Slovak University of Technology in Bratislava
Faculty of Mechanical Engineering
17th Conference on Applied Mathematics **APLIMAT 2018**

Proceedings

AUGMENTED REALITY IN MATHEMATICS EDUCATION FOR PRE-SERVICE TEACHERS IN PRIMARY LEVEL

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Abstract. New technologies with modern method of teaching must be implemented in the university studies. Important part is teacher training study. These technologies make the study more attractive and also it brings bigger motivation an understanding of notions by students. We present in our contribution augmented reality technology as a part of using of mobile technologies in mathematics education of future primary school teachers.

Keywords: Augmented reality, constructivism, constructionism, solids, digital technology, teacher training of pre-service teachers at primary level

Mathematics Subject Classification: Primary 97C30, 97D40, 97G40; Secondary 97U60, 97U70

1 Mobile Technologies and Virtual Reality in Education

The technological and social changes that accompany the current ubiquitous use of mobile devices brings the question, how it is possible to implement these technologies into the educational process. We can find several approaches to defining mobile learning in professional literature: Mobile learning refers to the use of mobile or wireless devices for the purpose of learning while on the move. Typical examples of the devices used for mobile learning include cell phones, smartphones, palmtops, and handheld computers; tablet PCs, laptops, and personal media players can also fall within this scope (see [6]). Taylor in [17] has defined mobile learning as "learning mediated by mobile devices, or mobility of learners (regardless of their devices), or mobility of content/resources in the sense that it can be accessed from anywhere".

Sharples in [14] has made a good summary on different views of defining mobile learning. Current perspectives on mobile learning generally fall into the following four broad categories:

1. Technocentric. This perspective dominates the literature. Here mobile learning is viewed as learning using a mobile device, such as a PDA, mobile phone, iPod, PlayStation Portable etc.
2. Relationship to e-learnig. This perspective characterises mobile learning as an extension of e-learning.
3. Augmenting formal education.
4. Learner-centered.

What is needed is clarity: in agreement with Traxler in [18], the technocentric/e-learning based definitions only seek to place "mobile learning somewhere on eLearning's spectrum of portability". Augmenting formal education. In the mobile learning literature, formal education is often characterised as face-to-face teaching, or more specifically, as a stereotypical lecture. However, it is not at all clear that this perspective is wholly correct. Forms of distance education (for example, distance correspondence) have existed for over 100 years, leading to the questions regarding the place of mobile learning in relation to all forms of "traditional" learning, not only the classroom.

2 Augmented Reality in Education for pre-service teachers in primary level

The term Augmented Reality (AR) was created around the year 1990 and it translates the integration of virtual images in the real world, i.e. the reality is augmented of virtual elements. The integration of such images is made by the use of Information and Communication Technologies (ICT), through a mobile device with a camera (computer, tablet, mobile phone with android or iOS operating systems) which allows the access to the available contents with AR. Furthermore, the development of such contents encourages higher learning autonomy and the use of systems that support mobile-learning. Besides, the exploration of ICT by the students can promote collaboration, innovation and creativity skills. One characteristic that AR applications offer is the integration and interaction between the real and the virtual, allowing a huge versatility and creativity in applications (see Coimbra, Cardoso, Mateus in [4]).

The basic goal of an AR system is to enhance the users perception of and interaction with the real world through supplementing the real world with 3D virtual objects that appear to coexist in the same space as the real world. Many recent papers broaden the definition of AR beyond this vision, but in the spirit of the original survey we define AR systems to share the following properties (see Azuma et al. in [2]):
1) Blends real and virtual, in a real environment
2) Real-time interactive
3) Registered in 3D
Augmented reality – AR is the supplementation of the reality perceived by the user with virtual elements. The use of mobile AR technology applications brings advantage that the work of educators can be made more effective, and in addition it enables pupils to become actively involved in the education process. Besides all this it makes studying both in the lesson and outside the lesson more attractive and more experiential. It is also suitable for arousing interest in subject matters for which the generation socialising on TV, on the Internet and on the Facebook portal is less receptive.

3 Constructivism and constructionism in the teacher training at university

The inductive (constructivist) approach in teaching comparing to deductive approach is characterized by distinctly different characteristics from the deductive approach, cognitive development and the learning process are defined as follows (see Kostrub [10]):

- to base always on the achieved level of learner's development,
- to provide meaningful learning,
- to enable learners to realize their own meaningful learning process
- to have influence on learners in that way that they will modify their own knowledge schemes,
- to create and maintain rich relationship among new knowledge and already existing knowledge schemes.

The constructivist theory of learning assumes that each person creates himself (constructs) his own knowledge of the world in which he lives. Constructivism tries to overcome the transmissiveness of traditional teaching - the transfer of "teacher's" knowledge to the student. It deals with learning with understanding (see Stoffová [15]).

According Ackermann [1] the word constructionism is a mnemonic for two aspects of the theory of science education. From constructivist theories of psychology, we take a view of learning as a reconstruction rather than as a transmission of knowledge. Then we extend the idea of manipulative materials to the idea that learning is most effective when part of an activity the students´ experiences as constructing a meaningful product.

