VISUALIZATION OF MATHEMATICAL CONCEPTS IN TEACHING LINEAR AND VECTOR ALGEBRA TO FIRST-YEAR ENGINEERING STUDENTS

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Abstract. A method based on the use of dynamical features of PowerPoint to teach linear and vector algebra to first-year students at the Faculty of Mechanical Engineering (FME) at the Riga Technical University (RTU) is proposed in the paper. Power Point has built-in animation features which can be quite helpful for students. This approach is used to explain such topics as matrix multiplication, computation of determinants, vector coordinates in three-dimensional space. The use of dynamical features of PowerPoint is positively evaluated by students.

Keywords: visualization technique, PowerPoint, higher mathematics

Mathematics Subject Classification: Primary 97B40, 97D40; Secondary 97U70.

1 Introduction

Teaching mathematics to first-year engineering students is a challenging task nowadays. In the present paper some approaches to teaching linear and vector algebra to first-year students at the Faculty of Mechanical Engineering (FME) at the Riga Technical University (RTU) are discussed. In order to assess the knowledge of high school mathematics all first-year students of RTU complete a test during the first tutorial session at RTU. The test is given during the last five years. The results [1] show a rather poor knowledge of mathematics among many first-year students (note that only a minimal passing grade in mathematics is required in order to be admitted to RTU). Starting from the year 2016, RTU is even forced to offer an additional (two-credit point) course on basics of high school mathematics for those students who have low or failing grades on the test.

In addition, only two or three semesters (depending on students' specialization) are devoted to linear algebra, vector calculus, analytic geometry and calculus at the FME of RTU. Thus, the challenge for an instructor is to deliver the material required for other engineering courses in a relatively short time under the assumption that many students do not have a strong background in mathematics.

In the present paper, the authors share their experience in teaching first year students at the FME of RTU. The major emphasis of the paper is given to visualization of mathematical concepts (using software) in explaining such topics as linear algebra and vector calculus. Active use of animation features of PowerPoint allows the instructor to save time (no need to draw figures of the board). In addition, students can work at their own pace at home to analyze the material covered during the lectures since the slides are available on the Moodle platform (called ORTUS at RTU). Moreover, the instructor can control the speed at which the drawing appear on the screen during the lecture and give comments while students are following the presentation. Lecture notes developed by the instructor are also available on ORTUS so that students can use the combination of the learning tools to understand the material better.

There are different approaches to visualization in the literature. Sometimes professors use hand-made visual aids to explain the concepts. Two pieces of plexiglas with wooden dowels are used in [2] to visualize two planes with normal vectors. Modeling clay is used in [2] to clarify the concept of a function of two variables. Development of computer graphics generated interest in visualization tools for teaching and learning mathematics. One of the first sources in this area is the collection of papers addressing the role of visualization in three-dimensional geometry, calculus and other fields [3]. Nowadays the use of special software such as Autograph [4] or Geogebra [5] becomes quite popular. There are examples where special software is developed by professors (see [6]). Video presentations are also gaining popularity [7]. Applications of special dynamical software in teaching and learning mathematics are discussed in the recent book [8]. In the present paper the use of PowerPoint dynamical features is proposed to visualize some concepts in linear and vector algebra which are difficult to explain to students (especially if their knowledge of high school mathematics is quite poor).

2 Organization of the course

There are no separate courses on linear algebra, analytic geometry, vector calculus and calculus at RTU. All these topics are taught in one course "Higher mathematics". The first semester covers linear algebra, analytic geometry, vector algebra and calculus of a function of one and two variables. This is a five-credit point course with 3x45 min. of lectures and 3x45 min. of tutorials per week. The lectures are given to large classes (typically, to 100 students or more), but the tutorial sessions are given to smaller groups (about 25-30 students in each group).

