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THE THREE-MEASURABLE PROBLEM CHANGE OF COORDINATES FUNCTIONS OF FLOATER SUSPENDED IN ACOUSTIC ENVIRONMENT

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Abstract. The constructed analytical model of elastic-stressed state of a floated gyroscope gimbal. Clarified the pattern of elastic displacement of the gyroscope gimbal under the action of the penetrating acoustic radiation. Analyzed the possibility of manifestation of local features. The obtained values of the coordinate functions of the floated gyroscope gimbal provide the possibility of manifestation, in the operating conditions, of the effect of selectivity of the angular motion of an aircraft, components of acoustic vibration of its surface in the form of the formation of a systematic measurement error.

Keywords: hypersonic technology, gyroscope gimbal, float-shell gimbal, acoustic emission, floated gyroscope

Mathematics Subject Classification: Primary 74J20, 74J25; Secondary 70Q05

1 Introduction

Nowadays, a country's borders are defended by intercontinental ballistic and space missiles of different classes and deployment types, nuclear submarines and strategic missile forces. The present-day launch vehicles (LV) are known to be able to deliver the means of destruction from the country's continental territory to any part of the globe not only with a great accuracy but also within the shortest time allowed.

Ballistic missiles (BM), while in flight, are controlled by inertial or radio inertial systems. The first-mentioned type should be considered the most reliable one it is only these systems which

offer the decisive advantage —self-containment. This feature makes it possible to implement a simultaneous launch of a large number of missiles, provides a sufficient degree of independence of the starting positions and, in addition, a high readiness and reliability of the on-alert means of tactical carrier-based aircraft (TPA), strategic bombers (SBA), the ship-toair missile systems (for example, class "Fort" with 64 C-300 missiles), which are deployed with cruisers that have gas turbine engines, missile launchers "Basalt" class with cruise missiles P-500 (superprecision shooting to a distance (radius) of 500 km), as well as unmanned aircrafts (autonomous robots) having freedom of movement, the explosive ordnance disposal (EOD) robots (Mini-Andros) and, finally, remotely controlled vehicles. Robots are finding the ever widening applications for military intelligence purposes, demining of the water area, destruction of troops in a zone of military conflicts, and others.

1.1 Diffraction of N-waves on the impedance surface of gimbal

Analyzing the interaction of an N-wave and mechanical systems of inertial autonomous positioning, we will consider, to be more specific, a commercially available single-axis gyroscope with a liquid-static gimbal. Assuming that the oscillation generated in the gimbal is not transmitted to the mating surfaces, the consideration can be limited to merely the shell part, while for sake of completeness we will construct a three-dimensional model. The results obtained can be fully used, inter alia, for the analysis of dynamics of a two-axis gyro case.

2 Analysis of the problem state

It is the spectral density of the process of energy distribution, which provide the most complete picture of the sonic boom [1, 2]. During the sound-barrier breaking, the level of N-waves may exceed ten times the level of starting from open positions [3, 4].

Acoustic radiation that penetrates an aircraft generates numerous vibration modes, including resonance one, in the hardware of inertial sensors [5, 6]. Together, they produce disturbance torques of Euler inertial forces, which give rise to errors (or drift) of inertial sensor output signals [7-14].

The simplest while also the least labor-intensive methods to eliminate this phenomenon are the methods of design and technological solutions, i.e., passive methods [15-17].

We consider partial cases of the analytical model of the influence of geometry of the floating gyroscope gimbal on the gimbal coordinate functions.

The **object** of the research is the process of elastic interaction of penetrating radiation and mechanical systems of on-board equipment.

The purpose of the investigations is to build an accurate computational model of elastic interaction of acoustic emission and the mechanical system of floated gyroscope under hypersonic flight conditions.

To this end, the following should be accomplished:

1. Build an analytical model of the movable portion of the gyroscope gimbal.

2. Clarify the pattern of elastic displacement of the gyroscope gimbal under the action of the penetrating acoustic radiation.

