strongly a geometric component typical of the works of those years. In the absence of the original drawings, we tried to reconstruct the coffee table prototype. We think that the original shapes of the top table and leg and the unusual materials were design experiments to evaluate market trends.
We propose to look into other achievements in the future such as the BICA handle of the company Olivari and the armchair prototype in Magnaghi-Delfino's home.

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