The Department of Engineering Mathematics (DEM) of RTU uses the following grading system for many years: 50% of the total grade is based on the midterm and final exams, 40% - on tests given during the semester and 10% - on homework assignments. There are 7 tests (given during tutorial sessions) and 7 homework assignments. Two homework assignments are performed in the form of online tests (based on Moodle system), two homework assignments are graded by the instructor, and the remaining three homework assignments are partially performed in the form of online tests and partially are graded by the instructor. After each test students have a chance to retake it once. About 25% of all students use this option. The marking is done by the instructor and requires a lot of time. The DEM is a service

department (there are no graduate students at the department who can help with marking). Assuming that the average time needed to grade one homework assignment (or a test) is 5 minutes per student (this is quite an optimistic estimate) the instructor devotes about 8 hours per week to grading only (this estimate is based on a class of 100 students). However, instructors have to do it, otherwise the results would be quite poor (a lot of students would fail the course). Thus, satisfactory results (in terms of gained knowledge) can be achieved only at the increase of the teaching load for the instructor. In summary, a closer collaboration between high school teachers and university professors is required in order to reduce dropout rate of first year students and raise the level of mathematical culture among the students who passed the course.

3 The use of dynamical features of PowerPoint

As the results of the survey among students have shown (see the next section) many first-year students of the FME at RTU use lecture notes and presentation slides as the primary sources of their gained knowledge in higher mathematics course. Knowing this trend, the instructors decided to use a combination of PowerPoint presentations and "classical blackboard" teaching to deliver lectures. PowerPoint has built-in animation features which can be quite helpful for students.

Let us consider how some topics covered in the first semester can be delivered with the help of PowerPoint. The first topic discussed below is matrix multiplication. The rule of matrix multiplication is dynamically explained as shown in Fig. 1. At first, the definition of matrix multiplication is presented (Fig. 1(a)). It is emphasized at the next step that the product exists if and only if the number of rows of the first matrix is equal to the number of columns of the second matrix (Fig. 1(b)). Next, a simple example is shown. The elements of two matrices *A* and *B* are displayed (Fig. 1(c)). It is checked that the product exists (Fig. 1(d)). Then the arrows appear on the screen showing the rows of matrix *A* and columns of matrix *B* (Fig. 1(e)). In addition, the 2x2 matrix *C* is also shown. Then the first row of *A* and second column of *B* is encircled with red and blue colors (Fig. 1(f)). Also, the product of Row 1 of *A* and Column 1 of *B* is explicitly shown. Similar procedure is repeated for other rows of *A* and columns of *B*. Note that all the information displayed on Figs. 1(a)-(g) appears on one slide when the example is solved. The instructor carefully measures the amount of time needed to explain a certain step in the calculation. The next feature on the screen appears only after time delay specified by the instructor. Thus, the process of matrix multiplication is visualized by students and allowed them to understand the concept better.

Another example of visualization is related to computation of determinants. First, the determinant of order three using Sarrus' rule is computed by visualization: the terms taken with the "+" sign are shown first and the terms taken with the "-" sign are shown later. Computation of determinants of higher order is also illustrated with the help of visualization. Similar visualization technique is used in the course to represent a system of linear algebraic equations in matrix form.

Fig. 1(a). The initial configuration.

Fig. 1(b). The next step.

Example. Given the matrices $A_{2 \cdot 3} = \begin{pmatrix} 1 & 3 & 8 \\ 2 & 1 & 0 \end{pmatrix}; B_{3 \cdot 2} = \begin{pmatrix} 5 & 4 \\ 3 & 1 \\ 2 & 0 \end{pmatrix}$ Find the product of the two matrices (if it exists).

Fig. 1(c). Example.

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- $A_{2 \cdot 3} \cdot B_{3 \cdot 2} = C_{2 \cdot 2} \implies$ the product exists

Fig. 1 (d). Check whether the product exists.

Example. Given the matrices $A_{2 \cdot 3} = \begin{pmatrix} 1 & 3 & 8 \\ 2 & 1 & 0 \end{pmatrix}; B_{3 \cdot 2} = \begin{pmatrix} 5 & 4 \\ 3 & 1 \\ 2 & 0 \end{pmatrix}$ Find the product of the two matrices (if it exists).