3. Analyze the possibility of manifestation of local features.

3 Gimbal with arbitrary outline of the meridian line

Boundary conditions of mathematical model under study are defined in the assumption that there is no transfer of flexural shell surface oscillation energy to the end surface and vice versa. This is observed in the hinged connection of these surfaces, which is assumed in the manuscript materials. In this case, simulation model of the shell and the flat end can be described as an infinite in length element – the shell and plates.

Initial conditions. Suppose the shell belongs to the curvilinear orthogonal coordinates α_1 and α_2 . They are regarded as lines of curvature with a radius R_1 and R_2 .

Denote Lame parameters of the middle surface of the π shell by A₁ and A₂. Then, adding inertial forces, we can use the shell equilibrium equations, which in expanded form are written as follows:

$$\begin{aligned} \frac{\partial A_2 T_1}{\partial \alpha_1} + \frac{1}{A_1} \frac{\partial A_1^2 S}{\partial \alpha_2} - \frac{\partial A_2}{\partial \alpha_1} T_2 + \frac{1}{R_1} \left(\frac{\partial A_2 M_1}{\partial \alpha_1} + \frac{1}{A_1} \frac{\partial A_1^2 H}{\partial \alpha_2} - \frac{\partial A_2}{\partial \alpha_1} M_2 \right) + \\ + \frac{\partial}{\partial \alpha_2} \left(\frac{A_1}{R_1} H \right) + \frac{1}{R_2} \frac{\partial A_1}{\partial \alpha_2} H = -A_1 A_2 q_1 + \rho A_1 A_2 h \frac{\partial^2 U_1}{\partial t^2} ; \end{aligned}$$

$$\begin{aligned} \frac{\partial \mathbf{A}_{1}\mathbf{T}_{2}}{\partial \alpha_{1}} + \frac{1}{\mathbf{A}_{2}} \frac{\partial \mathbf{A}_{2}^{2}\mathbf{S}}{\partial \alpha_{1}} - \frac{\partial \mathbf{A}_{1}}{\partial \alpha_{2}}\mathbf{T}_{1} + \frac{1}{\mathbf{R}_{2}} \left(\frac{\partial \mathbf{A}_{1}\mathbf{M}_{2}}{\partial \alpha_{2}} + \frac{1}{\mathbf{A}_{2}} \frac{\partial \mathbf{A}_{2}^{2}\mathbf{H}}{\partial \alpha_{1}} - \frac{\partial \mathbf{A}_{1}}{\partial \alpha_{2}}\mathbf{M}_{1} \right) + \\ + \frac{\partial}{\partial \alpha_{1}} \left(\frac{A_{2}}{R_{2}}H \right) + \frac{1}{R_{1}} \frac{\partial A_{2}}{\partial \alpha_{1}}H = -A_{1}A_{2}q_{2} + \rho A_{1}A_{2}h \frac{\partial^{2}U_{2}}{\partial t^{2}} ; \end{aligned}$$

$$\begin{aligned} \frac{T_1}{R_1} + \frac{T_2}{R_2} &- \frac{1}{A_1 A_2} \left\{ \frac{\partial}{\partial \alpha_1} \frac{1}{A_1} \left(\frac{\partial A_2 M_1}{\partial \alpha_1} + \frac{1}{A_1} \frac{\partial A_1^2 H}{\partial \alpha_2} - \frac{\partial A_2}{\partial \alpha_1} M_2 \right) + \right. \\ &+ \frac{\partial}{\partial \alpha_2} \frac{1}{A_2} \left(\frac{\partial A_1 M_2}{\partial \alpha_2} + \frac{1}{A_2} \frac{\partial A_2^2 H}{\partial \alpha_1} - \frac{\partial A_1}{\partial \alpha_2} M_1 \right) \right\} = q_n + \rho h \frac{\partial^2 W}{\partial t^2} \quad , (1) \\ & q_1 = p_1 + \frac{m_1}{R_1} \approx p_1 \; ; \qquad q_2 = p_2 + \frac{m_2}{R_2} \approx p_2 \; ; \end{aligned}$$

where

$$q_n = p_n + \frac{1}{A_1 A_2} \left(\frac{\partial A_2 m_1}{\partial \alpha_1} + \frac{\partial A_1 m_2}{\partial \alpha_2} \right) \approx p_n ,$$

as in most cases quantities m_i are of the hp order, so that by identifying q_i and p_i , the terms of order $\frac{h}{R}$ are thus dropped out in comparison with unity; T_1 , T_2 are normal forces, and S is tangential force; M1, M2 are bending moments; H is torque; ρ is density of shell material; h is shell thickness; u_i is elastic displacement of points of π surface towards the coordinate α_i .