According Papert, Harel [12] the idea of the constructionism is thinking of it as “learning-by-making”. It is nowadays in mathematics and informatics education used LEGO set and Logo program in primary and upper secondary level, which brings possibility for children to make some “cybernetic” activities, thus becoming as a part of the lives of young children as playing with toys and dolls. Future teachers for primary level need to have possibility of an experience with this play.

The opposite approach to constructionism is instructionism. There is typicall for this kind of teaching, that teacher makes instructions for children and they have small space for their own activity and personal way of thinking. Instructionism vs. conctructivism looks like a split about two strategies for education: two ways of thinking about transmission of knowledge. But behind this there is a split that goes beyond the acquisition of knowledge to touch on the nature of knowledge and nature of knowing.

An increasing number of researchers have come to the following view: The knowledge is essentially “situated” and it doesn’t not be detached from the situations in which it is constructed and actualized. This growing interest in the idea of situated knowledge, or knowledge as it lives and grows in context, has lead many researchers to look closely at individual people’s ways of knowing, or relating. Constructionism as an educational theory has many applications in the digital school environment (see Sabelli [13]).

The root for constructionism was constructivism. Constructivist instructional design according Kalaš in [8] aims to provide generative mental constructions embedded in relevant learning environments that facilitate knowledge construction by learners. Constructivists

approach has many applications in different areas. A good example in the field of languages is possible to find in Šipošová [16] and in the field of science education for disable children in Vančová, Sulovská [19]. We used in our following research these pedagogical theories, because students – pre-service teachers for primary level works with different applications used augmented reality. These applications were tools for better understanding of the base mathematical notions. In the field of the functional thinking see Fulier, Ďuriš, Frantová in [7].

4 Using of applications with augmented reality for teaching of base properties of solids

We conducted the research in the group of 27 students of the first year of the bachelor study in the teacher training program for future teachers in primary level. The pedagogical experiment was realized in November 2017 at Faculty of Education of the Comenius University in Bratislava.

Our research focused on analysing the use of mobile technology and manipulation activities in teaching mathematics based on constructivist and constructionist concepts. We looked into the problem of how to incorporate the use of smartphones and tablets in education. We believe that when these two methods are combined, the students´ understanding is deeper, their motivation is greater, and, last but not least, their creativity is strongly supported.

There was installed the AR application "Augmented Polyhedra" with cards on tablets and smartphones (see Fig. 1). This application is free of charge and the markers can be also printed free of charge. It works in a similar way as the AR Flashcards Shapes. It is good for increasing users' motivation. Site of this application is: http://mirage.ticedu.fr/?p=2635
The task for the students was to find under the every card the solid, his properties, volume and surface.

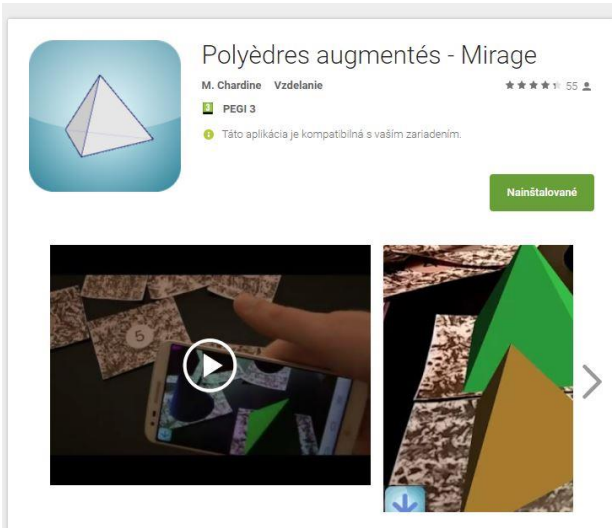

Fig. 1. Application "Augmented polyhedra".

Some students used the lineal, they tried to measure some lengths of different solids (see Fig. 2). They used it for calculating the surface and volume.

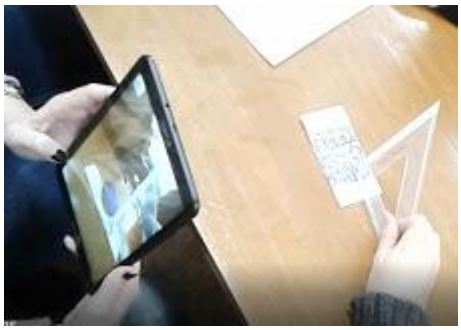

Fig. 2. Student working with lineal.

Some solids in the apllication were prisms. Students tried to make network of the prism with the geometric manipulative construction product Polydron (see http://www.polydron.co.uk). This network also helped them by searching of the volume and area of the prism (see Fig. 3).

Fig. 3. Student working with Polydron and AR application.

Students worked in the groups of 2 – 4 persons. During the lesson using the constructive method. In the course of the qualitative research, we would have liked to observe the effect of the AR Polyhedron application on the learning in a constructivist environment.

At the beginning of the activity, the students select randomly a few cards with a code that the application recognizes. Using the Polyhedron application on the tablet or on the smartphone, they can use the augmented reality to study the geometric shape that appears in three dimensions.