- $A_{2 \cdot 3} \cdot B_{3 \cdot 2} = C_{2 \cdot 2} \implies$ the product exists

- $\begin{pmatrix} 1 & 3 & 8 \\ 2 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 5 & 4 \\ 3 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} =$

Fig. 1(e). Visual explanation of the rule of matrix multiplication.

Example. Given the matrices $A_{2 \times 3} = \begin{pmatrix} 1 & 3 & 8 \\ 2 & 1 & 0 \end{pmatrix}; B_{3 \times 2} = \begin{pmatrix} 5 & 4 \\ 3 & 1 \\ 2 & 0 \end{pmatrix}$ Find the product of the two matrices (if it exists)

- $A_{2 \times 3} \cdot B_{3 \times 2} = C_{2 \times 2} \implies$ the product exists

- $\begin{pmatrix} 1 & 3 & 8 \\ 2 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 5 & 4 \\ 3 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} 1 \cdot 5 + 3 \cdot 3 + 8 \cdot 2 & \end{pmatrix} =$

$c_{ij} = \left\| \begin{matrix} \cdot i = 1 \\ \cdot j = 1 \end{matrix} \right\| = a_{11} b_{11} + a_{12} b_{21} + a_{13} b_{31}$

Fig. 1(f). Calculation of **c₁₁.**

Example. Given the matrices $A_{2 \times 3} = \begin{pmatrix} 1 & 3 & 8 \\ 2 & 1 & 0 \end{pmatrix}; B_{3 \times 2} = \begin{pmatrix} 5 & 4 \\ 3 & 1 \\ 2 & 0 \end{pmatrix}$ Find the product of the two matrices (if it exists)

- $A_{2 \times 3} \cdot B_{3 \times 2} = C_{2 \times 2} \implies$ the product exists

- $\begin{pmatrix} 1 & 3 & 8 \\ 2 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 5 & 4 \\ 3 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} 1 \cdot 5 + 3 \cdot 3 + 8 \cdot 2 & 1 \cdot 4 + 3 \cdot 1 + 8 \cdot 0 \\ 2 \cdot 5 + 1 \cdot 3 + 0 \cdot 2 & 2 \cdot 4 + 1 \cdot 1 + 0 \cdot 0 \end{pmatrix} =$

$c_{ij} = \left\| \begin{matrix} \cdot i = 1 \\ \cdot j = 1 \end{matrix} \right\| = a_{11} b_{11} + a_{12} b_{21} + a_{13} b_{31}$

$= \begin{pmatrix} 30 & 7 \\ 13 & 9 \end{pmatrix}$

Fig. 1(g). Calculation of all elements of C.

Visualization technique is also actively used to explain elements of vector algebra (to clarify the concepts of vector coordinates and an arbitrary vector expansion in terms of base unit vectors). It usually takes time to draw all figures on board in case of "classical blackboard" teaching. The concept of orthogonal projections of a vector is explained as follows (using PowerPoint). The coordinate system with three mutually orthogonal unit vectors is shown first (Fig. 2(a)).
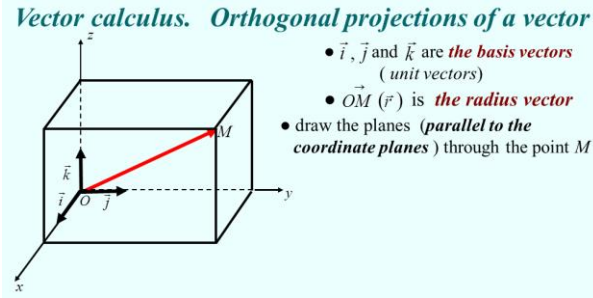
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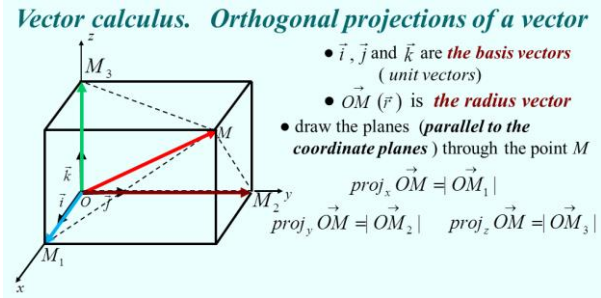Fig. 2(a). The initial configuration.