In the form presented, equations (1) are inconvenient to use. Therefore, a series of transformations shall be done over them, after which they should be written in a form that is acceptable for integration.

In dimensionless form, the differential equations of a shell with an arbitrary outline of the meridian line are written as [18]

$$\frac{\partial^{2}U_{z}}{\partial z^{2}} - (1+2\nu)\xi'(z)\frac{\partial U_{z}}{\partial z} + \left[(1+\nu\mu)\xi'^{2}(z) - \nu\xi''(z)\right]U_{z} + \frac{1}{R(1+\zeta)} \times \left[\frac{1+\nu}{2} + \nu(1+\mu)\xi(z)\right]\frac{\partial^{2}U_{\phi}}{\partial z\partial\phi} - \frac{1}{R(1+\zeta)}\left\{-\left[(1+\nu)\mu + 3\mu^{2}\right]\xi'(z)W + (\mu+\nu)\frac{\partial W}{\partial z}\right\} = \frac{1-\nu^{2}}{Eh}\left[1+2\mu\xi(z)\right]\left(-q_{1}+\rho h\frac{\partial^{2}U_{z}}{\partial t^{2}}\right)_{z}$$

$$(2)$$

$$\frac{\partial^{2} U_{\phi}}{\partial s^{2}} + \frac{1}{2} (1+\nu) \Big[1 - (1+\mu) \xi(z) \Big] \frac{\partial^{2} U_{z}}{\partial z \partial s} - \frac{1}{2} (1-\nu) \Big[1 - 2(1+\mu) \xi(z) \Big] \frac{\partial^{2} U_{\phi}}{\partial z^{2}} - \frac{1}{2} (1-\nu) (1+\mu) \xi'(z) \frac{\partial U_{\phi}}{\partial z} - \frac{1}{2} (3-\nu) \xi'(z) \frac{\partial U_{z}}{\partial s} + \frac{1}{2} (1-\nu) \xi''(z) U_{\phi} - \frac{1}{R(1+\zeta)} \Big\{ 1 + \nu \mu - \Big[(1+\nu) \mu + 3\nu \mu^{2} \Big] \Big\} \\ \xi(z) + \Big[(1+3\nu) \mu^{2} + \frac{15}{2} \nu \mu^{3} \Big] \xi^{2}(z) \Big\} \frac{\partial W}{\partial s} = \\ = \Big[1 - \xi(z) \Big]^{2} \Big(-q_{2} + \rho h \frac{\partial^{2} U_{\phi}}{\partial t^{2}} \Big) \frac{1-\nu^{2}}{Eh} ;$$

$$(3)$$

$$\frac{\partial^{2}}{\partial z^{2}} \Big\{ - \Big[1 - (1+2\mu) \xi(z) \Big] \frac{\partial^{2} W}{\partial z^{2}} + (\nu+\mu) \xi'(z) \frac{\partial W}{\partial z} - \nu \frac{\partial^{2} W}{\partial s^{2}} \Big\} - \frac{\partial}{\partial z} \Big\{ (1-\nu) \Big[2 + \xi(z) \Big] \frac{\partial^{3} W}{\partial z \partial s^{2}} + (\nu+\mu) \xi'(z) \frac{\partial W}{\partial z} - \nu \frac{\partial^{2} W}{\partial s^{2}} \Big\} - \frac{\partial}{\partial z} \Big\{ (1-\nu) \Big[2 + \xi(z) \Big] \frac{\partial^{3} W}{\partial z \partial s^{2}} + (\nu+\mu) \xi'(z) \frac{\partial W}{\partial z \partial s} \Big\} \Big\}$$