During the research, the task of the students was to find the name of the solids, his base properties and find his volume and area. Every group obtains 10 cards. They work with these 10 tasks for 90 minutes. After the students asked if they could use the Internet, the teacher allowed to use it in the course of the problem solving.

A particularly interesting part of the research was to see how students tried to find those geometric shapes on the Internet which were unknown for them. They were looking for pictures to identify the shape. Occasionally, only the English name was found in case of truncated pyramid and truncated cones. Some of them find on the Internet the expressions for volume and area of the solid surface and also the art of dimensions needed. The students as a

homework prepare presentations about solids, which they found through the cards with AR application.

Next block of teaching was oriented for presentation of the students' results. This block was one week after the first block. The students work at home with the material prepared during the first block. Every group choose 2 solids, which was presented for the whole class of students. Some students find wrong information on Internet and other students make corrections. If they cannot find the right answer, it was the case for the intervention of the teacher.

As a conclusion, students were motivated by the new method, they cooperated very well and learning was constructive. They gained new knowledge, collaborated, discussed their observations, argued and reasoned. The difficulties of problem solving were well managed, they had logical ideas. Our qualitative pedagogical research relies on the description of the teacher's observations and on the video recording of the students' work.

5 Conclusions

Our observations from the work of students we can conclude in the following table:

CATEGORY	SUBCATEGORY	EXPRESSIONS AND ACTIVITIES
1. Students´ mutual learning - constructionist approach	1. Students support each other	We are going to …, so that we can have, we will make, we have made, we have named, we have built, we are talking about, we are going to write, we are making a presentation
	2. Mutual learning helps them to understand terms and relations	We are creating together, to us, we will put, we are, let´s talk together, let´s take – so that we can, we are solving together
	3. They do not collaborate with each other	We were not able to explain it to each other, I am
2. Influence of mobile technology	1. It helps them when they can make several attempts thanks to the technology	Several tries, explanations, I have several attempts, several solutions, I can correct myself, with a simple click
	2. Mobile technology gives them feedback	We are verifying, she evaluated it on his/her own, so that it would be correct, I tried
	3. Thanks to mobile technology, they become aware of selected mathematical expressions and relations	We are thinking over, we are considering, we are thinking about, I can remember it
3. Manipulation activities	1. Discovery of new technologies	We are drawing, we are using a string, Polydron, cubes, we prepare a network, we calculate now the surface or volume
	2. Students are more motivated	We enjoy it, we are finding out, now we know, I am going to try it, we want to work
	3. Better awareness of mathematical knowledge and relations	A square, a rectangular, a triangle, a cube, construction, a diagram, a table, a network, expression for the volume or surface

Tab. 1. Categories of students activities.

The constructionist design of teaching involves constructionism, design, architecture. It is a conceptual art shared jointly and mutually by the students and their teachers as a result of the discursive and narrative essence of teaching in which they make proposals, discuss, describe, present, approve, evaluate, judge, agree, create and rework the non-material form of their products into a material form and vice versa (see Čavojský [5]). This takes place in the form of intentional but indirect student and teacher participation in teaching activities, controlled by the teacher, in which, however, the teacher acts as a consultant when he/she is invited by the students to comment on their ideas, while avoiding any reference to mistakes. Constructionist teaching takes place through didactically considered but conceptually open teaching activities, and through discourse (controlled argumentation, handling facts) in the form of individual as well as group exploration (learning groups), thanks to which common knowledge and understanding is established.

The students apply corresponding cognitive tools, such as thinking and speech in connection with the cognitive prostheses available in their surroundings. Their minds are thus formed in a different way, which means that digital technology delimits and structures cognitive schemes in a way that was unfamiliar to students of previous decades. The confidence arising from the availability of conventional information is pertinent; nevertheless, on the basis of such information, a conceptually reflective teacher supports the formation of unconventional knowledge which can be used for other teaching contexts. The students are able to perform both specific and unspecific transfers, which is a precondition for the teacher to lead the teaching process on the basis of the principles of constructionism (see also Kónya in [9]).

The goal of the activities for students from mathematical point of view was not to calculate the exact value of the surface area and volume of the solids, but to find the name and basic properties, network of the solid and all dimensions needed for the calculation of the surface area and volume of the solid. Students find that it was impossible to measure punctually dimensions of solids visible through AR application, because if they change the distance between smartphone, tablet and the card the solids change their dimensions. Only proportion of dimensions was constant. The advantage of this teaching was, that students must find which kind of dimensions they need for calculating of volume and surface area of solids. They use many modern sources and devices of information such AR application, Internet, smartphone, tablet, and computer, textbook. Group work of students' trains their collaborative abilities, inquiry-based teaching methods (see in Kovács [11]) which they can use in their future pedagogical practice. Similar collaborative activities is possible to find in the using of some educational software (see Beňačka, Čeretková in [3]).

Acknowledgment

The paper was written with the support of the grant KEGA 012UK-4/2018 "The Concept of Construcionism and Augmented Reality in the Field of the Natural and Technical Sciences of the Primary Education (CEPENSAR)".

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