Fig. 2(b). The parallelepiped.

Radius vector is drawn next (Fig. 2(b)). The parallelepiped is constructed as follows. Three mutually perpendicular planes that are parallel to the coordinate planes are drawn through the point M (Fig. 2(c)). The projections of the given vector are shown in different colors (Fig. 2(d)).
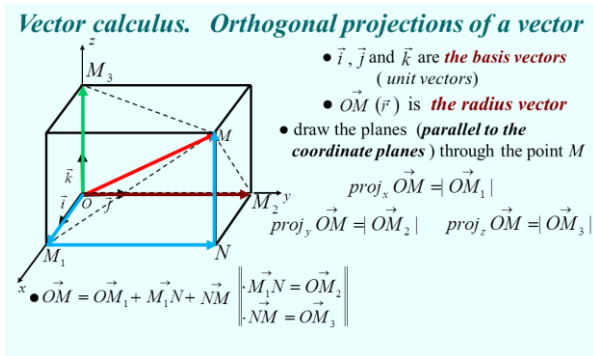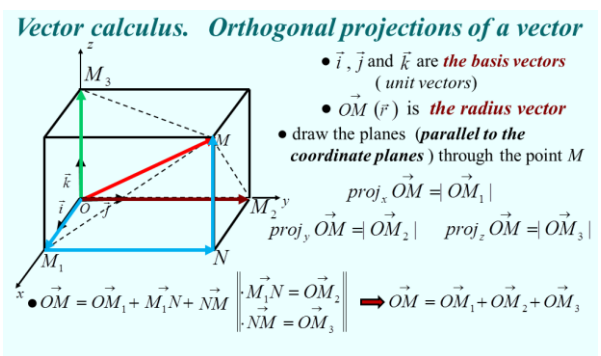

Fig. 2(c). The orthogonal projections.

Fig. 2(d). Radius vector expansion.

The rule of vector addition is illustrated next (Fig. 2(e)). The next figure shows how to represent the given vector in the form of the sum of three mutually perpendicular vectors (Fig. 2(f)).
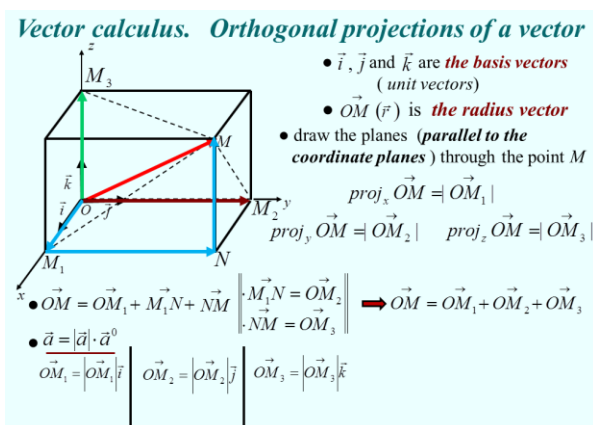

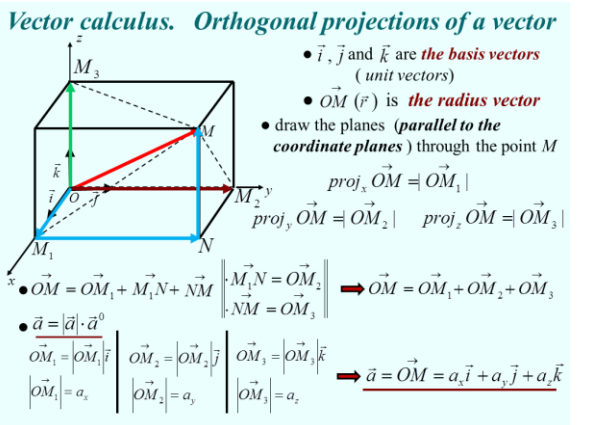Fig. 2(e). Definition of orthogonal components.