$$\left[1+2(1-\nu)\xi'(z)\right]\frac{\partial^2 W}{\partial s^2}+\nu\xi'(z)\frac{\partial^2 W}{\partial z^2}+\frac{1-\nu}{R(1+\zeta)}\times$$

$$\times \left[1 + \mu(\mu - 2)\xi(z)\right] \frac{\partial^{2}U_{\phi}}{\partial z \partial s} \right] - \frac{\partial^{4}W}{\partial s^{4}} - \nu \frac{\partial^{4}W}{\partial z^{2} \partial s^{2}} + \mu \xi'(z) \frac{\partial^{3}W}{\partial z^{3}} + \left[1 + \mu(1 - \nu)\right] \xi'(z) \frac{\partial^{3}W}{\partial z \partial s^{2}} - \mu(\nu + \mu) \xi'^{2}(z) \frac{\partial^{2}W}{\partial z^{2}} - (\nu + \mu) \mu \xi'(z) \xi''(z) \frac{\partial W}{\partial z} - \frac{1}{R(1 + \zeta)} \left[1 + \left(2 + \mu^{2}\right)\xi(z)\right] \frac{\partial^{3}U_{\phi}}{\partial s^{3}} - \frac{\nu \mu}{R(1 + \zeta)} \frac{\partial^{3}U_{z}}{\partial z \partial s^{2}} + \frac{\mu^{2}\xi'(z)}{R(1 + \zeta)} \frac{\partial^{2}U_{z}}{\partial z^{2}} + \frac{\mu(1 - \nu)}{R(1 + \zeta)} \xi'(z) \frac{\partial^{2}U_{\phi}}{\partial z \partial s} + \frac{1}{R(1 + \zeta)} \frac{\partial U_{\phi}}{\partial z} - \frac{1 + \mu\nu}{R(1 + \zeta)} \frac{\partial U_{\phi}}{\partial s} - \frac{1 + \mu\nu}{R(1 + \zeta)} \xi'(z) U_{z} \right] = -\frac{1}{D} \left[1 - (1 - \mu)\xi(z)\right] \left(q_{3} + \rho h \frac{\partial^{2}W}{\partial t^{2}}\right), \qquad (4)$$

where U_z , U_{φ} , W are the elastic displacements of the shell surface along the generatrix, along the parallel, and in the transverse plane; h is the shell thickness; ρ is the density of the material; E is the Young modulus; ν is the Poisson ratio; $R = f(0) = f(\ell) = const$ is the radius at the edges; ℓ is the shell length; r = f(z) is the distance from the axis of rotation to the point M; f(z) is the rotation curve (meridian line); $\frac{\partial}{\partial S} = \frac{1}{R(1+\zeta)} \frac{\partial}{\partial \varphi}$;

$$\eta = \frac{R}{\ell} \quad \zeta = \frac{\delta}{R} < 1; \quad \xi = \frac{\delta}{\ell} < 1; \quad \mu = 8\zeta (1+\zeta) \eta^2; \quad 2\mu \ll 1; \quad \delta \text{ is the ascent of the}$$

meridian line; provided that $\delta \to 0$, of course, we will have $\xi \to 0$ and $\mu \to 0$; ω_0 is the natural frequency.

Equations (2) provide an opportunity to further carry out a comparative analysis of the four types of float-shell gimbal: with an arbitrary outline of the meridian line; convex (Figure 1, *a*.); concave or *catenoid*, from Lat. *catena* (Fig. 1, *b*), and (iv) circular cylinder (Fig. 1, *c*). In all the cases, the curve f(z) that generates the shell is assumed to be symmetrical relative to the line *CM*.



Fig. 1. Float shell of special form: *a*) a convex shell of rotation; *b*) a concave shell; *c*) a circular cylinder

The coordinate systems $C_1 z_1 r_1$ and Ozr are related by the analytical expressions

$$r = r_1 + R; \quad z = z_1 + \frac{1}{2}\ell$$

In the coordinate system $C_1 z_1 r_1$ the meridian line is given by

$$r_1 = \pm f(z_1),$$

and the "+" and "-" signs refer to the schemes shown in Fig. 1, *a* and Fig. 1, *b*, respectively. Specify the class of the curves $f_1(z_1)$ for the implementation of the required shell. First of all, the following conditions must be necessarily met:

$$f_1(-z_1) = f_1(z_1);$$

$$f_1\left(\pm\frac{1}{2}\ell\right) = 0.$$

The function $\left[+f_1(z_1)\right]$ is considered to be strictly convex, and the function $\left[-f_1(z_1)\right]$ strictly concave. The function $f_1(z_1)$ is descending if $z_1 \in \left(0; \frac{\ell}{2}\right)$ (Fig. 1, *a*) and ascending

if
$$z_1 \in \left(0; \frac{\ell}{2}\right)$$
 (Fig. 1, *b*). Obviously,

$$f_1(-z_1)=f_1(z_1).$$

It follows from equations (1), (2), and (3) that the elastic displacement of gimbal surface in all three directions, influence each other to a certain extent. The degree of this influence will be determined later on.

For the convenience of integration we should turn to dimensionless coefficients. Considering that

$$\begin{aligned} \zeta &= \frac{\delta}{R}; \qquad \eta = \frac{R}{\ell}; \qquad \mu = 8\zeta \left(1 + \zeta\right) \eta^2; \qquad \qquad \xi(z) = \frac{\zeta}{1 + \zeta} \left(\frac{2z}{\ell}\right)^2; \\ \xi'(z) &= \frac{4\delta}{(R+\delta)\ell} \left(\frac{2z}{\ell} - 1\right); \ \xi''(z) = \frac{8\delta}{(R+\delta)\ell^2}, \end{aligned}$$

in the case of a circular cylinder we have: $\delta = 0$; $\zeta = 0$; $\mu = 0$; $\xi(z) = 0$.

So, we introduce the following notation:

$$\frac{z}{\ell} = \overline{z}; \frac{U_z}{h} = \overline{U}_z; \frac{U_{\varphi}}{h} = \overline{U}_{\varphi}; \frac{W}{h} = \overline{W}; \ \omega_0 t = \overline{t} \ . \tag{4}$$

In order to simplify further discussion, we omit the overbar. Neglecting the small terms, the equation of the float shell gimbal with an arbitrary outline of the meridian line can be finally written as:

$$\frac{\partial^{2}U_{z}}{\partial z^{2}} - a_{1}(2z-1)\frac{\partial U_{z}}{\partial z} - a_{2}U_{z} + a_{3}\frac{\partial^{2}U_{\varphi}}{\partial z\partial\varphi} - a_{4}\frac{\partial W}{\partial z} = \\ = \left[1 + \alpha_{1}(2z-1)^{2}\right]\left(-q_{1}^{*} + \alpha^{*2}\frac{\partial^{2}U_{z}}{\partial t^{2}}\right);$$
(5)

$$\frac{\partial^2 U_{\phi}}{\partial \phi^2} + b_1 \left[1 - \beta_1 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z \partial \phi} - b_2 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_{\phi}}{\partial z^2} - b_3 (2z-1) \frac{\partial U_{\phi}}{\partial z} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_{\phi}}{\partial z} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_{\phi}}{\partial z} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2 (2z-1)^2 \right] \frac{\partial^2 U_z}{\partial z^2} - b_4 \left[1 - \beta_2$$

$$-b_4(2z-1)\frac{\partial U_z}{\partial \varphi} + b_5 U_{\varphi} - b_6 \frac{\partial W}{\partial \varphi} = \left[1 - \beta_3 \left(2z-1\right)^2\right] \left(-q_2^* + \beta^{*2} \frac{\partial^2 U_{\varphi}}{\partial t^2}\right); \quad (6)$$