Fig. 2(f). The final step.

There are several advantages of using visualization techniques during lectures. First, students do not have to copy a drawing from the board after the instructor completed it (this certainly saves time for the instructor). Second, students can analyze the slides later at their own pace and go through the derivation as many times as needed. Third, the instructor controls the timing of appearance of different elements of a figure on the screen (as a result, comments and explanations are given to students while they are following the presentation without being disturbed by the necessity to copy the drawing from the board). Thus, more time is devoted to explanations and understanding the concept. This is very important since many students have rather poor background in math and they need more time to absorb all the aspects of the concept.

4 Results of students' survey

A survey is conducted among first-year students of the FME at RTU in the fall semester of 2016. The total number of participants was 62. The goal of the survey was the following: assess students' opinion on how mathematics is taught (for example, do lectures in the form of PowerPoint presentations are better than "classical" lectures with a blackboard?). The summary of the results is shown in Fig. 4 (five-point Likert scale ranging from "Completely agree" (label 1) to "Completely disagree (label 5) is used in the survey).
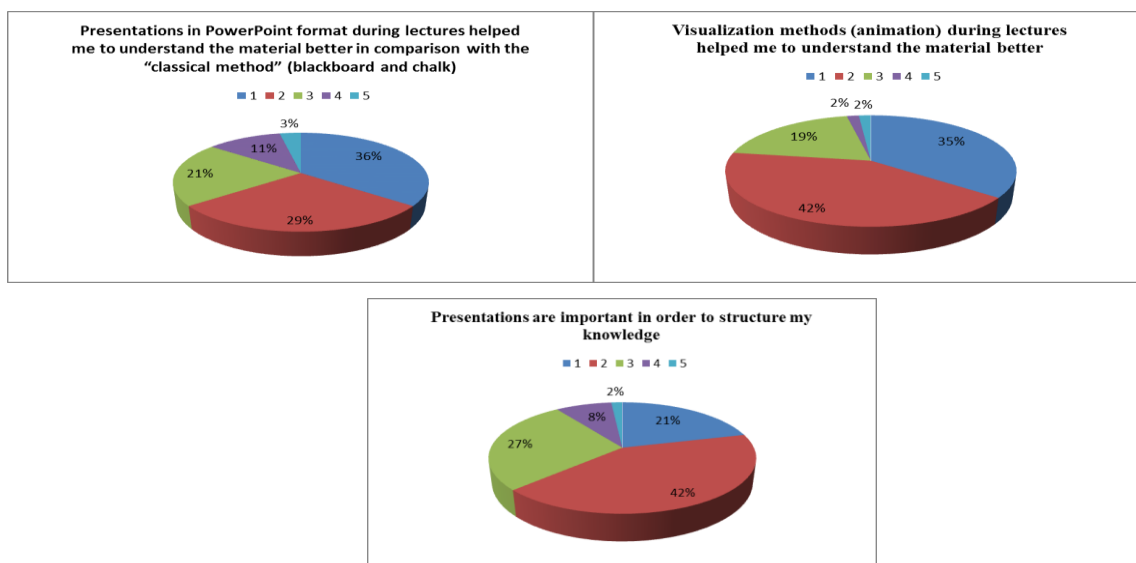

Fig.3. The results of the survey.

As can be seen from the results of the survey, students' responses to PowerPoint presentations and animations during lectures were quite positive.

5 Conclusions

1. A method based on animation features in PowerPoint is proposed in the paper to teach higher mathematics course to first-year students at the Faculty of Mechanical Engineering of RTU.
2. The use of dynamical features of PowerPoint during lectures on linear and vector algebra is positively evaluated by students.
3. The lecturer has a flexibility to control the time delay between appearances of different pieces of information on the slide. .
4. The author is currently developing the approach based on PowerPoint presentations to teach calculus of one variable and calculus of several variables.

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