$$\begin{bmatrix} -1 + \beta_4 (2z-1)^2 \end{bmatrix} \frac{\partial^4 W}{\partial z^4} - c_1 \frac{\partial^4 W}{\partial z^2 \partial \varphi^2} - c_2 \frac{\partial^4 W}{\partial \varphi^4} + c_3 (2z-1) \frac{\partial^3 W}{\partial z^3} - c_4 \frac{\partial^3 W}{\partial z \partial \varphi^2} + c_5 \frac{\partial^2 W}{\partial z^2} - c_6 \frac{\partial^2 W}{\partial \varphi^2} - c_7 (2z-1) \frac{\partial W}{\partial z} - c_8 \frac{\partial^3 U_{\varphi}}{\partial \varphi^3} - c_9 \frac{\partial^3 U_{\varphi}}{\partial z^2 \partial \varphi} - c_{10} \frac{\partial^3 U_z}{\partial z \partial \varphi^2} + c_{11} (2z-1) \frac{\partial^2 U_z}{\partial z^2} + c_{12} (2z-1) \frac{\partial^2 U_{\varphi}}{\partial z \partial \varphi} + c_{13} \frac{\partial U_z}{\partial z} + c_{14} \frac{\partial U_{\varphi}}{\partial \varphi} - c_{15} (2z-1) U_z =$$

$$= \left[1 - \beta_5 \left(2z - 1\right)\right] \left(q_3^* + \gamma^{*2} \frac{\partial^2 W}{\partial t^2}\right),\tag{7}$$

where

$$a_{1} = 4(1+2\nu)\frac{\delta}{R(1+\zeta)}; \quad a_{2} = 8\nu\frac{\delta}{R(1+\zeta)}; \quad a_{3} = \frac{1}{2}\frac{1+\nu}{1+\zeta}\frac{l}{R}; \quad a_{4} = \frac{\nu+\mu}{1+\zeta}\frac{h}{R};$$

$$\begin{split} q_1^* &= \left(1 - v^2\right) \frac{l^2}{E h^2} q_1; \quad \alpha^{*2} = \left(1 - v^2\right) \frac{\rho \omega_0^2 l^2}{E}; \quad \alpha_1 = 2 \mu \frac{\delta}{R(1 + \zeta)}. \\ b_1 &= \frac{1}{2} (1 + v) (1 + \zeta) \left(\frac{R}{l}\right); \quad b_4 = 2(3 - v) \frac{\delta}{l}; \\ b_2 &= \frac{1}{2} (1 - v) (1 + \zeta)^2 \left(\frac{R}{l}\right)^2; \quad b_5 = 4(1 - v) (1 + \zeta) \frac{\delta R}{l^2}; \\ b_3 &= 2(1 - v) (1 + \mu) (1 + \zeta) \frac{\delta R}{l^2}; \quad b_6 = 1 + v \mu; \\ \beta_1 &= \frac{1 + \mu}{1 + \zeta} \frac{\delta}{R}; \quad \beta_2 = 2 \frac{1 + \mu}{1 + \zeta} \frac{\delta}{R}; \quad \beta_3 = \frac{\delta}{R(1 + \zeta)}; \quad \beta_2 = 2\beta_1; \\ q_2^* &= \left(1 - v^2\right) \frac{R^2 (1 + \zeta)^2}{E h^2} q_2; \quad \beta^{*2} = \left(1 - v^2\right) \frac{\rho \omega_0^2}{E} R^2 (1 + \zeta)^2; \quad \beta_4 = \frac{1 + 2\mu}{1 + \zeta} \left(\frac{\delta}{R}\right); \\ c_1 &= \frac{2}{(1 + \zeta)^2} \left(\frac{l}{R}\right)^2; c_2 = \frac{1}{(1 + \zeta)^4} \left(\frac{l}{R}\right)^4; c_3 = 8 \frac{1 + 3\mu}{1 + \zeta} \left(\frac{\delta}{R}\right); \\ c_5 &= 8 \frac{(1 + v + 4\mu)}{1 + \zeta} \left(\frac{\delta}{R}\right); \quad c_6 = 16 \frac{(1 - v)}{(1 + \zeta)^3} \left(\frac{\delta}{R}\right) \left(\frac{l}{R}\right)^2; c_7 = \frac{32\mu(v + \mu)}{(1 + \zeta)^2} \left(\frac{\delta}{R}\right)^2; \\ c_8 &= \frac{1}{(1 + \zeta)^4} \left(\frac{l}{R}\right)^4; c_9 = \frac{1 - v}{(1 + \zeta)^2} \left(\frac{l}{R}\right)^2; c_{10} = \frac{v\mu}{(1 + \zeta)^3} \left(\frac{l}{R}\right)^3; c_{11} = \frac{4\mu^2}{(1 + \zeta)^2} \left(\frac{\delta}{R}\right) \left(\frac{l}{R}; \\ c_{12} &= \frac{4\mu (1 - v)(3 - \mu)}{(1 + \zeta)^3} \left(\frac{\delta}{R}\right) \left(\frac{l}{R}\right)^2; c_{13} = 12(v + \mu) \frac{l^3}{Rh^2}; c_{14} = 12 \frac{1 + v\mu}{(1 + \zeta)^2} \frac{l^4}{Rh^2}; \\ c_{15} &= \frac{4(1 + v\mu)}{(1 + \zeta)^2} \left(\frac{\delta}{R}\right) \frac{12l^3}{Rh^2}; \quad \gamma^{*2} = 12 \left(1 - v^2\right) \left(\frac{l}{h}\right)^4 \frac{\rho h \omega_0^2}{E}; \beta_5 = \frac{1 - \mu}{1 + \zeta} \left(\frac{\delta}{R}\right). \end{split}$$

4. Conclusions

The constructed analytical model of elastic-stressed state of a floated gyroscope gimbal provides the following:

Conduct a qualitative and quantitative analyses of the elastic displacements of the shell surface of the float in three directions - along the meridian lines, along the perimeter of the frame, and in the radial direction.

The constructed analytical model enables one not only to analyze the structure of the coordinate functions as a function of time and geometrical parameters of the surface, but also to solve the optimization problem of the gyroscope gimbal for the minimum surface acoustic vibration generated in the material.

By determining the value of sound transmission in the shell can clarify the causes and development of technological risks hypersonic aircraft in time, in particular the manifestation of the effect of "acoustic transparency" of the object as a result of the wave match.

The obtained values of the coordinate functions allow one to find the values of Euler inertial forces acting on the surface float under flight conditions, determine the value of additional errors of floated gyroscope to determine the magnitude of errors due to direct action of moments of couple of Euler inertial forces in accordance with the Rezal theorem, when velocity of vector end of gyroscope's angular moment acquires velocity directed in the same way as the vector of perturbing moment of couple of Euler inertial forces and movable part of the device would rotate relative to the output axis as long as there is parallelism between them. Additional error of a gyroscope, besides the mentioned one, would contain one more component, which is caused by the indirect action of perturbing moments of Euler inertial forces, more precisely,

Coriolis inertia forces due to elastic displacement velocities of float surface $(\dot{U}_{\varphi}, \dot{U}_z, \dot{W})$ on the aircraft body rotating relative to the cross axes of gyroscope suspension $(\omega_y, \omega_x, \omega_z)$. This

action is carried out through gyroscopic moments.

Numerical values of additional gyro errors in the operating conditions can thus be determined, as said above, on the basis of the known kinematics of aircraft body in the form of three angular velocities with respect to cross axes of the float and three velocities

 $\dot{U}_{\varphi}(t), \dot{U}_{z}(t), \dot{W}(t)$ of elastic deformation (surface coordinate functions), $(U_{\varphi}(t), U_{z}(t), W(t))$ of float surface under the influence of sound waves.

The obtained values of the coordinate functions of the floated gyroscope gimbal provide the possibility of manifestation, in the operating conditions, of the effect of selectivity of the angular motion of an aircraft, components of acoustic vibration of its surface in the form of the formation of a systematic measurement error.

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